Thermal processes in industrial furnaces

doc. Ing. Pavel Šonovský, CSc.
STUDY INSTRUCTIONS

Thermal processes in industrial furnaces

For the subject of the 10th semester of the study branch Metallurgical technology, you have received a study package containing integrated lecture notes for combined study, including also study instructions.

1. Prerequisites
For study of this subject, previous passing of subjects “Heat Transfer and Fluid Mechanics” and “Industrial furnaces” is required.

2. Aims of the subject and lectures
The aim of the subject is to get students acquainted with wider theoretical and practical knowledge in the fields of heating and cooling of metallic materials, with technologies of heating and heat treatment from the viewpoint of thermal processes and with thermal-technical conditions during heating in industrial furnaces, especially in pusher, walking-beam and carousel furnaces and in furnaces for heat treatment.

After study of this subject, students should be able to:

knowledge:
- Characterize technologies of heating in industrial furnaces and determine furnace type and conditions suitable for optimal heating of a particular material
- Calculate the basic parameters of heating of metals in industrial furnaces, for a given type of material and a consequent processing technology
- Formulate the basic thermal-technical phenomena occurring during heating of metals in industrial furnaces and determine their effects on quality and final mechanical properties of products

skills:
- Students will be able to apply their knowledge when deciding for the most suitable type of heating furnace, method and conditions of heating of individual metals in industrial furnaces
- Students will be able to apply their theoretical knowledge to design technologies for heating of metals during heat treatment in industrial furnaces and their consequent mechanical processing
- Based on the gained knowledge, students will be able to optimize a heating process from the viewpoints of heating technology and consequent technology of material processing

To whom is the subject addressed
The subject belongs to the master’s study of the Metallurgical Technologies study branch of the Thermal Engineering and Ceramic Materials study program, but it can also be studied by other students from other study branches, if they meet the required prerequisites.
The lecture notes are divided into sections – chapters – according to logical sectioning of the studied subject, but their contents are not equally long. The time suggested for study of each chapter can significantly differ. Therefore, large chapters are further sectioned in numbered sub-chapters with the below described structure.

**Study of each chapter is recommended to be done according to the following steps:**

- Get acquainted with content of the particular chapter
- Use recommended literature for a more detailed study
- Answer given questions, elaborate calculation of given task, if included at the end of a chapter
- Supplement already gained knowledge by answers and results of calculations

**Communication with lecturer:**

Communication of students with the lecturer will be performed in the following form:

- At presented lectures and practical/computational lessons, at which studied topics will be commented and given tasks will be consulted
- At consultation appointments
- Via electronic mail, by questions and answers
Lecture notes for study of the subject Thermal processes in industrial furnaces

1. Properties of metals and alloys in dependence on temperature

Time to study: 4 hours

Aim After study of this chapter students will be able to
- define basic thermal-technical parameters of metals and alloys and their influence on heating and cooling processes
- describe dependences of basic thermal-technical parameters of metals and alloys on temperature and determine the influences of their changes on heating and cooling processes
- determine values of basic thermal-technical parameters of metals and alloys for heating calculations

Lecture

To correctly determine technical possibilities and allowable heating rates, physical and mechanical properties of heated metals and their alloys in relation with temperature need to be known.

To the basic physical properties, which influence heat transfer in solid bodies, belong especially the following parameters:
- Thermal conductivity of heated material – \( \lambda \) (W.m\(^{-1}\).K\(^{-1}\))
- Specific heat capacity of heated material – \( c \) (J.kg\(^{-1}\).K\(^{-1}\))
- Density of heated material – \( \rho \) (kg.m\(^{-3}\))

To the basic mechanical properties, which influence the occurrence of mechanical stresses and characterize the transfer of metals and alloys to plastic state, belong:
- Linear temperature expansion coefficient – \( \beta \) (K\(^{-1}\))
- Elastic modulus – \( E \) (Pa)
- Tensile strength – \( \sigma \) (Pa)
- Relative elongation – \( \delta \) (1; %)
- Relative cross-section contraction – \( \phi \) (1; %)

1.1 Thermal conductivity of metals

Thermal conductivity \( \lambda \) (W.m\(^{-1}\).K\(^{-1}\)) characterizes the ability of a material to conduct heat. Thermal conductivity of solid materials depends on:
- Oscillative movement of molecules
- Distribution of atoms in crystal lattice
- Electrons free paths
Some practical knowledge of dependences of thermal conductivities of pure metals and their alloys on temperature, composition and heat (possibly mechanical) treatment:

- Pure metals have higher thermal conductivity than alloys, since they have longer electrons free paths and more regular crystal lattices.
- Increase in temperature results in shortening of electrons free paths – thermal conductivity of pure metals and carbon steels decreases.
- Thermal conductivity of carbon steels decreases with increasing contents of carbon, silicon, manganese, sulphur and phosphorus.
- Thermal conductivity of carbon steels decreases with increasing temperature, its minimum is in the temperature range from 800 to 1000°C, then it gradually increases.
- For alloyed steels and alloys of non-ferrous metals, thermal conductivity increases with increasing temperature. The cause is a more regular arrangement of atoms in crystal lattices and decrease in negative influences of alloying elements on the structure.

For alloyed steels, thermal conductivity is decreased the most by alloying elements chromium and nickel.

The values of thermal conductivities of carbon and alloyed steels are calculated according to relations, which were developed by generalization of a large number of experimental measurements. During their application is always necessary to consider the conditions, under which the experiments were performed and respect recommended ranges for their application, especially temperature range and chemical composition of the steel. The values of thermal conductivities can be found in tables, graphs and material lists of metals and alloys.

1.2 Specific heat capacity and thermal enthalpy of metals

The title describes the matter of this parameter exactly. Specific heat capacity \( c \) (J.kg\(^{-1}\).K\(^{-1}\)) serves to calculate enthalpy of materials.

\[
i_1 - i_2 = c_1t_1 - c_2t_2 \quad \text{(J.kg}\(^{-1}\))
\]

Where: \( c_1, c_2 \ldots \) mean specific heat capacity within the temperature range of \( 0 \div t_1 \), or \( t_2 \).

Generally, specific heat capacity depends on:

- Chemical composition of material. Changes in carbon content were shown to have only a negligible influence on steels.
- Structure of material. Changes in structure – in locations \( \text{Ac}_1, \text{Ac}_2, \text{Ac}_3 \) – are accompanied by a development of heat and thus also by significant changes in heat capacity.
- Temperature of material. Specific heat capacity increases with increasing temperature up to temperatures around 750°C, then it slightly decreases with increasing temperature. However, specific heat capacity increases constantly with increasing temperature for refractory metals.

Specific heat capacity of various compounds can be determined using tables, dependences and empirical relations, which were created by generalization of experimental measurements.

1.3 Material density

At first sight it seems that density has no influence on a heating process, but since the following applies:
\[ I = i \cdot m = i \cdot \rho \cdot V \quad \text{(J)} \quad \text{(2)} \]

From equation (2) ensues:

- For identical volumes, enthalpy increases with increasing density
- A compound with a higher density needs a higher heat supply for heating by identical \( \Delta t \)
- Heating time needs to be increased, especially in the phase of temperature equalization
- Density of steels depends on chemical composition, structure and temperature and the dependence of density of steels on temperature can be expressed as follows:

\[ \rho_t = \frac{\rho_0}{1 + 3 \beta t} \quad \text{(kg.m}^{-3}\text{)} \quad \text{(3)} \]

Where:
- \( \rho_t \)…density at given temperature
- \( \rho_0 \)… density at temperature \( t_0 = 0^\circ\text{C} \)
- \( \beta \)…linear thermal expansion coefficient

Density of compounds can be determined using tables, dependences and empirical relations.

### 1.4 Thermal diffusivity coefficient

Thermal diffusivity coefficient belongs to derived material parameters. Theoretical and experimental works have shown that rate of temperature changes is directly proportional to thermal conductivity \( \lambda \) and indirectly proportional to the product of specific heat capacity and density \( c \cdot \rho \ ). This has the following practical effects for metals:

- The higher is thermal diffusivity coefficient of a metal, the faster is changing of temperatures within the body and consequent equalization of temperatures.
- Specific heat capacity and density have opposite effects on changing of temperatures and their equalization within a material, i.e. the higher are specific heat capacity and density, the slower is equalization of temperatures within the material.

The parameter describing the rate of temperature changes is denoted as **thermal diffusivity coefficient** and is defined as:

\[ a = \frac{\lambda}{c \cdot \rho} \quad \text{(m}^2\text{.s}^{-1}\text{)} \quad \text{(4)} \]

The values of thermal diffusivity for steels are within the following ranges:

- For carbon steels \( a = (1,2 \div 1,7).10^{-5} \text{ m}^2\text{.s}^{-1} \)
- For alloyed steels \( a > 1,7.10^{-5} \text{ m}^2\text{.s}^{-1} \)

In accordance with knowledge about thermal conductivity, specific heat capacity and density, thermal diffusivity coefficient depends on temperature, structure and chemical compositions of metals and alloys.

### 1.5 Thermal inertia coefficient

**Thermal inertia coefficient** defined by the following equation can sometimes advantageously be used instead of thermal diffusivity coefficient:
\[
f = \sqrt{\lambda c \rho} \quad \text{(J.m}^{-2}.\text{s}^{-0.5}.\text{K}^{-1}) \quad (5)
\]

Advantages of application of thermal inertia coefficient:
- The dependence of thermal inertia coefficient on temperature is negligible – max. 4\(^\circ\)10\%
- This phenomena is not random, but regular, since decreasing (\(\lambda\) and \(\rho\)) is accompanied with increasing (\(c\)) for metals

### 1.6 Thermal expansion of metals

If temperature of a metal rod changes, its length changes as well. The change in length of a metal rod is dependent on its thermal expansion coefficient. Its definition ensues from the following consideration:
- For temperature \(t_1\), the length of the rod is \(l_1\).
- For temperature \(t_2\), the length of the rod is \(l_2\).

Linear thermal expansion coefficient is then given by the relation:

\[
\beta^{t_1}_{t_2} = \frac{l_2 - l_1}{l_1(t_2 - t_1)} \quad \text{(mm.m}^{-1}.\text{K}^{-1}) \quad (6)
\]

- For temperature \(t_1\), the volume of a body is \(V_1\)
- For temperature \(t_2\), the volume of a body is \(V_2\)

Volumetric thermal expansion coefficient is then given by the relation:

\[
A^{t_1}_{t_2} = \frac{V_2 - V_1}{V_1(t_2 - t_1)} \quad \text{(K}^{-1}) \quad (7)
\]

For linear and volumetric thermal expansion coefficients approximately applies:

\[
A^{t_1}_{t_2} \approx 3 \beta^{t_1}_{t_2} \quad \text{(K}^{-1}) \quad (8)
\]

The following facts were experimentally found for carbon steels:
- The value of \(\beta\) changes negligibly with a change in carbon content
- The value of \(\beta\) increases with increasing temperature up to structure changes
- The value of \(\beta\) decreases when structure changes occur

### Summary of terms

To the basic physical properties, which influence heat transfer in solid bodies, belong especially the following parameters:
- Thermal conductivity of heated material – \(\lambda\) (W.m\(^{-1}\).K\(^{-1}\))
- Specific heat capacity of heated material – \(c\) (J.kg\(^{-1}\).K\(^{-1}\))
- Density of heated material – \(\rho\) (kg.m\(^{-3}\))

To the derived physical properties, which influence heat transfer in solid bodies, belong especially the following parameters:
• Thermal diffusivity coefficient – \( a \) (m\(^2\).s\(^{-1}\))
• Thermal inertia coefficient – \( f \) (J.m\(^2\).s\(^{0.5}\).K\(^{-1}\))

### Questions to the discussed subject

- Draw the dependence of thermal conductivity on temperature for pure iron, carbon steel and alloyed steel and explain reasons for their differences.
- Define the influence of density of metals on their heating and its practical effects

### Tasks

- Calculate the overall elongation of rails between Ostrava and Prague, if the rails are made of carbon steel and the distance between both the cities is 300 km.

2. Heating and cooling of metals

**Time to study:** 5 hours

**Aim** After study of this chapter students will be able to

- define basic terms for metals heating and cooling processes
- determine basic components of heat transfer to heated material by radiation and convection within a furnace
- determine conditions for internal and external heat and determine, whether a heated body is thin or thick from thermal-technical viewpoint

**Lecture**

Heating and cooling of metals belong to nonstationary phenomena, during which temperatures of materials and heat fluxes on materials are variable in time.

**Basic facts**

- Cooling and heating are both driven by the same laws, the heat flux density \( q \) and enthalpy only have opposite signs
- During calculations of heating, we calculate body heating time \( \tau \)
  - If we know temperature, to which we have to heat the body
- During calculations of heating, we calculate temperatures at surface and center of a body
  - If we know heating time \( \tau \), within which we have to heat the body
- For comparison of heating of bodies with thicknesses \( b \) from various materials, **specific heating time** was defined:
Temperature gradient within a body $\Delta t$ is defined as the difference between temperatures in the hottest and coolest locations within the body.

- Temperature gradient depends on thermal resistance of the metal, i.e. it depends on physical properties of the metal and heating conditions.
- It is usually depicted using surface temperature $t_{\text{sur}}$ and center temperature $t_{\text{cen}}$, then
  \[ \Delta t = t_{\text{sur}} - t_{\text{cen}} \]  \((^\circ \text{C})\)  \((10)\)

### 2.2 Heat transfer on material in heating furnace

Within a furnace, heat is transferred to a material by radiation and convection, while heat transfer within a material is executed by conduction.

#### 2.2.1 Radiation of exhaust gas within furnace

From heat transfer theory ensues that three atomic gases can radiate within a furnace. They are CO$_2$, H$_2$O, possibly SO$_2$. Heat transfer to the heated material by exhaust gas radiation is described by the following equation:

\[
Q_{PM} = \varepsilon_{SP} C_0 \left[ \left( \frac{T_{SP}}{100} \right)^4 - \left( \frac{T_M}{100} \right)^4 \right] S_M \quad \text{(W)} \quad (11)
\]

Where:
- $\varepsilon_{SP}$ … specific exhaust gas absorption capacity
- $C_0$ … coefficient of ideal black body radiation \((\text{W.m}^{-2}.\text{K}^{-1})\)
- $T_{SP}, T_M$ … thermodynamic temperature of exhaust gas, body surface \((\text{K})\)
- $S_M$ … surface area contributing to heat transfer \((\text{m}^2)\)

For calculations is necessary to determine **specific exhaust gas absorption capacity** $\varepsilon_{SP}$, which is a function of partial pressures of CO$_2$, H$_2$O and SO$_2$, effective lengths of radiating rays and exhaust gas temperature. The Hottel – Egbert method is mostly used for practical calculations of $\varepsilon_{SP}$.

\[
\varepsilon_{SP} = \varepsilon_{CO_2} + \beta \varepsilon_{H_2O} + \varepsilon_{SO_2} - \Delta \varepsilon \quad (1) \quad (12)
\]

Where:
- $\Delta \varepsilon$ … correction for mutual absorption of radiations of gaseous components in certain regions of spectrum (can be neglected in practice)
- $\beta$ … correction factor representing a larger influence of partial pressure of H$_2$O than of effective ray length
Graphs specifying dependences of relative absorption capacities on gas temperatures and partial pressures and effective radiating ray lengths products are used to determine values of relative absorption capacities \( \varepsilon_{\text{CO}}, \varepsilon_{\text{H}_2\text{O}}, \varepsilon \).

- Combustion equation of a given fuel with a given excess air serves to determine temperature of exhaust gas and partial pressures of three atomic gases.
- The following equation serves to determine mean trajectory of a ray in a layer between radiant surfaces:

\[
1_{er} = 3.6 \cdot \frac{V_{SP}}{S_p} \quad \text{(m)}
\]

Where:
\( V_{SP} \) … volume of radiant exhaust gas layer \( \text{(m}^3 \text{)} \)
\( S_p \) … surface area of this exhaust gas volume \( \text{(m}^2 \text{)} \)

- If the flame is luminous, the total specific exhaust gas absorption capacity increases by \( \varepsilon_{\text{SAZ}} \). Then, specific flame absorption capacity can be calculated according to the modified equation:

\[
\varepsilon_{\text{PLAM}} = \varepsilon_{\text{SAZ}} + \varepsilon_{\text{CO}_2} + \beta \varepsilon_{\text{H}_2\text{O}} + \varepsilon_{\text{SO}_2}
\]

Where:
\( \varepsilon_{\text{SAZ}} \) … specific soot carbon absorption capacity \( \text{(1)} \)
in flame, can be determined only experimentally

2.2.2 Temperatures of furnace and furnace brickwork

Among exhaust gas, the material is influenced also by radiation from furnace brickwork and other material being heated. Conventional practical measurements using thermocouples are not able to exactly determine the exhaust gas temperature. However, we measure the average temperature between:

- Exhaust gas – \( t_{SP} \)
- Furnace brickwork – \( t_Z \)
- Surface of heated material – \( t_{PM} \)

This temperature is denoted as furnace temperature – \( t_{pee} \). Although it is an imaginary temperature, it is very useful for the purpose of controlling the heating. The correct temperature could be measured by a thermocouple, which would be located within a furnace under the following conditions:

- Hot thermocouple junction located close to heated material
- Hot thermocouple junction insulated from radiation of heated material

Then a thermocouple is affected only by exhaust gas and furnace brickwork radiations and for the effecting heat flux applies:
\[
\varepsilon_{ pec} C_0 \left( \frac{T_{ pec}}{100} \right)^4 = \varepsilon_{ sp} C_0 \left( \frac{T_{ sp}}{100} \right)^4 + \varepsilon_z C_0 \left( 1 - \varepsilon_{ sp} \right) \left( \frac{T_z}{100} \right)^4 \quad \text{(W.m}^{-2} \text{)} \quad (15)
\]

Where:

- \( T_{ pec} \) … furnace temperature measured under the above mentioned conditions (K)
- \( T_{ sp} \) … exhaust gas temperature measured by suction-type pyrometer (K)
- \( T_z \) … furnace brickwork temperature (K)

After adjustment, we get:

\[
\left( \frac{T_{ pec}}{100} \right)^4 = \frac{\varepsilon_{ sp}}{\varepsilon_{ pec}} \left( \frac{T_{ sp}}{100} \right)^4 + \frac{\varepsilon_z}{\varepsilon_{ pec}} \left( 1 - \varepsilon_{ sp} \right) \left( \frac{T_z}{100} \right)^4 \quad \text{(K}^4 \text{)} \quad (16)
\]

Specific furnace absorption capacity – \( \varepsilon_{ pec} \) – can be approximately determined from the equation:

\[
\varepsilon_{ pec} \approx \frac{\varepsilon_m (1 - \varphi_{mm})}{1 - \varphi_{mm} (1 - \varepsilon_m)} \quad \text{(17)}
\]

Where:

- \( \varphi_{mm} \) … coefficient of radiation of heated material on adjoining material (1)
  
  For \( \varphi_{mm} = 0 \) applies \( \varepsilon_{ pec} \approx \varepsilon_{ mat} \)

- \( \varepsilon_z \) … specific furnace brickwork absorption capacity, (1)
  
  the value is given by the brickwork producer in the form \( \varepsilon_z = f(t_z) \)

- \( \varepsilon_{ sp} \) … specific exhaust gas absorption capacity – see chap. 2.2.1

After substitution we can calculate furnace temperature – \( T_{ pec} \)

**What does furnace temperature serve for?** Heating of material in furnaces is in practice operated according to this temperature. This temperature is measured according to the above mentioned conditions in individual zones within a furnace and the power supply into furnace zones is controlled so that the required temperature of a given zone is kept. It is a simple and reliable system of controlling of heating, although it has one difficulty – measuring of furnace temperature \( T_{ pec} \) is inaccurate. Identical measuring conditions need to always be ensured. Identical positioning of thermocouples in individual furnace zones is especially necessary.

For cyclically operating chamber furnaces, the following empiric equations were experimentally determined to simply calculate furnace temperatures. For these furnaces apply:

- Furnace temperature, exhaust gas temperature and material temperature vary along the length of the chamber in dependence on time – heating time.

Furnace temperature can then be calculated e.g. as the average exhaust gas temperature using the equation:

\[
T_{ pec} = \sqrt{0.88 T_a \cdot T_{ odp}} \quad \text{(K)} \quad (18)
\]

Where:
Adiabatic fuel combustion temperature can be calculated using the relation:

\[ T_a = \frac{Q_n}{V_{sp,\text{min}} c_{p,sp}} \] (°C)  \hspace{1cm} (19)

Where:

- \( Q_n \) … fuel heating power (MJ.m\(^{-3}\))
- \( V_{sp,\text{min}} \) … exhaust gas volume for excess of combustion air \( \lambda = 1 \) (m\(^3\).m\(^{-3}\))
- \( c_{p,sp} \) … specific exhaust gas heat capacity at constant pressure (MJ.m\(^{-3}\).K\(^{-1}\))

If we know furnace temperature \( T_{pec} \), we can calculate heat flux \( Q \) to the surface of heated material from the following equation:

\[
Q = \varepsilon_{pec} C_0 \left[ \left( \frac{T_{pec}}{100} \right)^4 - \left( \frac{T_m}{100} \right)^4 \right] S_m \] (W) \hspace{1cm} (20)

This heat flux can also be expressed using furnace brickwork temperature - \( T_z \)

\[
Q = \varepsilon_z C_0 \left[ \left( \frac{T_z}{100} \right)^4 - \left( \frac{T_m}{100} \right)^4 \right] S_m \] (W) \hspace{1cm} (21)

Where:

- \( \varepsilon_z \) … radiation coefficient related to furnace brickwork temperature (W.m\(^{-2}\).K\(^{-4}\))

For a general case of \( \varphi_{zm} \neq 0 \), furnace brickwork temperature can be determined using the following equation:

\[
\left( \frac{T_z}{100} \right)^4 = \left( \frac{T_m}{100} \right)^4 + \left( \frac{T_{sp}}{100} \right)^4 - \left( \frac{T_{pec}}{100} \right)^4 \frac{\varepsilon_{sp}}{\varepsilon_{zm} \varepsilon_m (1 - \varepsilon_{sp})} \] (K\(^4\)) \hspace{1cm} (22)

Where:

- \( \varphi_{zm} \) … coefficient of brickwork radiation to heated material (1)

Brickwork temperature is important for designing and possibly checking of suitability of individual ceramic materials for heating furnaces brickwork working layers.

### 2.3 Regions of internal and external heat transfers in heating furnaces

The difficulty of heating in flame furnaces lies especially in the fact that the process of heat transfer proceeds between three, possibly even four, bodies and substances:

- Gas layer (exhaust gas)
- Furnace brickwork
- Heated material
- Flame of combusting fuel
Between these bodies and substances proceeds a complicated exchange of energies by radiation, convection and conduction within the heated material. Among the basic thermal processes, the following phenomena occur as well:

- Variable exhaust gas aerodynamics
- Physical-chemical phenomena
  - Fuel combustion within furnace
  - Scaling of heated material in furnace atmosphere

Among the above mentioned processes and phenomena, calculations of heating are also complicated by technological requirements and regulations, which complicate mathematic formulation of the whole process. Therefore, we try to find various simplifications for engineering calculations. These consist especially in categorization to:

- Region of external heat transfer – heat transfer by exhaust gas and furnace brickwork radiation and by exhaust gas convection to heated material
- Region of internal heat transfer – described by Fourier’s law for heat transfer, which describes changing of temperature field in dependence on time – \( t = f(\tau, x, y, z) \)

The value of ratio of both – the external and internal heat transfer – characterizes bodies from the thermal-technical viewpoint and enables their characterization as thin and thick. The criterion for this characterization is the Biot number. This is defined as the ratio of external and internal heat transfer and is described by the equation:

\[
Bi = \frac{\alpha_{\Sigma} \cdot b}{\lambda_m} = \frac{\lambda_m}{\alpha_{\Sigma} \cdot 1} \quad (1)
\]

Where:
- \( \alpha_{\Sigma} \ldots \) heat transfer coefficient of radiation and convection, in total \( (W.m^2.K^{-1}) \)
- \( \lambda_m \ldots \) thermal conductivity of heated material \( (W.m^{-1}.K^{-1}) \)
- \( b \ldots \) characteristic dimension of heated material \( (m) \)

Characterization to thin and thick bodies is then determined according to the following values of Biot number:

- \( Bi \leq 0.25 \) – thin body
- \( Bi \geq 0.50 \) – thick body
- \( 0.25 < Bi < 0.50 \) – transition between thin and thick body
  - For exact calculations, a body is considered as thick
  - For approximate calculations, a body is considered as thin

Characterization as thin and thick bodies can be performed also according to Stark radiation law. This criterion can advantageously be used for heating, for which heat transfer by radiation prevails. Stark law is defined as:
Sk = \frac{c_{sp}}{100} \left( \frac{T_{sp}}{100} \right)^3 \cdot \frac{b}{\lambda} \tag{1} \tag{24}

Where:

- $c_{sp}$ … specific exhaust gas heat capacity
- $T_{sp}$ … exhaust gas temperature

Since the relation $\frac{c_{sp}}{100} \left( \frac{T_{sp}}{100} \right)^3$ is analogic to the total heat transfer coefficient $\alpha$, Stark law is also analogic to Biot number. Characterization to thin and thick bodies according to Stark law is determined according to the following values.

- Sk $\leq 0.1 \div 0.15$ – thin body
- Sk $\geq 0.15$ – thick body

From the thermal-technical viewpoint, thin and thick bodies are characterized with the following properties and parameters:

**Thin bodies**

- Heating is equal along the whole cross-section
- Temperature gradient can be neglected
  - Temperature differences along the cross-section do not have to be respected in calculations
  - In any moment of heating and cooling, we consider temperatures in all the locations along the cross-section to be equal to surface temperature
- For heating and cooling, **external heat transfer** is crucial

**Thick bodies**

- Heating and cooling are accompanied with temperature gradients between body surface and center
- **Internal and external heat transfers** need to be considered in calculations
- Temperature gradient during heating and cooling depends on:
  - Body thickness
  - Thermal conductivity
  - Heating (cooling) rate

**Summary of terms**

Heating and cooling of metals belong to nonstationary phenomena, during which temperatures of materials and heat fluxes on materials are variable in time.

- Cooling and heating are both driven by the same laws, heat flux density $q$ and enthalpy only have opposite signs.
During calculations of heating, we calculate body heating time $\tau$
  - If we know temperature, to which we have to heat the body
During calculations of heating, we calculate temperatures at surface and center of a body
  - If we know heating time $\tau$, within which we have to heat the body

The following parameters are important during heating:
- Exhaust gas temperature, furnace brickwork temperature, furnace temperature
- Total heat transfer coefficient of radiation and convection

For calculations of heating and cooling, we characterize bodies as thin and thick according to Biot number and Stark law.

### Questions to the discussed subject

- How do we determine specific exhaust gas absorption capacity for calculation of external heat transfer in a flame furnace, what does this parameter depend on?
- Define “furnace temperature”, conditions under which it can be measured and practical application of this parameter?
- Define criterions which serve to characterization of heated bodies to thin and thick and how are they characterized from the viewpoint of heating calculations, define the difference between thick and thin bodies.

### 3. External heat transfer to batch in heating furnace

#### Time to study: 5 hours

#### Aim
After study of this chapter students will be able to
- determine balance of heat fluxes on surface of heated body and furnace brickwork within furnace – by radiation and convection
- determine external heat fluxes by radiation and convection using a graphic-analytic method
- calculate external heat fluxes by radiation and convection using approximate engineering calculations
- characterize heating and cooling in liquid environments and calculate basic parameters of heating and cooling in liquid environments

#### Lecture
External heat transfer to batch is, from the viewpoint of mathematic formulation, a very difficult phenomenon, which characterizes exchange of thermal energy between:
- Exhaust gas – final product of fuel combustion
They have the highest temperature during heating and from the second law of thermodynamics ensues that they transfer their thermal energy to material and furnace brickwork.

Heat transfer is executed by radiation and convection.

- Furnace brickwork
- Heated material
- Combusting fuel flame – usually not considered during calculations

The ratio of convective and radiant heat transfers to material and furnace brickwork is dependent on exhaust gas temperature and velocity. With a certain simplification, the contribution of convection to the overall external heat transfer in industrial furnaces can be determined as follows:

- 50 ÷ 60% - ovens
- 10 ÷ 30% - heating furnaces for heat treatment and for heating in rolling mills
- 5 ÷ 10% - melting furnaces

3.1 Balance of heat fluxes to surface of heated materials and furnace brickwork

The aim of solving of balance thermal equations is to determine the final heat flux to material – effective heat, which can be described by balance equation (25):

$$\sum Q_{už} = \sum Q_{dop} - \sum Q_{odr} + \sum Q_{kony} - \sum Q_{sál} \quad (W) \quad (25)$$

3.1.1 Balance of heat fluxes to surface of heated materials

Schematic depiction of heat fluxes within a heating furnace is shown in Fig. 1.

![Fig. 1 - Schematic depiction of heat fluxes in industrial furnace heating chamber](image)

Gradually we determine the balance of heat fluxes to:

- Surface of heated material by radiation
- Surface of furnace brickwork by radiation
- Surface of heated material by convection heat transfer
- Surface of furnace brickwork by exhaust gas convection

From determined components of the balance, we compose the final equation for heat transfer to the heated material.

Factors getting to or from a heated material surface:

- Radiant exhaust gas heat flux – $Q_{sp}^m$
- Radiant brickwork heat flux – $Q_z^m$
- Radiant heat flux of adjoining heated material – $Q_m^m$
- Convective heat flux – $Q_k^m$
- Radiated heat flux – $Q_{sp}^e$

Balance energetic equation for surface of material heated in a flame furnace has the form:

$$Q_z = Q_{sp}^m + Q_z^m + Q_m^m + Q_k^m - Q_{sp}^e$$

(W) (26)

3.1.2 Balance of heat fluxes on furnace brickwork surface

On surface of furnace brickwork gets:

- Radiant exhaust gas heat flux to brickwork – $Q_{sp}^z$
- Convective exhaust gas heat flux to brickwork – $Q_k^z$
- Radiant material heat flux to brickwork – $Q_m^z$

From brickwork gets:

- Effective brickwork radiation to furnace area – $Q_{sp}^e$
- Waste heat flux from brickwork conduction (resultant heat flux) – $Q_r^z$

Resultant brickwork heat flux outside the furnace is expressed as:

$$Q_z^r = Q_{sp}^z + Q_k^z - Q_{sp}^e$$

(W) (27)

Effective brickwork heat flux is expressed as:

$$Q_{sp}^e = Q_{sp}^z + Q_k^z - Q_r^z$$

(W) (28)

3.1.3 Convective heat transfer to heated material

Heat transferred to material during heating by convection contributes to the total transferred heat as follows:

- In melting furnaces - 5 ÷ 10 %
- In high-temperature heating in rolling and forging plants - 10 ÷ 30%
- In ovens – 60% and more
A correction factor, by which the calculated heat transferred by radiation is multiplied, is used to express convection in approximate calculations. The correction factor values can be:

\[ k_{kon} = 1.1 \div 1.4 \]

Newton’s relation is used for more precise calculations of heat transfer by convection:

\[ Q_k^m = \alpha_k^m S_m = \alpha_k (t_{sp} - t_m) S_m \quad \text{(W)} \quad (29) \]

Where:

- \( \alpha_k \) … convective heat transfer coefficient \((\text{W.m}^{-2}.\text{K}^{-1})\)
- \( t_{sp} \) … exhaust gas temperature \(\text{(°C)}\)
- \( t_m \) … material temperature \(\text{(°C)}\)
- \( S_m \) … heated material surface area \((\text{m}^2)\)

Accuracy of calculations according to the Newton’s equation is dependent on accuracy of determination of convective heat transfer coefficient \( \alpha_k \). Determination of \( \alpha_k \) is very complicated since its value is dependent on a variety of parameters. We can write:

\[ \alpha_k = f(t_{sp}, v_{sp}, P_{sp}, ...) \]

Generally, the value of heat flux coefficient varies within the range of:

\[ \alpha_k = 3.5 \div 14000 \text{ W.m}^{-2}\text{.K}^{-1}, \]

In flame furnaces it can be:

\[ \alpha_{k,\text{max}} = 60 \text{ W.m}^{-2}\text{.K}^{-1}, \]

although according to some authors it can reach 175 W.m\(^{-2}\). K\(^{-1}\) and for rapid heating even 240 W. m\(^{-2}\). K\(^{-1}\). These are all very different values.

Convective heat transfer coefficient can be determined from criterion equations derived using similarity theory. For melting and heating furnaces applies:

\[ \text{Nu} = f(\text{Re}, \text{Pr}) \quad (1) \quad (30) \]

Where:

- \( \text{Nu} \) … Nusselt number \((1)\)
- \( \text{Re} \) … Reynolds number \((1)\)
- \( \text{Pr} \) … Prandtl number \((1)\)

According to Michejev, for calculations of Nusselt number applies:

\[ \text{Nu} = 0.032 \text{.Re}^{0.8} \quad (1) \]

Heat transfer coefficient \( \alpha_k \) is then after adjustments calculated using the following relations for Nu, Pr, and Re numbers:

\[ \text{Nu} = \frac{\alpha_k l}{\lambda} \quad (1) \quad (31) \]

\[ \text{Pr} = \frac{v}{a} = \frac{\text{Pe}}{\text{Re}} \quad (1) \quad (32) \]
Re = \frac{w.l}{v} \quad (1) \quad (33)

Where:
- \( \alpha_k \) … convective heat transfer coefficient \( (W.m^{-2}.K^{-1}) \)
- \( \lambda \) … thermal conductivity of medium \( (W.m^{-1}.K^{-1}) \)
- \( l \) … characteristic dimension \( (m) \)
- \( v \) … kinematic viscosity of medium \( (m^2.s^{-1}) \)
- \( w \) … medium convection rate \( (m.s^{-1}) \)
- \( a \) … thermal diffusivity \( (m^2.s^{-1}) \)

### 3.2 Final equation of external heat transfer

The aim is to determine final heat transfer to heated material as a dependence of:
- Exhaust gas temperature
- Material temperature

The basic balance equations (26) and (27) can be used for solving:

\[
Q_{\Sigma} = Q_{sp}^m + Q_{z}^m + Q_{m}^m + Q_{k}^m - Q_{m}^{ef} \\
Q_{z}^{ef} = Q_{sp}^r + Q_{k}^r - Q_{z}^{ef}
\]

Two unknown variables are here introduced - \( Q_{m}^{ef}, Q_{z}^{ef} \)

Analysis of the equation shows that we have three equations for three unknown variables, which signifies that the equation is solvable.

- Using simple mathematic adjustments we:
  - exclude \( Q_{m}^{ef} \)
  - determine \( Q_{z}^{ef} \)
  - introduce substitution members - \( \chi, \beta_{sp}, \beta_{m} \)

And get the equation

\[
Q_{\Sigma} = \chi \cdot 5.67 \left[ \beta_{sp} \left( \frac{T_{sp}}{100} \right)^4 - \beta_{m} \left( \frac{T_{m}}{100} \right)^4 \right] S_m + Q_{k}^{m} + \chi \left( Q_{k}^{r} - Q_{z}^{r} \right) \quad (W) \quad (34)
\]

in which for the substitution members applies:

\[
\chi = \frac{\varepsilon_m + (1-\varepsilon_{sp})(1-\varepsilon_m)\varphi_{mn}}{1-(1-\varepsilon_m)(1-\varepsilon_{sp})(1-\varphi_{mn}) + \frac{\varepsilon_{sp}}{\alpha(1-\varphi_{mn})(1-\varepsilon_{sp})}} \quad (1) \quad (35)
\]

\[
\beta_{sp} = \frac{\varepsilon_{sp}}{\chi} \left[ \frac{\chi}{\omega} + \varepsilon_m + (1-\varepsilon_{sp})(1-\varepsilon_m)(\varphi_{mn} + \chi - \varphi_{mn} \chi) \right] \quad (1) \quad (36)
\]
\[
\beta_m = \frac{e_m}{\chi} \left[ 1 - \chi(1 - \varphi_{nm})(1 - e_{sp}) - \varphi_{nm}(1 - e_{sp}) \right]
\]  

(1) 

In equation (34), the individual members represent:

Member 1 – component representing heat exchange between exhaust gas and material by radiation

Member 2 – heat transferred to material by convection

Member 3 – heat transferred to furnace brickwork \( Q^z_k \)

– heat loses by furnace brickwork convection \( Q^z_t \)

We consider simplification:

\( Q^z_k = Q^z_t \)

From the accepted simplifying consideration ensues that member 3 of equation (34) is equal to zero. This is possible for approximate calculations, the error is not substantial.

Substitution member \( \chi \) is then equal to: \( \chi \geq 0.3 \)

### 3.3 Methods of solving the external heat transfer equation

Practical solution of the external heat transfer equation (34) can be performed:

- Analytically – only very easy cases
- Graphically-analytically – common engineering calculations
- Approximate calculations – approximate calculation using empiric equations

#### 3.3.1 Graphical-analytical solution of external heat transfer

If we divide equation (34) by material surface area \( S_m \), after adjustments we get heat flux to unit of heated material surface area. Equation for calculation of heat flux has then the form:

\[
q_\Sigma = 5.67 \chi \beta_m \left[ \frac{\beta_{sp}}{\beta_m} \left( \frac{T_{sp}}{100} \right)^4 - \left( \frac{T_m}{100} \right)^4 \right] \alpha_k (t_{sp} - t_m) 
\]  

(W.m\(^{-2}\))  

(38)

After substitution

\[
C_\Sigma = 5.67 \chi \beta_m 
\]  

(W.m\(^{-2}\).K\(^{-4}\))

We get equation for a partial solution by graphical method.

\[
q_\Sigma = C_\Sigma \left[ \frac{\beta_{sp}}{\beta_m} \left( \frac{T_{sp}}{100} \right)^4 - \left( \frac{T_m}{100} \right)^4 \right] + \alpha_k (t_{sp} - t_m) = q_{sal} + q_k 
\]  

(W.m\(^{-2}\))  

(39)

### Calculation of radiant component

Considering necessary accuracy of calculation of radiant component and wide temperature ranges of exhaust gas and heated material, the influence of which on heat transfer by radiation is given by a fourth power function, it is suitable to:
• Solve the radiant component by a combination of graphical and analytical method, i.e. not only by graphical nomogram

Graphical-analytical method lies in:

Determination of $C_\Sigma$ and $\frac{\beta_{se}}{\beta_m}$ from intersection diagrams, the example of which is for specific absorption capacity

• Analytical calculation of radiant heat flux $q_{\text{salt}}$

The fact that specific absorption capacity of heated materials $\varepsilon_m$ changes significantly with chemical composition and material needs to be pointed out. It depends on surface, especially on its roughness and temperature. For example, the following approximate values can be mentioned for common carbon steels:

• Specific absorption capacity changes significantly within temperatures of $300 \pm 440^\circ\text{C}$
  - $t_m < 300^\circ\text{C}$ – max. value reaches $\varepsilon_m \approx 0,35$
  - $t_m < 440^\circ\text{C}$ – max. value reaches $\varepsilon_m \approx 0,95$
  - $600 < t_m <1200^\circ\text{C}$ – values within $\varepsilon_m \approx 0,85 \div 0,95$

• Dependences for $C_\Sigma$ and $\frac{\beta_{sp}}{\beta_m}$ have been elaborated for the most common materials and most common $\varepsilon_m$ values. For example in [].

• To exclude $\omega = \frac{S_m}{S_z}$, $\varphi_{mm} = \frac{S_m}{S_m + S_z}$ was introduced.
  - The relation is somehow illogical
  - Does not correspond to theory
  - Enables easy calculation of $\omega$

$$\omega = \frac{S_m}{S_z} = \frac{\varphi_{mm}}{1 - \varphi_{mm}} \quad (1) \quad (40)$$

• Equation (40) cannot be used for limit case of $\varphi_{mm} = 0$

• Practice has shown sufficient accuracy of calculations according to the above mentioned relation

• To determine $C_\Sigma$ and $\frac{\beta_{sp}}{\beta_m}$, we substitute into equation (39) and calculate $q_{\text{salt}}$

**Calculation of convective component**

Convective component is calculated from the known Newton’s relation

$$q_k^m = \alpha_k \cdot (t_{sp} - t_m) \quad (W) \quad (41)$$

and the accuracy of calculation depends on accuracy of determination of convective heat transfer coefficient $\alpha_k$
Convective heat transfer coefficient $\alpha_k$ can be determined from criterion equations:

- When using the relation

$$Nu = 0.032 \cdot Re^{0.8}$$

We get after substitution:

$$\frac{\alpha_k \cdot d_h}{\lambda} = 0.032 \left( \frac{w \cdot d_h}{v} \right)^{0.8}$$

After adjustment, we express

$$\alpha_k = R^* \cdot \frac{w_{sp}^{0.8}}{d_h^{0.2}}$$

Where:

$$R^* = 0.032 \cdot \frac{\lambda_{sp}^{0.8}}{v_{sp}}$$

Parameter $R^*$

- characterizes thermal-technical properties of flowing exhaust gas in dependence on temperature

- the main difficulties during its determination
  - determination of exhaust gas velocity
  - determination of free cross-section for exhaust gas flow
- $\alpha_k$ can be determined using dependences

For more complicated cases, modelling is used to determine
- analogy of transfer of mass, energy and momentum

### 3.3.2 Approximate calculation of external heat transfer

Relatively difficult equation (34) for determination of $Q_\Sigma$ is substituted by an adjusted equation for convective heat transfer in the form:

$$Q_\Sigma = \alpha_{s+k} \cdot \left(t_{sp} - t_m\right) S_m$$

(W) \hspace{1cm} (45)

The issue of solving of this equation is thus limited to determination of composed heat transfer coefficient $\alpha_{s+k}$

$$\alpha_{s+k} = C_{\Sigma} \cdot \left[ \left( \frac{T_{sp}}{100} \right)^4 - \left( \frac{T_m}{100} \right)^4 \right]_{sf} + \alpha_k$$

(W.m$^{-2}$.K$^{-1}$) \hspace{1cm} (46)

Graphical dependences or empiric equations can be used for quick and approximate calculations:

- For heating of semi-products in chamber furnaces up to temperatures of 700 to 900°C can be used
\[ \alpha_{s+k} = 0.105 \left( \frac{T_{sp}}{100} \right)^3 + (11.5 + 17.5) \quad \text{(W.m}^{-2}\text{.K}^{-1}) \quad (47) \]

- For heating of semi-products in continuous furnaces up to temperatures of 1000 to 1300°C can be used

\[ \alpha_{s+k} = 58 + 0.35 \left( t_{sp} - 700 \right) \quad \text{(W.m}^{-2}\text{.K}^{-1}) \quad (48) \]

- For heating of non-ferrous metals alloys in chamber furnaces up to temperatures of 700 to 800°C can be used

\[ \alpha_{s+k} = 0.044 \left( \frac{T_{sp}}{100} \right)^3 + (11.5 + 17.5) \quad \text{(W.m}^{-2}\text{.K}^{-1}) \quad (49) \]

Composed heat transfer coefficient \( \alpha_{s+k} \) determined by this process is then substituted into equation (45) and heat transferred to heated material is then calculated.

### 3.4 Heating and cooling in liquid environment

When and why do we sometimes decide for heating of material in liquid environment? The answer is primarily in the required quality of material heating, especially as regards uniformity of heating and surface quality after heating.

Liquid environment provides the following favorable conditions and qualitatively higher heating results when compared to conventional flame furnaces:

- Higher temperature uniformity during heating, especially for low heating temperatures
- Heating is performed without any contact with furnace atmosphere and thus without any occurrence of scales on heated material surfaces
- Heat transfer is very efficient and can be very intense

For heating (cooling), the following basic liquid environments are used:

- Water and salt water-based solutions
- Melted salt
- Oils

A very important technological process is materials cooling, especially during heat treatment. Cooling is then a part of the whole process and is selected according to requirements given by technological specifications, especially as regards cooling rate and course of the process, absolute temperature decrease and requirements on surface quality.

### 3.5. Cooling of batch on air

The process, during which products heated up to various temperatures cool down after taking from heating furnace on air, either via natural or forced convection, occurs quite often in technological practice. This mostly occurs after heat treatment in chamber furnaces, but also in storages of pre-cast products after continuous steel casting. Heat flux between body surface and surrounding air during cooling on air can be calculated using the Newton’s relation.
Heat flux between body surface and surrounding air

\[ q_{\text{chl}} = \alpha_{\text{chl}} (t_m - t_v) = (\alpha_s + \alpha_k)(t_m - t_v) \]  
\[ \text{(W.m}^{-2}\text{)} \]  \hspace{1cm} (50)

Where:

\( \alpha_{\text{chl}} \) ... heat transfer coefficient for cooling involving convective and radiant components \( \text{(W.m}^{-2}\text{.K}^{-1}) \)

\( \alpha_s \) ... heat transfer coefficient for cooling involving radiant component \( \text{(W.m}^{-2}\text{.K}^{-1}) \)

\( \alpha_k \) ... heat transfer coefficient for cooling involving convective component \( \text{(W.m}^{-2}\text{.K}^{-1}) \)

\( t_m \) ... cooled body surface temperature \( \text{(°C)} \)

\( t_v \) ... surrounding air temperature \( \text{(°C)} \)

Heat transfer coefficient for cooling involving convective and radiant components can approximately be determined from dependences and empiric equations. A more accurate solution ensues from individual determination of individual components – radiant and convective.

**Heat transfer coefficient for cooling involving convective component**

\[ \alpha_s = \frac{q_s}{t_m - t_v} \]  
\[ \text{(W.m}^{-2}\text{.K}^{-1}) \]  \hspace{1cm} (51)

Heat transfer coefficient for cooling involving convective component

Convective heat transfer coefficient is determined using dependences from criterion equations. Generally applies:

\[ \text{Nu} = f(Re, Pr, Gr) \]  
\[ \text{(1)} \]  \hspace{1cm} (52)

**Heat transfer coefficient for free convection**

\[ \text{Nu} = f(Pr, Gr) \]

An often used relation for Nu has the form of:

\[ \text{Nu} = C(Pr.Gr)^n \]  
\[ \text{(1)} \]  \hspace{1cm} (53)

The C and n values in equation (53) can be determined from tables. For the most common case of cooling on air applies for C and n values:

- vertically or horizontally positioned bodies of cylindrical or quadratic cross-sections
- vertically positioned bodies of plate-like shapes

C = 0.54 and n = 0.25, then equation (53) is in form:

\[ \text{Nu} = 0.54(Pr.Gr)^{0.25} \]  
\[ \text{(1)} \]

For approximate calculations we can often use empiric relations – for example:

\[ \alpha_k = D^4(t_m - t_v) \]  
\[ \text{(W.m}^{-2}\text{.K}^{-1}) \]  \hspace{1cm} (54)
Where:

D … coefficient dependent on body shape and way of positioning \( (W.m^{-2}.K^{-4}) \)

**Heat transfer coefficient for forced convection**

If products are cooled intensively, by air via ventilators, then the following can occur:

- Combined free and forced convection – then the following relation applies for calculation of convective heat transfer coefficient:

\[
\text{Nu} = f(\text{Pr}, \text{Gr}, \text{Re})
\]

(1)

- Forced convection – then the following relation applies for calculation of convective heat transfer coefficient:

\[
\text{Nu} = f(\text{Pr}, \text{Gr})
\]

(1)

For approximate calculations we can often use empiric and semi-empirc relations created by generalization of experimental results. However, during their application, we have to consider their range of applicability.

For laminar convection of cooling air, we can use equation:

\[
\alpha_k = A_1 \left( \frac{\rho \cdot w_0}{L} \right)^{0.5}
\]

\( (W.m^2.K^{-1}) \)

(55)

For turbulent convection of cooling air, we can use equation:

\[
\alpha_k = A_2 \left( \frac{\rho \cdot w_0}{L^{0.2}} \right)^{0.8}
\]

\( (W.m^2.K^{-1}) \)

(56)

Where:

\( A_1, A_2 \ldots \) coefficients derived from criterion equations

\( w_0 \ldots \) convective air velocity under normal conditions \( (m.s^{-1}) \)

\( \rho \ldots \) air density \( (kg.m^{-3}) \)

\( L \ldots \) region of body for which we determine \( \alpha_k \) \( (m) \)

### Summary of terms

External heat transfer to batch is, from the viewpoint of mathematic formulation, a very difficult phenomenon, which characterizes exchange of thermal energy between:

- Exhaust gas – final product of fuel combustion
  - They have the highest temperature during heating and from the second law of thermodynamics ensues that they transfer their thermal energy to material and furnace brickwork
  - Heat transfer is executed by radiation and convection
- Furnace brickwork
- Heated material
Combusting fuel flame

The ratio of heat transfer by convection and radiation to material and furnace brickwork is dependent on exhaust gas temperature and velocity.

**Basic external heat transfer balance equation**

The aim of solving of balance thermal equations is to determine the final heat flux to material – effective heat, which can be described by balance equation

\[
\sum Q_{uz} = \sum Q_{dop} - \sum Q_{odr} + \sum Q_{konv} - \sum Q_{sál} \quad (W)
\]

Practical solution of the external heat transfer equation (34) can be performed:

- Analytically – only very easy cases
- Graphically-analytically – common engineering calculations
- Approximate calculation – approximate calculation using empiric equations

**Questions to the discussed subject**

- Graphically determine basic heat fluxes on material during heating in a flame furnace and write basic external heat transfer balance equation.
- Write the basic equation for heat transfer by convection and radiation during heating in flame furnaces.
- When would you use heating, possibly cooling, of material in liquid environment, determine advantages and disadvantages of the process.

4. Theoretic calculation of heating of thin bodies

**Time to study:** 3 hours

**Aim** After study of this chapter students will be able to

- Perform calculation of heating of bodies in furnaces with constant temperature
- Perform calculation of heating of bodies in furnaces with variable temperature

**Lecture**

We already know that heated bodies can be characterized from the thermal viewpoint as thin and thick. The characterizing criteria are Biot number and Stark law. Thin bodies are those, for which applies \( Bi \leq 0,25 \) or \( Sk \leq 0,1 \div 0,15 \).
Calculations of heating of thin bodies can be performed according to two model cases - heating of bodies in furnaces with constant temperature and heating of bodies in furnaces with variable temperature.

4.1 Heating of thin bodies in furnaces with constant temperature

It is the most common case of thin bodies heating. During calculation according to this case we ensue from the balance between:

- External heat flux to material within a certain time $d\tau$
- Thin body enthalpy

The balance equation of both the heats has then the form:

$$q_{sp-m}S_m \cdot d\tau = m \cdot c \cdot dt$$ \hspace{1cm} (W) \hspace{1cm} (57)

Where:

- $q_{sp-m}$ ... external heat flux between exhaust gas and material (W.m$^{-2}$)
- $m$ ... heated body mass (kg)
- $c$ ... specific heat capacity of heated material (J.kg$^{-1}$.K$^{-1}$)

When convective heat flux is prevailing, we express $q_{sp-m}$ by Newton’s relation and after substitution to equation (57) and its solving we get:

- Necessary heating time, if the final heating temperature is given
- Final heating temperature, if the necessary heating time is given

**Heating time for thin bodies**

$$\tau = \frac{b \cdot \rho \cdot c}{k \cdot \alpha_s} \cdot \frac{\ln \left(\frac{t_{pec} - t_0}{t_{pec} - t_k}\right)}{t_{pec} - t_k}$$ \hspace{1cm} (s) \hspace{1cm} (58)

**Temperature at the end of heating**

$$t_k = t_{pec} - \left(t_{pec} - t_0\right) \cdot \exp\left[-\frac{k \cdot \alpha_s \cdot \tau}{b \cdot \rho \cdot c}\right]$$ \hspace{1cm} (°C) \hspace{1cm} (59)

Where:

- $t_{pec}$ ... furnace temperature (°C)
- $t_0$ ... heated material initial temperature (°C)
- $t_k$ ... heated material final temperature (°C)
- $b$ ... heated body thickness for calculations (half of thickness of plate, radius of cylinder or sphere) (m)
- $\rho$ ... heated material density (kg.m$^{-3}$)
- $k$ ... body shape coefficient (1)

For body shape coefficient, representing the ratio of heated metal volume and its area, applies the following relation:

$$\frac{m}{S_m} = \frac{b \cdot \rho}{k}$$ \hspace{1cm} (kg.m$^{-2}$) \hspace{1cm} (60)
For basic shapes, body shape coefficient can easily be calculated. For both-sided heating of a plate with infinite surface of area $2S_m$ and thickness $2b$, the ratio can be written as:

$$\frac{m}{2S_m} = \frac{\rho \cdot 2b \cdot S_m}{2S_m} = b \cdot \rho$$

Body shape coefficient for a plate is then: $k = 1$

For cylinders with radius $r$ applies:

$$\frac{m}{S_m} = \frac{r \cdot \rho}{2}$$

Body shape coefficient for a cylinder is then: $k = 2$

For a sphere with radius $r$ applies:

$$\frac{m}{S_m} = \frac{r \cdot \rho}{3}$$

Body shape coefficient for a sphere is then: $k = 3$

Body shape coefficients for bodies with shapes different than plates, cylinders or spheres are then $1 < k < 3$. Exact values can be found in dependences.

The following equation was derived to calculate the average value of thin body temperature during a given heating time.

$$t = t_{pec} - \frac{t_0 - t_k}{\ln \frac{t_{pec} - t_0}{t_{pec} - t_k}} \quad (°C) \quad (61)$$

In industrial furnaces used in mechanical engineering and metallurgy it is performed usually within temperatures of $t_{pec} > 700 \div 800°C$, radiant heat transfer prevails and thus is more suitable to express heat transfer by Stefan-Boltzmann law. Subsequently, by solving of differential equation (59) we get radiant equation for determination of heating time or final body temperature:

$$\tau = \frac{b \cdot \rho}{k} \cdot \frac{c}{C_{pec}} \cdot \frac{100}{\left(\frac{T_{pec}}{100}\right)^3} \left[\psi \left(\frac{T_k}{T_{pec}}\right) - \psi \left(\frac{T_0}{T_{pec}}\right)\right] \quad (s) \quad (62)$$

Where applies:

$$\psi \left(\frac{T}{T_{pec}}\right) = \frac{1}{4} \ln \frac{1 + \frac{T}{T_{pec}}}{1 - \frac{T}{T_{pec}}} + \frac{1}{2} \arctg \frac{T}{T_{pec}} \quad (63)$$

Radiant equation is more difficult than convective equation, calculation is tougher and thus calculations of high-temperature heating are often simplified. Calculation is performed
according to convective equation and a correction according to the $(t_{pec} - t_k)$ divergence value is performed subsequently. Corrections are performed according to tables.

4.2 Heating of thin bodies in furnaces with variable temperature

Furnace temperature is not supposed to be constant, but changing linearly according to equation:

$$t_{pec} = t_{pec,0} + Z \tau \quad (°C) \quad (64)$$

Where:

- $Z$ … rate of furnace temperature increase ($°C.h^{-1}$)
- $T_{pec,0}$ … initial furnace temperature ($°C$)

Considering the presupposition according to relation (64), we get the final relation for calculation of thin body temperature by solving differential equation (57):

$$t = t_{pec} + Z \tau - Z \frac{1}{m} \left( t_{pec,0} - t_0 - Z \frac{1}{m} \right) \exp(-m \tau) \quad (°C) \quad (65)$$

Where:

- $t_0$ … initial body temperature ($°C$)
- $m$ … is expressed by:

$$m = \frac{\alpha.K}{b.\rho.c} \quad (66)$$

Since for $m.\tau \geq 4$ is $\exp(-m.\tau) = 0$, for $\tau \geq \frac{4}{m}$ body temperature is different than furnace temperature by a constant value according to:

$$t_{pec} - t = Z \frac{b.\rho.c}{k.\alpha} \quad (°C) \quad (67)$$

If furnace temperature does not change linearly, but along a random curve, we can substitute the curve of furnace temperature change with a broken line in engineering calculations and use equation (65) for its individual sections.

### Summary of terms

Heated bodies can be characterized from the thermal viewpoint as thin and thick. The characterizing criteria are Biot number and Stark law. Thin bodies are those, for which applies:

- $\text{Bi} \leq 0.25$ or $\text{Sk} \leq 0.1 \div 0.15$

Calculations of heating of thin bodies can be performed according to two model cases:

- heating of bodies in furnaces with constant temperature and heating of bodies in furnaces with variable temperature. For calculations, we come from differential equation:
We calculate two basic heating parameters:
- Necessary heating time, if the final heating temperature is given
- Final heating temperature, if the necessary heating time is given

Questions to the discussed subject
- Define basic differential equation expressing balance between external heat transfer to heated material within a heating time and enthalpy of a thin body.
- Define presuppositions, which are considered during solving of the differential equation, and how results obtained from the simpler convective equation are corrected for furnaces with prevailing radiant heat transfer.
- Characterize body shape coefficient, its definition and practical impact on heating of materials. Compare courses for prism, cylinder and sphere of identical mass.

5. Theoretic calculation of heating of thick bodies

Time to study: 5 hours

Aim  After study of this chapter students will be able to
- Define conditions for solving of differential equation of heat transfer
- Define and apply conditions of explicitness during solving of heat transfer equation – initial and surface conditions
- Define influence of geometric body shape on heating and determine their average temperature and enthalpy
- Ways of solving of Fourier heat transfer equation

Lecture

Thick heated bodies are characterized with the Biot number value of Bi ≥ 0.50, possibly with Stark law of Sk ≥ 0.15. The basis for solving of heating and cooling of thick bodies is Fourier differential equation.

5.1 Fourier heat transfer differential equation - derivation

To derivate heat transfer differential equation via Fourier series, the following presuppositions were considered:
- Heat transfer proceeds in an elemental cube – see Fig. 10
- Heat transfer is a non-stationary process
- For simplicity, heat transfer proceeds only in the direction of x axis, the other 4 limiting surfaces are ideally insulated and enable no heat transfer
During heating of the cube, the heat entering the cube \( dQ_1 \) has to be different from the heat exiting the cube \( dQ_2 - dQ_1 \neq dQ_2 \)

Fig. 10 – Schematics of non-stationary heat transfer via elementary cube

According to Fourier series, both the heats can be expressed in dependence on temperature gradient:

\[
dQ_1 = -\lambda \frac{\partial t}{\partial x} dS dt \\
dQ_2 = -\lambda \frac{\partial t}{\partial x} dS dt
\]

(W) \hspace{1cm} (68)

(W) \hspace{1cm} (69)

In equation (69), temperature gradient \( \frac{\partial t}{\partial x} \) is in parentheses to differentiate from temperature gradient \( \frac{\partial t}{\partial x} \) in equation (68).

If we consider heating of the cube, then applies:

- More heat enters the cube than exits from it
- A certain portion of heat accumulates in the cube

Then, due to negative signs of variables applies:

\[
\frac{\partial t}{\partial x} < \left( \frac{\partial t}{\partial x} \right)
\]

This inequality can be expressed also as follows:

\[
\left( \frac{\partial t}{\partial x} \right) = \frac{\partial t}{\partial x} + \Delta \frac{\partial t}{\partial x} \hspace{1cm} (K.m^{-1}) \hspace{1cm} (70)
\]

Or the inequality can be expressed as follows:

\[
\left( \frac{\partial t}{\partial x} \right) = \frac{\partial t}{\partial x} + \frac{\partial^2 t}{\partial x^2} dx \hspace{1cm} (K.m^{-1}) \hspace{1cm} (71)
\]

Then heat exiting the cube is equal to:
\[
dQ_2 = -\lambda \cdot dS \left( \frac{\partial t}{\partial x} + \frac{\partial^2 t}{\partial x^2} \right) d\tau \quad (W) \quad (72)
\]

Heat, which remained accumulated in the cube, can be expressed in two ways:

- **As difference between entering and exiting heats:**

  \[
  dQ = dQ_1 - dQ_2
  \]

  After substituting for \(dQ_1\) and \(dQ_2\) we get:

  \[
  dQ = \lambda \cdot dS \cdot \frac{\partial^2 t}{\partial x^2} \cdot dx \cdot d\tau \quad (W) \quad (73)
  \]

  After substituting for \(dS\) we get equation:

  \[
  dQ = \lambda \cdot dx \cdot dy \cdot dz \cdot \frac{\partial^2 t}{\partial x^2} \cdot d\tau \quad (W) \quad (74)
  \]

- **From change of enthalpy of cube**, the temperature of which increased during heating:

  \[
  dQ = c \cdot dx \cdot dy \cdot dz \cdot \rho \cdot \frac{\partial t}{\partial \tau} \cdot d\tau \quad (W) \quad (75)
  \]

By comparison of equations (74) and (75) we get relation for time change of elementary cube thermal field in x axis direction:

\[
\lambda \cdot \frac{\partial^2 t}{\partial x^2} = c \cdot \rho \cdot \frac{\partial t}{\partial \tau} \quad (W.m^{-2}) \quad (76)
\]

After adjustment we finally get Fourier heat transfer differential equation in x axis direction, which is the basis for further solving of heating or cooling of materials:

\[
\frac{\partial t}{\partial \tau} = \frac{\lambda}{c \cdot \rho} \cdot \frac{\partial^2 t}{\partial x^2} = a \cdot \frac{\partial^2 t}{\partial x^2} \quad (K.s^{-1}) \quad (77)
\]

Where:

\[
a \ldots \text{thermal diffusivity coefficient} \quad (m^2.s^{-1})
\]

\[
a = \frac{\lambda}{c \cdot \rho}
\]

The value of thermal diffusivity coefficient determines rate of temperatures equalization within body during heating or cooling. The higher is \(a\) coefficient, the faster is heating or cooling of material and the faster is temperature equalization within the body after heating has been finished.

If heat is not transferred only in the x axis direction, but also in other directions, i.e. directions of axes y and z, then the final three-dimensional heat transfer differential equation is in form:

\[
\frac{\partial t}{\partial \tau} = a \left( \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} \right) \quad (K.s^{-1}) \quad (78)
\]

If we introduce the following using calculation operators:
\[ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} = \nabla^2 \]

Where:
\( \nabla^2 \) … Laplace operator

We get the heat transfer differential equation in a shortened form:
\[ \frac{\partial t}{\partial \tau} = a \nabla^2 t \quad \text{(K.s}^{-1} \text{)} \quad (79) \]

Sometimes is advantageous to calculate in cylindrical coordinates. Therefore also the three-dimensional heat transfer differential equation can be expressed in cylindrical coordinates. This can be performed by substitution of \( x \) and \( y \) by the following in equation (78):
\[ x = r \cos \varphi \]

and
\[ y = r \sin \varphi \]

After adjustment, we get the three-dimensional heat transfer differential equation in cylindrical coordinates:
\[ \frac{\partial t}{\partial \tau} = a \left( \frac{\partial^2 t}{\partial r^2} + \frac{1}{r} \frac{\partial t}{\partial r} + \frac{1}{r^2} \frac{\partial^2 t}{\partial \varphi^2} + \frac{\partial^2 t}{\partial z^2} \right) \quad \text{(K.s}^{-1} \text{)} \quad (80) \]

For a cylinder with indefinite length and symmetrical thermal field distribution along the cylinder axis, the heat transfer equation has a simpler form:
\[ \frac{\partial t}{\partial \tau} = a \left( \frac{\partial^2 t}{\partial r^2} + \frac{1}{r} \frac{\partial t}{\partial r} \right) \quad \text{(K.s}^{-1} \text{)} \quad (81) \]

Heat transfer differential equation can similarly be derived for a sphere:
\[ \frac{\partial t}{\partial \tau} = a \left( \frac{\partial^2 t}{\partial r^2} + \frac{2}{r} \frac{\partial t}{\partial r} \right) \quad \text{(K.s}^{-1} \text{)} \quad (82) \]

### 5.2 General solution of heat transfer differential equation

During heating and cooling of thick bodies, temperature gradient develops along their cross-sections and thus their heating proceeds under more complicated conditions, when compared to thin bodies. Besides external heat transfer, internal heat transfer, i.e. heat transfer within the heated (cooled) material, is also necessary to be considered.

Analytical thermal theory elaborated by Joseph Fourier enables to investigate temperature field during heating and cooling of a thick body. Partial heat transfer differential equation can be solved:

- By methods of mathematical physics
- By approximate methods

**Solving of heat transfer differential equation by mathematical physics**
• Analytical solving methods – separation of variables
  o Transfers the solution of partial differential equations to a system of ordinary
differential equations
  o The final solution is obtained by integrating of differential equations
  o Advantages of the method:
    ▪ Too abstract
    ▪ Direct and in simple solutions leads to concrete results
  o Disadvantages of the method
    ▪ Suitable only for simple cases
• Method of thermal impulses
  o The basis of the method lies in division of heat transfer process into a number
of elementary impulses. The process then proceeds as a summary of
temperature equalizations between a number of elementary thermal impulses
divided in time and space
• Laplace transform method – method of calculation operators. For unidimensional tasks,
Laplace transformation transfers characteristic heat transfer differential equation to a
normal algebraic equation
  o For two-dimensional tasks, Laplace transformation transfers partial differential
equation to a normal differential equation
  o The method is suitable for tasks with more difficult boundary conditions
  o Solving of heat transfer differential equation usually leads to sum of an
indefinite progression, the calculation of which is possible only using modern
computational technics

5.3 Conditions of explicitness during solving of heat transfer differential equation

To get an explicit solution of heat transfer differential equation is necessary to
determine explicit conditions for its solving and necessary to know:
• Geometric shape of heated body
• Boundary conditions
• Intensity and distribution of internal thermal impulses
• Thermal-technical characteristics of body material

5.3.1 Heat sources (impulses)

One of the basic terms in conductive heat transfer theory is the term heat sources.
Since there is a variety of different heat sources, there is also a variety of basic data for heat
transfer calculations.

Each heat source is characterized by seven signs, according to which the sources are
characterized into groups, which are characterized as follows:
• Sources with given temperature – I_t or with given heat flux density or heat volume – I_q
• Positive sources – heat emitters, negative sources – heat absorbers
• Point, line, planar and volume sources
• Internal sources – inside a body, external sources – on body surface
• Momentary or continuous sources
• Sources constant or variable in time
• Movable and stable sources

**Example of a heat source of type Iₜ**

• Heating or cooling of a body by convective heat transfer from surrounding environment
  o Heating of metal within furnace
  o Heating of material in salt baths and melted salts
  o Cooling of products freely on air after taking off bogie-hearth furnace
  o Cooling of products in water cooling bath

**Example of a heat source of type Iₚ**

• Thin resistance wire located within a body
  o A current with constant value flows through the wire, the source has the following characteristics:
    • Positive thermal source
    • Line thermal source
    • Internal thermal source
    • Continuous thermal source
    • Stable thermal source
    • Constant intensity thermal source

• Initial thermal characteristics within a body
  o This source can be considered as a result of the influence of:
    • Internal sources
    • Volume sources
    • Stable sources
    • Momentary sources
  o Instantaneous power of thermal sources can be described as:

\[ P = t.c_v \quad (W) \]

Final word to the problematics of sources – the type of a source is independent of its physical basis, but depends on knowledge about the given source, its influence on the body and phenomena which it implies within the body.

**5.3.2 Initial condition**

The initial condition characterizes temperature field within a body in a random moment during heating or cooling process.
At the beginning of heating in time $\tau = 0$

We consider an important moment during heating, possibly heat treatment – since this moment we calculate the time of the whole process.

Since most cases in general practice can be solved as unidimensional tasks of body temperature field changing, the initial condition can sufficiently be given as temperature change:

- Along the cross-section, in the direction of normal to the surface
- In radial direction for cylinder and sphere

The simplest case of determination of initial condition is when the temperature of a body before heating is identical (e.g. 10°C) in all its locations. Then the initial condition is equal to:

$$ t_{\tau=0} = t_0 = 10^\circ C \quad (82) $$

Similarly easy is mathematic description of initial condition if we know total body enthalpy before heating or cooling. Initial condition has then the form of:

$$ t_{\tau=0} = t_0 = \frac{Q_{\tau=0}}{c_p \cdot V} \quad (83) $$

This case occurs in practice quite often, especially when heating of cold batch or during heat treatment, if the heating or cooling process is solved as a multi-phase one.

During heating of a warm batch, e.g. after exiting the device for continuous steel casting, the temperature field along the body cross-section (pre-cast product) is variable, which has to be considered in the given initial condition. The initial condition is then given, including the initial course of temperature along cross-section, as a function:

- Temperature change along cross-section – linear
- Temperature change along cross-section – parabolic
- Temperature change along cross-section – along a cosinusoid

More difficult cases are in practice solved by substitution of curves by:

- Broken lines
- Combination of various curves – parabola, hyperbola etc.

**5.3.3 Surface conditions**

The surface condition is characterized by thermal conditions on body surface and consists of two basic parts:

- External (surface) heat sources
- Conditions of heat transfer between source and body surface

Four types of surface conditions can be characterized – surface conditions of first to fourth type. Individual conditions are defined as follows:

- **1st type condition** – temperature distribution on body surface is defined as a function of coordinates and time - $t_{pov} = f(x, y, z, \tau)$,
Surface temperature can be described by linear function, exponential function, multinominal etc.

**II\textsuperscript{nd} type condition** – external heat flux intensity is given as a function of time. From Fourier’s law ensues:

$$-\lambda \frac{\partial t}{\partial x} \bigg|_{x=0} = q$$  \hspace{1cm} (84)

Temperature gradient is related to a location of a body being in a direct vicinity of body surface, which can be seen from index $x = +0$.

**III\textsuperscript{rd} type condition** – temperature of environment (gas or liquid) surrounding a body, as well as mathematic relation describing heat transfer between the environment and body surface, are known. Heat transfer from gaseous or liquid environment to a body is mostly determined on the basis of Newton’s law:

$$-\lambda \frac{\partial t}{\partial x} \bigg|_{x=0} = \alpha (\theta - t_{x=0})$$  \hspace{1cm} (85)

**IV\textsuperscript{th} type condition** – is used in cases the investigated body is in a direct contact with another body having different physical properties than the investigated one. Surface contact is perfect to ensure identical temperature of contacting points. Then applies:

$$t \bigg|_{x=0} = t \bigg|_{x=0}$$

Balance equation on boundaries of both the bodies is in form:

$$-\lambda_1 \frac{\partial t}{\partial x} \bigg|_{x=0} = -\lambda_2 \frac{\partial t}{\partial x} \bigg|_{x=0}$$  \hspace{1cm} (86)

From analyses of equations (84), (85) and (86) ensues:

- Left sides of equations (84), (85), (86) are identical and represent heat flux intensity within the investigated body
- Right sides of equations (84), (85), (86) differ and represent external heat transfers to body surface

In general case, the boundary layer affects heat transfer from a source to body surface. This layer: Absorbs a portion of penetrating heat due to its accumulative ability, i.e. offers accumulative resistance against heat transfer to body surface. This can be expressed as:

$$C = c' \cdot h' \cdot \rho' \hspace{1cm} (J.m^{-2}.K^{-1})$$  \hspace{1cm} (87)

- Decreases surface temperature, since it performs temperature resistance against heat transfer. This can be expressed as:

$$\Omega = \frac{h'}{\lambda'} = \frac{1}{\alpha} \hspace{1cm} (m^2.K.W^{-1})$$  \hspace{1cm} (88)

This results in decreased source effectivity, which results in decreased temperature and decreased heat flux intensity.

If, in a general case, a thermal source also effects during heat transfer, heat from both the sources impacting body surface is given by equation:

$$q_\Sigma = q + \alpha (\theta - t_{x=0}) \hspace{1cm} (W.m^{-2})$$  \hspace{1cm} (89)
As a result of accumulative resistance, a portion of heat is consumed by change of boundary layer enthalpy:

\[ q' = c' \rho' h' \frac{\partial t}{\partial x} \bigg|_{x=0} \quad (W.m^{-2}) \quad (90) \]

The final balance equation on body surface can then be written in the general form:

\[ -\lambda \frac{\partial t}{\partial x} \bigg|_{x=-0} = q + \alpha (9 - t_{x=-0}) - c' \rho' h' \frac{\partial t}{\partial x} \bigg|_{x=0} \quad (W.m^{-2}) \quad (91) \]

From general equation (91), a variety of relations for surface conditions can be created.

### 5.3.3.1 Simplification of surface conditions

If a surface condition has been selected, we have to find whether it can be simplified by Biot number \( Bi \), which represents a ratio of internal and external resistances:

\[ Bi = \frac{\alpha_h}{\lambda} = \frac{h}{\lambda} \frac{1}{\alpha} \quad (1) \]

Where:

- \( h \) … characteristic body dimension (m)

Biot number can also be expressed as:

\[ Bi = \frac{h}{\lambda} = \frac{h}{h'} \frac{1}{\alpha} \quad (1) \quad (92) \]

Where \( h' = \frac{\lambda}{\alpha} \) is length dimension and represents thickness of boundary layer, the free (external) surface temperature of which is equal to surrounding environment temperature. Biot number thus also represents a ratio of characteristic body dimension to boundary layer thickness.

### 5.4 Influence of geometric body shape, average body temperature and enthalpy

Calculations can be performed using so called **computational diagrams**. Calculations can relatively easily be performed for 4 basic body types:

- Planar plane limited only by its thickness, its width and length are not limited
- Cylinder with indefinite length, given is only radius – \( r \)
- Sphere with limited dimensions, given is radius – \( r \)
- Semi-bounded body – limited only in one direction

#### 5.4.1 Possibility to simplify shape for calculations

Sometimes is suitable to perform a simplification by substitution of body shape with another imaginative one via the method:

- Volume of fluid method
- Deformation method – while maintaining body volume. During this method, a difficult shape is transferred to a basic shape (plane, cylinder, sphere), while body volume and affecting heat flux remain identical.

The fact that surface area of a body with simple shape $S_j$ is always smaller than of a body with complex shape $S_s$ (considering identical volumes) is known. Heat flux intensity thus needs to be increased when submitting boundary conditions of $II^{nd}$ type. Then applies:

$$ q_j = \varepsilon q_s \quad \text{(W.m}^{-2}) \quad (93) $$

Where:

$$ \varepsilon = \frac{S_s}{S_j} \geq 1 \quad (1)_{pr} $$

When having boundary conditions of $III^{rd}$ type, we have to increase the calculative value of Bi number in the following way:

$$ Bi_j = \varepsilon Bi_s $$

Characteristic dimensions of basic shape bodies and $\varepsilon$ value can be calculated from equations according to given dimensions of complex shapes:
- For plate:
  $$ h_j = \frac{V}{S_{pr}} ; \quad \varepsilon = \frac{S_j}{2S_{pr}} $$
  Where:
  $S_{pr}$ … area of mean plane of complex body (largest cross-section area)
- For cylinder
  $$ R_j = \sqrt[3]{\frac{S}{\pi}} ; \quad \varepsilon = \frac{p_s}{\sqrt[4]{4\pi S}} $$
  Where:
  $S$ … area of cross-sectional cut (m$^2$)
  $p_s$ … circumference of complex body cross-sectional cut
- For sphere
  $$ R_j = \sqrt[3]{\frac{3V}{4\pi}} ; \quad \varepsilon = \frac{S_j}{\sqrt[3]{36\pi V^2}} $$
  Where:
  $V$ … bodies volumes (m$^3$

5.4.2 Average temperature and body enthalpy
Computational diagrams enable to calculate either temperature of selected body location using argument $\Theta$ or average body temperature using argument $\bar{\Theta}$. This average temperature is equal to:

$$\bar{t} = \frac{1}{V} \int_V t \, dV \quad \text{(°C)} \quad (94)$$

Where:

$V$ … volume of whole body or its part \hspace{1cm} (m$^3$)

If we know the average body temperature, we can calculate its enthalpy:

$$i = c_\rho V \bar{t} = c_\rho \int_V t \, dV \quad \text{(J)} \quad (95)$$

To heat a metal, metal enthalpy change $\Delta i$, which is defined as a difference of enthalpies in two different momentary heatings, usually needs to be known. If we know temperature change $\Delta t$ of each location of the body within a given time, enthalpy change is given as:

$$\Delta i = c_\rho \int_V \Delta t \, dV \quad \text{(J)} \quad (96)$$

or

$$\Delta i = c_\rho V \Delta t$$

Heat flux change can be calculated from heat transferring within a certain time into a body or its layer by surface area $S$:

$$\Delta i = \lambda \int_{t_2}^{t_1} \frac{\partial t}{\partial n} \, dS \, dt \quad \text{(J)} \quad (97)$$

Where:

$n$ … normal direction to body surface

\[ Σ \]

**Summary of terms**

Thick heated bodies are characterized with the Biot number value of $Bi \geq 0.50$, possibly with Stark law of $Sk \geq 0.15$. \hspace{1cm} (K.s$^{-1}$)

During heating and cooling of thick bodies, temperature gradient develops along their cross-sections and thus their heating proceeds under more complicated conditions, when compared to thin bodies. Besides external heat transfer, internal heat transfer, i.e. heat transfer within the heated (cooled) material, is also necessary to be considered.

Analytical thermal theory elaborated by Joseph Fourier enables to investigate temperature field during heating and cooling of a thick body.

The basis for solving of heating and cooling of thick bodies is application of Fourier series on differential equations. For three-dimensional heat transfer, it is in the following form:
Partial heat transfer differential equation can be solved:
- By methods of mathematical physics
- By approximate methods

To get an explicit solution of heat transfer differential equation, explicit conditions for solving need to be determined and the following needs to be known:
- Geometric shape of heated body
- Boundary conditions
- Intensity and distribution of internal thermal impulses
- Thermal-technical characteristics of body materials

**Questions to the discussed subject**

- Write Fourier heat transfer differential equation for three-dimensional heat transfer
- Define methods by which can Fourier heat transfer differential equation for three-dimensional heat transfer be solved
- Characterize conditions, which are necessary to be defined to obtain an explicit solution of heat transfer differential equation
- Define presuppositions, which are considered to simplify solution of thick bodies heating process

6. Oxidation and decarburization of steel during heating

**Time to study:** 4 hours

**Aim** After study of this chapter students will be able to
- Define conditions of steel oxidation development and influence of temperature, heating time, furnace atmosphere and chemical composition of steel on scales formation
- Calculate amounts of scales for individual heating cases
- Define conditions for occurrence of steel decarburization
- Characterize methods leading to limiting of oxidation and decarburization of steels and heating optimization
Heating of steels in common types of heating flame furnaces and in electric furnaces with air atmospheres is always accompanied by metal surface oxidation. Since steel needs to be heated repeatedly during a steel product production process, from a pre-cast product or ingot to the final product, metal loses by oxidation are high and reach to 3 to 5% of metal mass. Oxidation of steel thus causes loses in production and increases cost of a final product.

Oxidation of steel during heating does not result only in metal loses due to burn-out, but also in waste products due to rolling or forging of scales into products and exposing of sub-surface voids. It also causes increased wear of rolls and dies. Within a furnace, scales deteriorate heat-resistant materials and cause growth of furnace hearth ground. Scales thus in various ways complicate technological processes and increase idle times in production.

6.1 Theoretical bases of steel oxidation

Oxidation of steel occurs as rusting already at room temperature. Temperature increase to 200 - 300°C and higher results in covering of steel surfaces with reaction colors. Temperature increase results also in increase in scale-formation rate; the process is the most intensive at temperatures above 1000°C.

From the chemical viewpoint, it is a reaction between oxidation gases (O_2, CO_2, H_2O) and iron with its admixtures, resulting in a generation of mixture of iron oxides with various oxidation degrees and its admixtures. During oxidation, three iron oxides form:
- Ferrous oxide FeO, stable at temperatures above 570°C with melting temperature of 1377°C
- Ferrous-ferric oxide, stable within the whole range of temperatures from room temperature to melting temperature – 1538°C
- Ferric oxide Fe_2O_3, stable from low temperatures to melting temperature – 1565°C

A more detailed study showed that metal oxidation is actually a two-dimensional diffusion. It is primarily diffusion of oxygen and gas stream core to heated material surface, its absorption on the surface, diffusion of reaction components through scale layer and crystallic-chemical transformation connected with change of composition and lattice structures of solid phases.

Simultaneously, diffusion proceeds also in the opposite direction – diffusion of metal atoms through scales on metal surface. Iron oxide with the highest oxidation degree forms on metal surface and oxides with lower oxidation degrees follow.

6.2 Influence of individual factors on metal burn-out

Diffusion through oxide layer has the essential influence on oxidation rates of steel and iron. Therefore, burn-out of metals depends primarily on factors influencing diffusion – time and temperature of heating, furnace atmosphere composition and chemical composition of steel.

6.2.1 Influence of heating temperature and time

Scales generation rate is dependent on material heating temperature – the following have been observed:
- Practically observable scales generation – from temperatures of 600 - 650°C
• Slow oxidation – to 900°C
• Rapid oxidation rate increase – above 900°C
• Abrupt oxidation rate increase – at temperatures 1275 ÷ 1300°C, is related to melting of scales, uncovering of metal surface occurs
  o Oxidation rate is six time faster than at the temperature of 900°C

Oxidation process proceeds according to diffusion laws, therefore the dependence of thickness or mass of scales and oxidation time can be expressed using a parabolic equation:

\[ y^2 = k \cdot \tau \]  \hspace{1cm} \text{(kg}^2\text{)} \hspace{1cm} (98)

Where:

\( y \) … mass or thickness of scales
\( k \) … constant
\( \tau \) … oxidation time

Constant \( k \) representing oxidation rate change in dependence on temperature can be expressed by Arrhenius equation:

\[ k = a \cdot e^{\frac{b}{R \cdot T}} \] \hspace{1cm} (99)

Where:

\( a, b \) … constants
\( R \) … gas constant
\( T \) … absolute temperature

The concrete dependence of generated scales mass on heating temperature and time within temperatures from 600 to 1200°C is for carbon steels expressed by equation:

\[ y = 6.3 \cdot \sqrt{\tau} \cdot e^{-\frac{9000}{T}} \] \hspace{1cm} (g.cm\(^{-2}\)) \hspace{1cm} (100)

### 6.2.2 Influence of furnace atmosphere

Furnace atmosphere chemical composition has a great influence on metal burn-out. Composition of furnace atmosphere depends on:

• Fuel chemical composition
• Excess, possibly lack of air
• Constructional design of burners

Gasses forming within furnace atmosphere can, in dependence on temperature and mutual ratios, influence material:

• Oxidatively – \( O_2, CO_2, H_2O, SO_2 \)
• Reductively – \( CO, H_2 \)
• Be neutral – \( N_2 \)
• Have carburizing effect – \( CO, CH_4 \)
• Have decarburizing effect – CO₂, H₂O, H₂

It was theoretically and practically proven that reductive furnace atmosphere ensuring scaleless steel heating has to contain 11 to 12% CO and 15 to 17% H₂ for imperfect gas fuel combustion. During combustion of coke-oven gas or natural gas, exhaust gas with this composition corresponds to air excess of n = 0.4 ÷ 0.5.

However, from practical viewpoint, such a composition is not possible for casual heating furnaces operating with overpressure in combusting and heating chambers, primarily from safety reasons. Fuel losses by chemical waste would also be high.

The volume of scales forming on surfaces of steels of various compositions is not identical for identical conditions. The volume of generating scales is influenced by:
- Shape and dimensions of heated body
- Method and conduction of heating
- Type and properties of generating scales

Composition of steel influences character and properties of scales, which then influence the further process of scales generation. The following is characteristic for individual types of steels:

- Low-carbon steels
  - Form porous scales with low adhesiveness
  - They enable further O₂ diffusion and scales formation
- Steels with carbon content C > 0.3%
  - Form a thin CO layer above surface, which limits penetration of O₂ to surface and protects steel against oxidation
  - The steels thus have lower burn-out
- Alloyed steels - Cr, Al, Si, Ni
  - Oxides of alloying elements form scales with good adhesiveness
  - Compact scales protect steel surface against further oxidation

π criterion enables to evaluate characteristic properties of scales.

\[
\pi = \frac{M \rho_k}{x A \rho_{ox}} \quad (1)
\]

Where:
- M … oxide molecule mass
- A … metal atomic mass
- \(\rho_k, \rho_{ox}\) … density of metal and oxide
- x … number of metal atoms in an oxide molecule

If:
- \(\pi < 1\) – scales are porous and do not protect metal against oxidation
6.3 Decarburization of steel

Heating of steels is accompanied not only by oxidation of their surface layers, but also by decrease in surface layers carbon contents – the layers are decarburized. Decarburization is especially observed during the process of:

- Heat treatment
- Heating of alloyed steels before rolling

Decarburization of steel causes:

- Decrease in resistance against static load
- Decrease in fatigue limit
- Increased tendency to cracking

Oxidation and decarburization is related to each other and decarburization precedes oxidation. Generated scales layer protects steel against further decarburization.

6.4 Ways to decrease burn-out and increase heating efficiency

Each decrease in burn-out means not only decrease in loses and production costs, but also resources and energy savings and reduction of negative influence of industrial production on the environment.

Decrease in burn-out can be achieved by:

- Improvement of furnace thermal work
  - Maintain optimum combustion ratio – $n = 1.05 \div 1.1$
  - Not exceed technologically necessary heating temperature
  - Minimize heating time, especially at temperatures above 900°C
  - Maintain heating furnaces in good technical conditions, heat with overpressure in furnace area and limit sucking of additional air
  - Maintain uniform furnaces operation as concerns temperatures, as well as output powers
- Apply rapid-heating

Summary of terms

Heating of steels in common types of heating flame furnaces and in electric furnaces with air atmosphere is always accompanied by metal surface oxidation.
Oxidation of steel during heating does not result only in metal loses due to burn-out, but also in waste products due to rolling or forging of scales into products and exposing of sub-surface voids. It also causes increased wear of rolls and dies.

From the chemical viewpoint, it is a reaction between oxidation gases (O₂, CO₂, H₂O) and iron with its admixtures, resulting in a generation of mixture of iron oxides with various oxidation degrees and its admixtures. During oxidation, three iron oxides form:

- **Ferrous oxide FeO**, stable at temperatures above 570°C with melting temperature of 1377°C
- **Ferrous-ferric oxide**, stable within the whole range of temperatures from room temperature to melting temperature – 1538°C
- **Ferric oxide Fe₂O₃**, stable from low temperatures to melting temperature – 1565°C

Oxidation is influenced especially by:

- Steel heating temperature and time
- Furnace atmosphere
- Chemical composition of material

Oxidation and burn-out can primarily be reduced by:

- Heating process quality improvement
- Application of rapid heating technology

**Questions to the discussed subject**

- Define the most important influences on metal burn-out during heating, mechanisms which contribute to scales formation and how can they be limited.
- Define laws applying for scales formation calculations
- Characterize the main methods to limit burn-out in industrial furnaces

**7. Technological basics of metals heating**

**Time to study:** 4 hours

**Aim** After study of this chapter students will be able to

- Determine final temperatures for metals heating
- Calculate heat stress during heating
- Determine allowable non-uniformity of heating and heating degree
- Design optimum metals heating regimes
Lecture

Metals heating is an important production process, which usually precedes hot mechanical processing – rolling, forging, stamping – or forms a substantial part of metals heat treatment.

To important factors in metals heating technology belong:

- Final metals heating temperature
- Allowable metals heating rate in relation to heat stress
- Way of material lodgment in furnace
- Temperature and thermal regimes of heating of a given metal

7.1 Final metals heating temperature

Two main factors influence the heating process – heating temperature and heating time or rate. Heating temperature is of a higher importance, especially as regards a higher amount of admixtures.

During heat treatment of steel, heating temperatures and equalization temperatures are determined by critical points in the iron-carbon diagram. These temperatures are within the range of 750 to 1000°C. Selection of heating temperature for any steel before hot mechanical processing depends on:

- Properties of heated steel
- Method of subsequent mechanical processing - rolling, forging, stamping

Optimum heating temperature needs to be selected not only from the viewpoint of energy consumptions for heating and mechanical processing, but also from the viewpoints of mechanical properties of final products and wear of tools – rolls, dies.

Increased heating temperature results in:

- Decrease in flow stress during mechanical processing
  - Decrease in consumption of energy for mechanical processing
  - Possibility to increase reductions during mechanical processing
  - Increase in lifetime of tools for mechanical processing
  - Increase in thermal energy consumption for metals heating
  - Grain growth and deterioration of mechanical properties of final product
  - Increased oxidation, decarburization and burn-out

These factors determine the upper heating limit. It is given to be 100 to 150°C below the solidus curve.

There is also a lower heating limit, which is determined from the allowable temperature at the end of mechanical processing for each metal.
7.2 Heat stress during heating and cooling of steel

Heat stress, which in some cases can be limiting factor for heating rate, develops as a consequence of inhomogeneous temperature field during metal heating.

If heat stress does not exceed yield strength, heat stress is eliminated by equalization of temperature differences.

If plastic deformation occurs, i.e. yield stress is exceeded, a new distribution of heat stress develops in locations which were not plastically deformed. Residual stress remains in metal after equalization of temperature differences. When yield strength is exceeded in heated metal, cracks start to develop.

Mechanical properties of steels change with increasing temperature. It was found that steel is elastic to temperatures of 500 to 550°C and above this temperature transforms to plastic state. For carbon steels, this temperature is around 400°C and soft steels transform to plastic state at temperatures of around 350°C.

From the above mentioned ensues that heat stress under concrete heating conditions occurs for steels only in the first stage of heating, within temperatures from 0 to 500°C. Above this temperature they are plastic and stress decreases and scatters.

Heat stress can be calculated using:

\[ \sigma = \frac{\beta E}{1 - \nu} (\bar{t} - t_x) \]  

Where:

- \( \sigma \) ... heat stress (Pa)
- \( \beta \) ... linear expansion coefficient (K\(^{-1}\))
- \( E \) ... elasticity modulus (Pa)
- \( \nu \) ... Poisson number (1)
- \( \bar{t} \) ... average body temperature (°C)
- \( t_x \) ... temperature in observed body location (°C)

7.3 Allowable heating rates

Knowledge of the allowable heating rate for an individual case is especially important for the issue of heating intensification.

- For thin bodies, heating rate is limited by conditions of external heat transfer
- For thick bodies, conditions of internal heat transfer, thus heat transfer within a heated body, apply substantially together with conditions of external heat transfer for many steels

To increase heating rate of thick bodies, the maximum allowable increase of surface temperature to the final value and subsequent equalization of temperatures within body cross-section need to be ensured.

Allowable heating rate is directly connected with allowable heat stress, which can be applied to body material during heating. According to the solution proposed by N.J.Tajc, in which he considers also residual stress, the allowable stress is calculated via equation:
\[
\sigma_d = \frac{\sigma_{pt}}{1,5 \div 2} = (0,7 \div 0,5)\sigma_{pt} \quad \text{(Pa)} \quad \text{(103)}
\]

Where:

\(\sigma_{pt}\) … ultimate strength of heated material \((\text{Pa})\)

### 7.4 Allowable non-uniformity and heating degree

An important factor in a heating process is non-uniformity of heating - the final difference of heated body surface and center temperatures related to its thickness. Non-uniformity is especially related to equalization phase. Temperatures equalization can generally proceed at:

- Constant furnace temperature – \(t_{pec} = \text{const.}\)
- Constant investigated body surface temperature – \(t_{pov} = \text{const.}\)
- Constant metal enthalpy – \(\Delta i = 0\)

Calculations can prove that the least time consuming is equalization of temperatures at constant metal enthalpy, during which the source of heat necessary for temperatures equalization is surface layers. However, this regime counts with surface overheating and thus the danger of its damage arises.

Equalization time is usually calculated as:

\[
\tau_e = \frac{j b^2}{a} \quad \text{(s)} \quad \text{(104)}
\]

Where:

- \(j\) … coefficient of temperature equalization dependent on heating degree \(\delta\) and body shape
- \(b\) … heated material computational thickness \((\text{m})\)
- \(a\) … thermal diffusivity coefficient \((\text{m}^2\cdot\text{s}^{-1})\)
- \(\delta\) … material heating degree - \(\delta = \frac{\Delta t_1}{\Delta t_0}\)

### 7.5 Design of metals heating regimes via graphic-analytic method

During design of temperature and heat regimes of heating, the following factors and parameters need to be determined:

- External heat flux within furnace – discussed in detail in chaps. 2 and 3
- Internal heat flux within heated material
- Time of heating phase
- Time of initial heating phase – i.e. time of irregular phase, sometimes also time of heating delay and inertia time during heating
• Time of equalization phase

7.5.1 Calculation of internal heat flux within heated material

Internal heat flux within heated material changes during heating. For a given moment, in which temperature gradient within the body is known, heat flux can be described by a simple Fourier equation for stationary state. For plate or prism applies:

\[ q = \frac{2 \lambda_i \Delta t}{b} \quad (W.m^2) \quad (105) \]

For cylinder:

\[ q = \frac{2 \lambda_i \Delta t}{R} \quad (W.m^2) \quad (106) \]

Where:

- \( \Delta t \) … temperature gradient within body \((°C)\)
- \( \lambda_i \) … thermal conductivity of heated material \((W.m^{-1}.K^{-1})\)
  - for average metal temperature in given moment
- \( b, R \) … computational body thickness, radius \((m)\)

7.5.2 Calculation of heating phase time

The basic equation for heat transfer representing temperature field change within an observed time, Fourier-Kirchhoff for a boundary condition of \( q = \text{const.} \), can after adjustments be written in a form expressing material heating time within the regular phase.

\[ \tau = \frac{b \rho \Delta i}{kq} \quad (s) \quad (107) \]

Where:

- \( b \) … computational body thickness, radius \( R \) for cylinders \((m)\)
- \( \rho \) … body density \((kg.m^{-3})\)
- \( k \) … body shape coefficient \((1)\)
- \( \Delta i \) … material enthalpy increment during heating time \((kJ.kg^{-1})\)
- \( q \) … heat flux within heated body \((W.m^{-2})\)

To solve equation (109), nomograms for determination of heating time within heating phase have been elaborated.

7.7.3 Calculation of equalization phase time

Equalization phase is important for quality of heating, as well as for course of the subsequent technological process. Equalization phase is usually the last phase of heating and is therefore very important, especially for thick bodies with \( \text{Bi} > 0.5 \).

To determine equalization phase time, nomograms have been elaborated as well. To determine equalization time from nomograms, the following needs to be known:

- Heated material thermal conductivity coefficient
- Heated material thermal diffusivity coefficient
Heated material computational thickness
Heating degree at entrance to equalization phase
Heated material average temperature at entrance to equalization phase

Summary of terms

Metal heating is an important production process, which usually precedes hot mechanical processing – rolling, forging, stamping – or forms a substantial part of metals heat treatment.

To important factors in metals heating technology belong:

- Final metals heating temperature
- Allowable metals heating rate in relation with heat stress
- Way of material lodgment in furnace
- Temperature and thermal regimes of heating of a given metal

Questions to the discussed subject

- Define the most important heating phases and criteria, according to which temperature and heat material heating are selected.
- Define the bases for determination of the final temperature of material heating?
- Define the main reasons for determination of allowable heating rate and possible consequences of its exceeding?

Literature

Mandatory

Recommended