Material forming

Such an unnumbered chapter is added here because of the fact that more and more students of a master’s degree are interested in the material forming and have no preview bachelor’s degree in this field; even any other theoretical, especially, technological background in the materials area. It is like explaining an engine principle to a student who has no idea what is a car (overstatement).

Time to study: 2 hours

Aim

After study the unnumbered chapter you become familiar with the English version of the materials’ forming introduction. It is added here because of the fact that more and more students applying for the master’s degree have no bachelor background and it provides them with the introduction of the forming theory; without a basic technology overview it is quite unfavorable. One of the essential conditions in exercising of the forming theory subject will be getting a credit from the study of the English literature (eventually, the other languages), (mostly a selected article and/or a part of book’s chapter). Details will be provided in a lecture. The articles describe a real forming process and therefore your knowledge of the technical English will be verified.

- Define basic forming principles in the Czech language
- Describe and draw some selected forming methods
Introduction

- Practically, all metals, which are not used in a cast form, are reduced to some standard shapes for subsequent processing.
- Manufacturing companies producing metals supply metals in a form of continuous casting methods which are obtained by casting liquid metal into a square cross section.
- Slab (500-1800 mm wide and 50-300 mm thick)
- Billets (40 to 150 sq mm)
- Blooms (150 to 400 sq mm)
- These shapes are further processed through hot rolling, forging or extrusion, to produce materials in standard forms such as plates, sheets, rods, tubes and structural sections.

Primary Metal Forming Processes

- Rolling
- Forging
- Extrusion
- Tube and wire drawing
- Deep drawing
Rolling is the most extensively used metal forming process and its share is roughly 90%  
☐ The material to be rolled is drawn by means of friction into the two revolving roll gap.  
☐ The compressive forces applied by the rolls reduce the thickness of the material or change its cross sectional area.  
☐ The geometry of the product depends on the contour of the roll gap.  
☐ Roll materials are cast iron, cast steel and forged steel because of the high strength and wear resistance requirements.  
☐ Hot rolls are generally rough so that they can bite the work, and cold rolls are ground and polished for good finish. In rolling the crystals get elongated in the rolling direction. In cold rolling crystals more or less retain the elongated shape but in hot rolling they start reforming after coming out from the deformation zone.  
☐ The peripheral velocity of rolls at entry exceeds that of the strip, which is dragged in if the interface friction is high enough.
In the deformation zone the thickness of the strip gets reduced and it elongates. This increases the linear speed of at the exit.

Thus there exists a neutral point where roll speed and strip speeds are equal. At this point the direction of the friction reverses.

When the angle of contact $\alpha$ exceeds the friction angle $\lambda$, the rolls cannot draw fresh strip.

Roll torque, power etc. increases with increase in roll work contact length or roll radius.

![Fig. 2 Various Roll Configurations](image)

(a) Two-high (b) Three-high (c) Four-high (d) Cluster mill (e) Tandem mill
Forging is perhaps the oldest metal working process and was known even during prehistoric days when metallic tools were made by heating and swaging.

Forging basically involves a plastic deformation of material between two dies to achieve desired configuration. Depending upon a complexity of the part, forging is carried out as open die forging and closed die forging.

In the open die forging, the metal is compressed by repeated blows by a mechanical swage and shape is manipulated manually.

In closed die forging, the desired configuration is obtained by squeezing the workpiece between two shaped and closed dies. On squeezing the die cavity gets completely filled and excess material comes out around the periphery of the die as flash which is later trimmed.

Press forging and drop forging are two popular methods in closed die forging.
In press forging the metal is squeezed slowly by a hydraulic or mechanical press and the component is produced in a single closing of die, hence the dimensional accuracy is much better than drop forging.

Both open and closed die forging processes are carried out in hot as well as in cold state.

In forging favorable grain orientation of metal is obtained.

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Direct and Indirect Extrusion

In direct extrusion the metal flows in the same direction as that of the ram. Because of the relative motion between the heated billet and the chamber walls, the friction is severe and is reduced by using molten glass as a lubricant in case of steels at higher temperatures. At lower temperatures, oils with graphite powder are used for lubrication.

In indirect extrusion process metal flows in the opposite direction of the ram. It is more efficient since it reduces friction losses considerably. The process, however, is not used extensively because it restricts the length of the extruded component.
Large quantities of wires, rods, tubes and other sections are produced by the drawing process which is basically a cold working process. In this process the material is pulled through a die in order to reduce it to the desired shape and size.

In a typical wire drawing operation, once end of the wire is reduced and passed through the opening of the die, gripped and pulled to reduce its diameter.
Tube drawing

- Tube drawing is also similar to the wire drawing, except that a mandrel of appropriate diameter is required to form the internal hole.
- Here two arrangements are shown in figure (a) with a floating plug and (b) with a moving mandrel.
- The process reduces the diameter and thickness of the tube.

Deep Drawing

- This operation is extensively used for making cylindrical shaped parts such as cups, shells, etc. from sheet metal.
- As the blank is drawn into the die cavity compressive, stress is set up around the flange and it tends to wrinkle or buckle the flange.
Material forming – principle types of primary forming.

**Principle forming technologies** – rolling, forging, extrusion, tube and wire drawing and deep drawing.

**Rolling** – a number of rolls, directions of travel, a length of deformation zone, a structure of rolled material, continuous rolling, manufacture of threads, rings flaring, punching by oblique rolling.

**Forging** – free forging, stamping, die forging.

**Extrusion** – direct, indirect, a bin, a ram, friction conditions.

**Drawing** – wires, tubes, rod drawing and floating plug.

**Deep drawing** – material, a blank holder.

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1. Explain the reasons why the material forming belongs to the basic finishing processes of the production.
2. Name basic types of obtained semi-finished products forming.

3. Describe and draw the basic figures of the individual forming technologies.

4. Describe why the work flow of the rolls occurs (a friction force?) during the rolling process.

5. What happens with the material structure during the rolling process?

6. Is the material speed of the strip from the rolls the same as the one identical to the peripheral speed of the rolls?

7. Try to describe another special forming technologies based on the rolling.

8. Describe the difference between free forging and stamping.

9. What is the direct and indirect extrusion? Find some examples from civil life on which you could explain it.

10. Which technological terms do pop in your mind when it is said the drawing process?

11. Do you know any product which has been apparently made by the deep drawing process?

1. **Application of stress while rolling**

   **Time to study:** 25 hours

   **Aim**  The chapter is the essential part of forming theory. It will be divided into subchapters and further to subparts. If it is necessary, there will be solved tasks, eventually, the terms’ summary.

   You will be introduced with the options of an application of forces to a body in space. A possibility of another mathematic application results from a cut method. You will understand a way of stress determination in coordinate planes, its stress distribution to normal and tangential stress and their quantities. The basic equation composition is based on the balance equations and you will be able to compose them in selected directions. You
will find out that you must include even the tangential stress under determined angles into the resulted partial equations. In the end, you will derive the result normal stress coming from the center and it is perpendicular to an octahedral plane. Gradually, you will come from the general coordinate system to the main coordinate system and the issue will become easier. You will derive a main normal stress and draw a stress ellipsoid. Apparently, you will get familiar with a main tangential stress. You will understand how to make the Mohr’s circles. The stress is presented as a tensor which might have invariants. You will be able to express the stress in the octahedral plane. You will be able to express the stress intensity mathematically and graphically, both ways. You will understand a meaning of a stress condition indicator and express a surface and space stress condition. You will also learn about differential equations of stress in rectangular, cylindrical and spherical coordinate systems. You will derive an equilibrium equation for flat rolling and, consequently, a composition of deformation resistance longitudinal a deformation zone.

1.1 Stress and tensity

At the beginning of the forming theory we meet with two terms. A stress and a tensity. In this case, we understand the tensity as a state of body being under the application of force and stress; it is a magnitude defined as a ratio of force to surface and afterwards we will intensively deal with the magnitude. As the tensity we understand and identify the state of body which was made by the means of force application to an examined material. These forces might be inner or outer. Finally, the tensity causes a deformation in the body. The kinds of forces, especially the outer ones, are mostly caused by tools, eventually friction forces. The forces are transferred to the body from the tools actively by means of loading, while the friction forces follow the active actions. Outer forces and their application cause a reaction in the body, which is displayed by the change of the body shape, as was already said. The inner forces occur. It is important to understand a size of these forces and their location. The time
is also an essential part of the game. The time course might be static or dynamic. The issues of forces investigation are solved currently by the cut methods.

Let’s come back to the application of outer forces. They might be surface ones and/or volume ones. They might be centralized evenly or spread unevenly. The volume forces are these, which are applied to all body points and they are related to the weight, as is a gravity force. We avoid the application of such forces in practice. Within the study of stress and tensity conditions we assume that the body is isotropic, i.e. it has the same structure and characteristics features in all directions. We assume that the body is continuous and homogeneous. If the system of the various points is in equilibrium, we assume that also the outer forces are in equilibrium in the system. It is a principle or a definition of a rigid body. The equilibrium might exist in such a rigid elastic state under various ratio of outer forces. On the contrary, it is necessary to determine relationships and forces’ size in a plastic relationship. The cut methods mean to make a virtual cut through the selected body. We fit the plane and half-plane, which are characterized by a n-normal towards the selected coordinate system through the point of the body. Although, we are going to learn about the other coordinate systems during study, as a principle, we will use a rectangular coordinate system with x, y and z axes.

In Fig. 9, in the middle of the small flat, there is fitted a plane, which normal is identified by a vector n. The outer forces F applied to the body are compressive and tractive ones. The force is the vector and that is why it is identified not only by the point of action but also its direction. If it is related to the
compressive forces, the arrow points inside the body, the tractive forces are identified by the arrow pointing outside of the body. The flat around A point is identified as \( \Delta S \). A resultant of a force flow \( F_1, F_2 \) up to \( F_n \) and then \( \Delta F' \) applies towards the flat \( \Delta S \). The forces quantity projected to the stress (or rather tensity) in point A is determined by the intensity, i.e. a force per a cut flat unit as stated in the equation 1.

\[
p = \frac{\Delta F'}{\Delta S}
\]

(1)

\[
\sigma = \frac{\Delta F'}{\Delta S}
\]

(2)

\[
p = \sigma = \lim_{\Delta S \to 0} \frac{\Delta F'}{\Delta S}
\]

(3)

The identification \( p \) represents a pressure in the equation. The value \( p \) is quite often used as a pressure expressed in the ratio of force/flat (explanation: the force unit is N; the flat unit is \( m^2 \), \( p \), or rather, the stress \( \sigma \) is actually Pascal). Because of a fact the unit is too large, in the field of material forming there is used a unit MPa, which is \( 10^6 \) Pascal and it is N/mm\(^2\). We assume the uniformly distributed stress, because it is the vector. The stress vector represents a general orientation towards the examined flat and it is usually distributed into the normal and tangential directions. Each point in the body is connected with the other points and therefore a stress with a certain quantity and the direction will be applied towards a random flat.

In mechanics, there is a known term of an absolute rigid body. It is not deformed by the application of outer forces and transfers the application to the other bodies. In this case, we are able to move the point of application to any point in the direction of its vector. In contrary, the application of outer forces depends on the point of application at plastic bodies. If we transfer the point of application in the direction of its forces, its application to the body will change. During the deformation processes which are the principle of technological forming processes, the plastic material and the body are exposed to shape and dimensions modifications, but the total body volume is not changed. If the outer forces are applied they will produce the elastic and plastic deformations in the body. If the outer forces
stop their application, the elastic deformations disappear and the plastic ones remain. In the technical deliberations, there is also added the deliberation of various modifications of the rigid body:

a) It might be an ideally elastic body which is deformed only elastically by the application of outer forces. The entire energy which is applied for the deformation and it is accumulated in the body as a potential energy increases the level of the energetic condition. The body is under the energetic unequal condition after the deformation. If the force stops its application, the body will return to its original energetic equal condition. It is possible to make a general assumption that between the stress caused by the application of outer force towards the body and the deformation there is a linear functional relationship in terms of Hook’s Law. There is the linear relationship between the pressure and the deformation. Additionally, the deformation does not depend on the loading time.

b) Ideal plastic body. If the stress caused by the application of outer forces reaches a certain limit quantity, the induced plastic deformation keeps the pressure in the mentioned limit quantity. There is no metal strengthening. The limit quantity of the tensity needed for the induction of the plastic condition does not depend on the deformation speed. It is possible to express such a relationship between the pressure and the deformation mechanically by a constant pressure value, or rather the value of pressure intensity.

c) Real rigid body. It is the contrast to the ideal rigid body and it is expressed by the first elastic deformation by means of outer forces application and if the certain level is reached, it is deformed permanently, i.e. plastically. In the real body the plastic deformations are always accompanied with the elastic deformations, which are minor in case of metal while real formed and mostly insignificant and they disappear, if the outer forces stop their application. We derive the deformation of real rigid body from the pressure equilibrium equations of notional elementary particles, whereas the time flow of the deformation might be understood as a space transfer of these elementary particles. They have the same physical characteristics as the forming body and they continuously fill out its entire volume. So, they create a kind of continuum. From the mechanics point of view, we identify these particles as a sort of physical points. The examination of tensity condition and deformation is actually a detection of these conditions in mentioned physical points. From the kinematics point of view, we notice a travel of elementary physical particles and a speed of their movement along their travels. Within mathematic examination of tensity condition and deformation in the physical point of the body we assume that such a point represents the average values of physical characteristics of the body, the body is homogenous, i.e. isotropic, close to the structure including the mechanical characteristics of metals,
which are the crystalline substances; it is a kind of static average of mechanical characteristic of crystalline grains.

1.2 Stress on coordinate planes

On the flats of general tetrahedron, later we will imagine a cube, in the general system of x, y, z axes, which do not have to form a right angle (again, to make it easier, we consider the rectangle coordinate system), we will illustrate the elements of perpendicular stress; they are three of them. On the flats there are also tangential stress, they are 6 of them. (The tangential stresses which point to the common center from the sites are called combined and they have the same quantity). Caution! The stress is the vector; so, it might have the same quantity but usually the different place of application and the direction.

![Image of stress distribution on tetrahedron](image)

Fig. 10 Principle of stress distribution on tetrahedron

1.3 Tensity in general plane

- The physical point A lies in the triangle flat identified by letters A, B, C. It is the octahedral plane characterized by the normal n. The direction angles $\alpha_x$, $\alpha_y$, $\alpha_z$ are the angels between the normal and axes x, y, and z. The vector $\sigma_n$ lies in the direction of the normal, in words the
normal stress. The vector of actual stress $\tau_n$ lies in the given octahedral plane and it is perpendicular to the direction $\sigma_n$.

- The final value $p$ is actually the stress and it is a vector summary of $\sigma_n$ and $\tau_n$.
- The stress elements are indicated in Fig. 11.

Although we have the general coordinate system here, in the future we will try to use main axes and there the stress will be indicated by the settlement based on the equation (4).

We will indicate the flat $\Delta S$ as the size 1 and other flats have the sizes based on the equation (5).

\begin{align}
\delta_1 &> \delta_2 > \delta_3 \\
\Delta S_x &= \Delta S \cdot \cos \alpha_x \\
\Delta S_y &= \Delta S \cdot \cos \alpha_y \\
\Delta S_z &= \Delta S \cdot \cos \alpha_z
\end{align}  

---16---
The tetrahedron flats are identified as $\Delta S_x$, $\Delta S_y$, $\Delta S_z$ and they are determined by the mentioned equation. We get the values of $p_x$, $p_y$, $p_z$ (to make it easier, we will use the values $p_1$, $p_2$, $p_3$ in the future) by the marginal projection of the final vector $p$ back to the individual directions.

To get the final summary we create an equation of force equilibrium in the individual directions as a summary of all stress multiplied by given flats. The consequent mathematical relationship in all three axes expresses that it is necessary the forces in the individual axes to equal 0 to get the equilibrium.

\[
\begin{align*}
\Sigma_x &= p_x \cdot \Delta S_x - \delta_x \cdot \Delta S_x - \tau_{xy} \cdot \Delta S_y - \tau_{xz} \cdot \Delta S_z = 0 \\
\Sigma_y &= p_y \cdot \Delta S_y - \delta_y \cdot \Delta S_y - \tau_{yx} \cdot \Delta S_x - \tau_{yz} \cdot \Delta S_z = 0 \\
\Sigma_z &= p_z \cdot \Delta S_z - \delta_z \cdot \Delta S_z - \tau_{zx} \cdot \Delta S_x - \tau_{zy} \cdot \Delta S_y = 0
\end{align*}
\] (6)

From this equation it is possible to determine mathematically the values of given elements $p_x$, $p_y$, $p_z$ mathematically in the following configuration.

\[
\begin{align*}
p_x &= \delta_x \cdot \cos \alpha_x + \tau_{xy} \cdot \cos \alpha_y + \tau_{xz} \cdot \cos \alpha_z \\
p_y &= \tau_{yx} \cdot \cos \alpha_x + \delta_y \cdot \cos \alpha_y + \tau_{yz} \cdot \cos \alpha_z \\
p_z &= \tau_{zx} \cdot \cos \alpha_x + \tau_{zy} \cdot \cos \alpha_y + \delta_z \cdot \cos \alpha_z
\end{align*}
\] (7)

For a notation, there is a generally accepted statement that the stress values $\sigma_x$, $\sigma_y$, $\sigma_z$ lie on the diagonal of these three equations mentioned below. These three linear equations represent a general condition of tension. The final resultant is determined by the vector summary as follows

\[
p^2 = p_x^2 + p_y^2 + p_z^2
\] (8)

A very significant magnitude in this field is the quantity of normal stress $\sigma_n$. As has been said, it is a normal line to the octahedral plane (the octahedral is the space body composed of 8 equilateral triangles; we only analyze one of them). The value $\sigma_n$ of normal stress is sometimes indicated as $\sigma_8$ or $\sigma_{okt}$. We can express the value of normal stress as the summary of the projection of forces application towards the components $p_x$, $p_y$, $p_z$ to the direction of the normal $n$. So, the equation is as follows

\[
\delta_m = p_x \cdot \cos \alpha_x + p_y \cdot \cos \alpha_y + p_z \cdot \cos \alpha_z
\] (9)
In the practical application of the equation we usually avoid the values of tangential stress, especially at the transition to the system of main axes 1, 2, 3.

Another magnitude which we need to find out is the quantity of tangential stress $\tau_n$, which we easily calculate from the knowledge of total resultant $p$ and investigated volume $\sigma_n$, by means of the equation (11) and Fig. 12.

\[
\delta_n = \delta_x \cos^2 \alpha_x + \delta_y \cos^2 \alpha_y + \delta_z \cos^2 \alpha_z + 2 (\tau_{xy} \cos \alpha_x \cos \alpha_y + \tau_{yz} \cos \alpha_y \cos \alpha_z + \tau_{zx} \cos \alpha_z \cos \alpha_x) 
\]

\[
p^2 = \delta_n^2 + \tau_n^2 \\
\tau_n^2 = p^2 - \delta_n^2
\]
1.4 Main normal stress

In each point of the body (if we are still in the normal state) we can find three reciprocal perpendicular flats where the normal stress only operates and the shear stresses equal 0. These stresses are called the main normal stresses and flats, where these stresses operate, are main flats and the directions of the main normal stresses are the main ones. We indicate them as the main axes 1, 2, 3. We initiate the identification of coordinate system to reach the same direction of 0x (the center → a direction of axis x) with the direction of the stress $\sigma_1$. If we slightly swivel the coordinate system to comply with the following equation from which the value of resultant is in form of components and consequently the value of the main normal stress $\sigma_n$ is logically indicate as $p_n$ – see Fig. 13.
If we slightly swivel the coordinate system having it parallel to the main directions then the only main, normal stress operates in the given planes determined by the main axes. From above stated it implies that the tensity condition in the body point is fully determined by the direction of three main axes and the volumes of the main stresses are indicated as follows

\[
\begin{align*}
\sigma_1 & = \delta_1 \\
\sigma_2 & = \delta_2 \\
\sigma_3 & = \delta_3
\end{align*}
\]

In parallel with the normal \( n \) we create the quantity of so called radius vector based on the equation

\[
\begin{align*}
p_n &= \delta_1 \cos^2 \alpha_1 + \delta_2 \cos^2 \alpha_2 + \delta_3 \cos^2 \alpha_3 \\
\delta_n &= \delta_1 \cos^2 \alpha_1 + \delta_2 \cos^2 \alpha_2 + \delta_3 \cos^2 \alpha_3 \\
\tau_n &= \delta_1 \cos^2 \alpha_1 + \delta_2 \cos^2 \alpha_2 + \delta_3 \cos^2 \alpha_3 - \\
&\quad - (\delta_1 \cos^2 \alpha_1 + \delta_2 \cos^2 \alpha_2 + \delta_3 \cos^2 \alpha_3)^2
\end{align*}
\]
consequently, there is the projection of vector $r$ to axes $x, y,$ and we are coming back to the general coordinates; in fact, these are the coordinates of the final vector $r$ based on the given equation. The final resultant is the second-order one; it is curved flat described from the center of coordinates. However, if we change the flat of general axes $x, y, z$, the final radius always lies on the surface plane and determines the stress condition (Cauchy).

$$\cos \alpha_x = \frac{x}{r} \quad \cos \alpha_y = \frac{y}{r} \quad \cos \alpha_z = \frac{z}{r}$$

$$c = \delta u \cdot r^2 = \delta x \cdot x^2 + \delta y \cdot y^2 + \delta z \cdot z^2 + \ldots$$

1.5 Stress ellipsoid

Let’s take the theoretical assumption that between the angle of normal $n$ and the main direction of 1, 2, 3 there are the angles $\alpha_1, \alpha_2, \text{and} \alpha_3$.

Fig. 14 Stress ellipsoid
These angles might reach different values, so we have three options, in total.

\[ \alpha_1 = 0 \Rightarrow \cos \alpha_1 = 1 \quad \alpha_2 = \alpha_3 = 90^\circ \Rightarrow \cos \alpha_2 = \cos \alpha_3 = 0 \]

\[ p_1 = \delta_1 \cdot \cos \alpha_1 \quad p_2 = \delta_2 \cdot \cos \alpha_2 \quad p_3 = \delta_3 \cdot \cos \alpha_3 \]

\[ p_1 = \delta_1 \quad p_2 = p_3 = 0 \]

- Under the first condition the result is \( p_1 = \sigma \).
- Under the second and third conditions \( p_2 = \sigma_2 \) and \( p_3 = \sigma_3 \).

Because of the fact it is possible to change the flats positions that in total together with the direction and stress in the flats, the final point of the vector \( r \) radius reaches a certain geographical surface and the terms and conditions comply with the following equations (20-23).

\[ p_1^2 = \delta_1^2 \cdot \cos^2 \alpha_1 \quad p_2^2 = \delta_2^2 \cdot \cos^2 \alpha_2 \quad p_3^2 = \delta_3^2 \cdot \cos^2 \alpha_3 \]

\[ \left( \frac{p_1}{\delta_1} \right)^2 = \cos^2 \alpha_1 \quad \left( \frac{p_2}{\delta_2} \right)^2 = \cos^2 \alpha_2 \quad \left( \frac{p_3}{\delta_3} \right)^2 = \cos^2 \alpha_3 \]

\[ \left( \frac{p_1}{\delta_1} \right)^2 + \left( \frac{p_2}{\delta_2} \right)^2 + \left( \frac{p_3}{\delta_3} \right)^2 = \cos^2 \alpha_1 + \cos^2 \alpha_2 + \cos^2 \alpha_3 = 1 \]
Under the condition that the space plane is the space ellipsoid, it is the stress area with the half-axes \( \sigma_1 \), \( \sigma_2 \), \( \sigma_3 \) and the tension in the point is determined by the stress ellipsoid – Fig. 14. If we, for example, change the value \( p \) with changed outer loading, the values \( p_1 \), \( p_2 \), \( p_3 \) will change too together with the radiuses and \( \sigma_1 \), \( \sigma_2 \), \( \sigma_3 \). (Usually, as mentioned in the beginning, let’s keep the statement of \( \sigma_1 > \sigma_2 > \sigma_3 \)). There might occur special cases, i.e. a spherical ellipsoid, a globe, the space transforms to the flat and finally it is an abscissa under the stated conditions of the main stress quantity.

### 1.6 Summary of basic equations

The main tangential stress lies in the octahedral plane and it is perpendicular to the main octahedral stress \( \sigma_n \).

If we consider the option of \( \alpha_1 = \alpha_2 = \alpha_3 \) and summarize them we get the equation as follows (the relationship for \( \cos \alpha = \frac{1}{\sqrt{3}} \)), and then \( \alpha \approx 55^\circ \)

\[
\delta_4 = \delta_{0\, \text{kr}} = \delta_8 = \delta_1 \cdot \cos^2 \alpha_1 + \delta_2 \cdot \cos^2 \alpha_2 + \delta_3 \cdot \cos^2 \alpha_3 = \\
= \cos^2 \alpha (\delta_1 + \delta_2 + \delta_3) = \frac{\delta_1 + \delta_2 + \delta_3}{3} \tag{24}
\]

\[
\delta_4 = \frac{\delta_1 + \delta_2 + \delta_3}{3} = \frac{\delta x + \delta y + \delta z}{3} \tag{25}
\]

\[
\delta_1 > \delta_2 > \delta_3
\]
The angle is the gradient of the normal stress towards the coordinate main axes. Mathematically the value follows up in the equation (24). The resulted equation comes from a proper tensity analysis of the stress conditions of given selected elementary body. To reach it was quite complicated. Finally, it is quite simple arithmetic average of three main stresses.

1.7 Mohr’s circles

One of the options how to illustrate the mutual application of three stresses towards the plane is the Mohr’s circles. We apply the stress quantity on the positive (or negative axe x) and by means of semicircles we determine the volumes of the tangential stress. The largest tangential stress based on the equation (26) is between the largest and the smallest main stresses.

The illustration of all 3 main stresses is possible to transfer into the plane by this method where the normal stress in on the axe x and the tangential stresses on the axe y. After that it is possible to create the semicircles, see the Fig. 14. The given example, chosen only with 3 main positive and/or tensile stresses, is quite unique, as we see later; the Mohr’s circles might be used for any combination of stresses, their volumes and directions and/or tensile or compressive ones. The tangential stress is between the points of stress 1, 3.
\[
\tau_{1,3} = \frac{\sigma_1 - \sigma_3}{2} = \tau_n
\]

Fig. 14 Mohr’s circles
1.8 Formation of plastic deformation in uniaxial axes application

The preview chapters were theoretical and mathematical and we assume it is necessary to deal with a kind practical application as well; therefore in the following part we are going to imagine a quantity of necessary tangential stress causing the plastic deformation in the uniaxial application of forces.

In the Fig. 15, the cylindrical body is loaded by the force $F$ in axe and gradually the given stresses occur and operate in the flats $S_1$ and $S_2$. In general inclined plane $S_2$ the stress $\tau$, causing the plastic deformation, lies towards the axe of force application.

\[ S_2 = \frac{S_1}{\cos \alpha} \quad \frac{F}{S_2} = \frac{F \cdot \cos \alpha}{S_1} \]  
\[ \tau_0 = \frac{F}{S_1} \cdot \cos \alpha \cdot \cos \phi \]  
\[ \delta = \frac{F}{S_2} \quad S_2 = \frac{S_1}{\cos \alpha} \quad \delta = \delta_1 \cdot \cos \alpha \]

Our aim is to investigate the stress $\tau$ in form of $\tau_{\text{max}}$ and also determine the angle of the plane inclination which is going to be in action towards the applied force. We will use following
relationships. We will take an advantage of following simple relationships. We will perform the cut of front view. We will decompose the vector $\sigma$ into values $\sigma_{\alpha}$ and $\tau_{\alpha}$ and count their maximal quantities – Fig. 16 and 17.

![Cut of cylinder](image1)

**Fig. 16 Cut of cylinder**

![Stress distribution](image2)

**Fig. 17 Stress distribution**

\[
\delta_{\alpha\max} = \delta \cdot \cos \alpha = \delta_1 \cdot \cos^2 \alpha
\]

\[
\tau_{\alpha\max} = \delta \cdot \sin \alpha = \delta_1 \cdot \cos \alpha \cdot \sin \alpha = \frac{1}{2} \delta_1 \cdot \sin 2\alpha
\]
The plastic deformation in uniaxial application of forces is caused by the stress $\tau_{\text{max}}$, which equals to the half of the normal stress quantity and the plane where is the plastic deformation inclines towards the axis of applied forces of 45°.

\[
\tau_{\text{max}} = \frac{\sigma_1}{2} \quad \text{(34)}
\]

\[
\tau_{\text{max}} = \frac{\sigma_1}{2} \quad \text{(35)}
\]

1.9 Tensorial expression of tensity

The condition tensity in the point of deformed body is defined by 9 stresses, from which 3 are normal and 6 shear including 3 associated shear stresses. It is possible to illustrate these 9 magnitudes in the mathematic notation as a tensor (a magnitude, which is constant in the given space and it is the only one, called a scalar; it is, for example, a temperature in the closed room; the magnitude which has 3 values, i.e. a point of action, a quantity, a direction. It is called a vector and it is for example a speed of body travel, but also a force and stress). If there are more magnitudes, specifically 9, we can indicate the mathematic notation as a tensor.

\[
T_6 = \begin{pmatrix}
\delta_x & \tau_{\text{xy}} & \tau_{\text{xz}} \\
\tau_{\text{yx}} & \delta_y & \tau_{\text{yz}} \\
\tau_{\text{zx}} & \tau_{\text{zy}} & \delta_z
\end{pmatrix} \quad \text{(36)}
\]
In the square matrix there are centralized the values of normal stresses along the main diagonal; in the rows there are the values of stress in direction of the given axe, in the columns there are stresses on walls and flats of given elements. It is necessary to express the general tensor of stress by following figure.

\[
T_6 = \begin{bmatrix}
\delta_1 & 0 & 0 \\
0 & \delta_2 & 0 \\
0 & 0 & \delta_3
\end{bmatrix} \quad K_6 = \begin{bmatrix}
\delta_8 & 0 & 0 \\
0 & \delta_8 & 0 \\
0 & 0 & \delta_8
\end{bmatrix}
\]

The tensors are possible to summarize, subtract, multiply and divide. It is possible to express also a spherical tensor. If the tensity condition is defined only by the main stresses, the stress tensor will become simpler to, so called, tensor of main stresses, which expresses the scheme of the main stresses. Although, in the preview chapter we derived the value \( \sigma_n \) as a value summarizing three stresses \( \sigma_1 + \sigma_2 + \sigma_3/3 \) and called it as a kind of \( \sigma \) normal, \( \sigma \) octahedral, from the point of arithmetic a potential middle stress, the value practically shows that it does not have any significant affection to the metal transmission to the plastic state. The transmission is better to monitor in the point of tensity and it composed of two parts: the equal versatile stress, or rather tension, which results from the stress coming from \( \sigma_n \) and of the second part, which composes of a tensity deviator and a tensor of the main stresses. Then it is possible to break it down into the figure of the spherical tensor, which has the same meaning, the quantity and the tensity deviator in all directions.

\[
D_\delta = \begin{bmatrix}
\tau_{xx} & \tau_{xy} & \tau_{xz} \\
\tau_{yx} & \tau_{yy} & \tau_{yz} \\
\tau_{zx} & \tau_{zy} & \tau_{zz}
\end{bmatrix} \quad D_\delta^\prime = \begin{bmatrix}
d_1 & 0 & 0 \\
0 & d_2 & 0 \\
0 & 0 & d_3
\end{bmatrix}
\]

From this definition and the breakdown it implies that the value \( \sigma_n \) represents the material resistance towards the change of body volume. The resultant mathematical relationships for the stress tensor distribution to the spherical tensor and the stress deviator are written in both figures, in the general system as well as the system of main axes – equations from (38) up (41).
From the mathematic point of view the important characteristics is resulted; the summary of stress deviators equals to zero (42).

\[
\sigma_1 + \sigma_2 + \sigma_3 = d_1 + d_2 + d_3 = 0
\]
The preview picture represents the general tensity condition in the left cube, the middle cube represents the spherical tensor result and the last cube in right represents the deviator scheme. To make it simple, the positive values are chosen in case of the tensity condition tensor to get a very simple result of $\sigma$ normal value, i.e. $\sigma_8$. In the bottom part of the Figure the values $\sigma_1$, $\sigma_2$, and also $\sigma_3$, representing the tensor of main stress are gradually applied in all horizontal axes 1, 2, 3 from the center of coordinates 0. It equals to the spherical tensor and the value of tensity deviator value is added to it. So, the spherical tensor affects only the quantity of stress. Actually, it is the versatile space compression and tension and it does not cause the change of shape. It might be understood as the only small volume changes in the area of plastic deformation. The stress deviator, which simultaneously manages the tensity quality, determines the tensity characteristics. The body changes its plastic shape permanently without volume change from the point of various tensity deviator conditions. Thanks to previously determined value of $\sigma_8$ in form of $\sigma_8 = (\sigma_1 + \sigma_2 + \sigma_3)/3$ the summary on of tensity deviator, in general or main formula, equals 0.

1.10 Invariance of tensity condition

In the chapter we are going to explain the dependency of the tensity condition on three-dimensional orientation. We select the basic coordinate system in axes 1, 2, 3 together with the new system in axes $x$, $y$, $z$.

New and old axes are going to make the angles of $\beta_1$, $\beta_2$, $\beta_3$. The main components of normal stresses are possible to express by these equations and consequently to break them down into the general formula.

\[
\begin{align*}
\delta n_x &= \delta n \cdot \cos \beta_1 \\
\delta n_y &= \delta n \cdot \cos \beta_2 \\
\delta n_z &= \delta n \cdot \cos \beta_3
\end{align*}
\]

\[
\begin{align*}
\delta n_x &= \delta n \cdot \cos \beta_1 + \tau_{xy} \cdot \cos \beta_2 + \tau_{xz} \cdot \cos \beta_3 \\
\delta n_y &= \delta n \cdot \cos \beta_2 + \tau_{yx} \cdot \cos \beta_1 + \tau_{yz} \cdot \cos \beta_3 \\
\delta n_z &= \delta n \cdot \cos \beta_3 + \tau_{zx} \cdot \cos \beta_1 + \tau_{zy} \cdot \cos \beta_2 + \delta_2 \cdot \cos \beta_3
\end{align*}
\]

The solution of independence relies on the putting the matrix including nine stresses to 0.
The solution is 9 equations being the subject of the matrix calculation rules. As the result there is stated the equation of given formula, which has 3 radices \( \sigma_x, \sigma_y, \sigma_z \), which are always real and they are determined by the means of a coefficient depending on angles \( \beta_1, \beta_2, \beta_3 \), thus independent on the three-dimensional orientation. The first tensor linear invariant of the main stress after the tensor of general tension condition has the formula as follows:

\[
\begin{bmatrix}
\delta_x - \delta_1 & \tau_{xy} & \tau_{xz} \\
\tau_{yx} & \delta_y - \delta_2 & \tau_{yz} \\
\tau_{zx} & \tau_{zy} & \delta_z - \delta_3
\end{bmatrix} = 0
\]

\[
-I_1(T6) = \delta_x^3 + I_1(T6) \cdot \delta_x^2 - I_2(T6) \cdot \delta_x + I_3(T6) = 0
\]

The first tensor linear invariant of the main stress after the tensor of general tension condition has the formula as follows:

\[
I_1(T6) = \delta_x + \delta_y + \delta_z = \delta_1 + \delta_2 + \delta_3 = \text{const}
\]

If we remember the so-called octahedral stress \( \sigma_n \) or rather \( \sigma_8 \) we can see the certain analogy and the right linear invariant has the meaning of some middle main normal stress and expresses the resistance towards to change of body volume.

The quadratic invariant of second-order is a subdeterminant after the distribution of the third-order determinant to three second-order determinants, generally it is expressed as follows:

\[
I_2(T6) = \begin{vmatrix}
\delta_x & \tau_{xy} \\
\tau_{yx} & \delta_y
\end{vmatrix} + \begin{vmatrix}
\delta_y & \tau_{yz} \\
\tau_{zy} & \delta_z
\end{vmatrix} + \begin{vmatrix}
\delta_z & \tau_{zx} \\
\tau_{zx} & \delta_x
\end{vmatrix} =
\]

\[
= \delta_x \delta_y + \delta_y \delta_z + \delta_z \delta_x - \tau_{xy}^2 - \tau_{zx}^2 - \tau_{yz}^2 =
\]

\[
= \sigma_n \delta_x + \sigma_z \delta_y + \sigma_y \delta_z = \text{const}
\]

It has been for the first time in the theory that we are getting to the term which has a certain connection with the intensity of stress and/or the intensity of tensity condition which expresses the resistance towards the change of body shape.
Finally, the invariant of third-order cubic determines the position of a tangible component in the plane perpendicular to the normal stress. It is closed to the value of a later defined tensity condition. It determines a kind of deformation based on the tensity condition (tension, compression and shear).

\[ I_3(\tau) = \delta_x \cdot \delta_y \cdot \delta_z + 2 \left( \tau_{xy} \cdot \tau_{yz} \cdot \tau_{zx} \right) - \]
\[ - \delta_x \cdot \tau_{y2}^2 - \delta_y \cdot \tau_{z2}^2 - \delta_z \cdot \tau_{x2}^2 = \]
\[ = \delta_1 \cdot \delta_2 \cdot \delta_3 = \text{konst}. \quad (49) \]

1.11 Stress in octahedral plane

The sub-chapter will require your own preparation. We can see the octahedral plane and the vector of normal stress \( \sigma_n \) in the picture – in words the octahedral normal stress. (Note: The stress vector is also called and indicated as \( \sigma_B \), sometimes also \( \sigma_{okt} \). Why? Because there are eight octahedral planes in total, four above the plan view, four below it and they create together the solid body composed of eight equilateral triangles, i.e. the octahedral. Do you know other solid bodies composed of the regular plane-spherical diagrams?). Let’s assume that the stress vector makes the same angle with every axe, i.e. all alphas are equal. Derive yourself \( \cos \alpha = \ldots \) and you reach the expression for the octahedral normal stress.
1.11.1 Tangible octahedral stress

Based on the knowledge of resultant $p$ and the normal stress $\sigma_n$ we can calculate the tangible stress based on the following formula. We will introduce only a beginning of the calculation and a final formula.

\[
\tau_n^2 = p^2 - \delta_n^2
\]

\[
\tau_n^2 = \delta_1^2 \cos^2 \alpha_1 + \delta_2^2 \cos^2 \alpha_2 + \delta_3^2 \cos^2 \alpha_3 - 
\qquad - (\delta_1 \cos \alpha_1 + \delta_2 \cos \alpha_2 + \delta_3 \cos \alpha_3)^2
\]

\[
\tau_n^2 = \frac{1}{3} (\delta_1^2 + \delta_2^2 + \delta_3^2) - \frac{1}{9} (\delta_1 + \delta_2 + \delta_3)^2
\]

\[
\tau_n = \tau_8 = \pm \frac{1}{3} \left[ (\delta_1 - \delta_2)^2 + (\delta_2 - \delta_3)^2 + (\delta_3 - \delta_1)^2 \right]^\frac{1}{2}
\]

\[
\delta_1 - \delta_2 = 2 \tau_1 \quad \delta_2 - \delta_3 = 2 \tau_2 \quad \delta_3 - \delta_1 = 2 \tau_3
\]

\[
\tau_n = \tau_8 = \frac{2}{3} \left( \tau_1^2 + \tau_2^2 + \tau_3^2 \right)^\frac{1}{2}
\]

1.11.2 Graphical determination of tangible stress
In this case and another derivation including the graphical determination of other magnitudes we have to imagine the essential rectangular coordinate system 1, 2, 3 in quite different form. Let’s imagine a corner of a room with a floor, or rather a projection plan, a front wall, or rather a front view, and a side wall, or rather a side view. The octahedral plane, let’s say on the floor, in both axes and in the direction of ceiling, always reaches the same distance and an equilateral triangle is created. From the center of coordinates, thus the corner of the room, a radius vector, which is perpendicular to the octahedral plane, comes out; it is a normal and the normal stress lies inside. Let’s have a look at the normal in space and look down to the corner of the room. We see 3 axes 1, 2, 3 under 120°.

By this method we are going to solve the graphical determination of the tangible stress. First of all, it is necessary to determine the values of \( p_1, p_2, p_3 \) as a constituent of the resultant \( p \), which is given by the vector summary of the normal and the tangential stresses values.

\[
\begin{align*}
\tau_8 &= \delta_1 \cdot \Delta S_1 = \delta_1 \Delta S \cdot \cos \alpha_1 = \frac{\delta_1}{\sqrt{3}} \\
p_1 &= \frac{\delta_1}{\sqrt{3}} \\
p_2 &= \frac{\delta_2}{\sqrt{3}} \\
p_3 &= \frac{\delta_3}{\sqrt{3}}
\end{align*}
\]

(52)

Afterwards we have to project these values to the flats \( \Delta S_1, \Delta S_2, \Delta S_3 \) and calculate the values \( p_1, p_2, p_3 \) with an apostrophe based on the following formulas.

---35---
We reach the expression of the value of $\tau_{n}^{2}$, or rather $\tau_{h}^{2}$, with the application of Pythagoras triangles and the knowledge of cosines angles and we can continue with the mathematical formula as follows:

\[
\tau_{8}^{2} = \tau_{h}^{2} = \left( p_{1}' - \frac{p_{2}'+p_{3}'}{2} \right)^{2} + \left[ \frac{\sqrt{3}}{2} \left( p_{2}' - p_{3}' \right) \right]^{2}
\]

\[
\tau_{8}^{2} = \tau_{h}^{2} = p_{1}'^{2} - p_{4}'\left( p_{2}'+p_{3}' \right) + \frac{1}{4} \left( p_{2}'^{2} + 2p_{2}'p_{3}' + p_{3}'^{2} \right) + \frac{3}{4}p_{2}'^{2} - \frac{3}{2}p_{2}'p_{3}' + \frac{3}{4}p_{3}'^{2}
\]

\[
m = p_{1}' - n = p_{1}' \cdot \cos 60^\circ + p_{3}' \cdot \cos 60^\circ = \frac{P_{2}'+P_{3}'}{2}
\]

\[
\ell = p_{2}' \cdot \sin 60^\circ - p_{3}' \cdot \sin 60^\circ = (p_{2}' - p_{3}').\frac{\sqrt{3}}{2}
\]

The similar way as in case of the calculation of tangential stress we will show only the constituents and consequently the final result, which formula will be similar to the preview mathematical derivation.

\[
\tau_{8}^{2} = \tau_{h}^{2} = \pm \frac{1}{3} \left[ (\delta_{1}'-\delta_{2})^{2} + (\delta_{2}'-\delta_{3})^{2} + (\delta_{3}'-\delta_{1})^{2} \right]^{\frac{1}{2}}
\]
Later we will find out that if we want to express the total stress application, so rather tensile relationship, we choose the tensile, so called the stress intensity, which will have also the mathematical coincidence with the tensile stress.

\[
\tan \omega \tau = \frac{\tau}{\sigma} = \frac{\sqrt{3}(\sigma_2 - \sigma_3)}{2\sigma_1 - (\sigma_2 + \sigma_3)}
\]

(Reward and relax)

It is not possible to determine exactly how long did it take to get here, but you are certainly exhausted by continuously repeating equations. It is not possible to memorize them. It is quite simple mathematics; it is enough to realize where it comes from and where we want to go get. Anyway, I do think you were not being able to get here at one stroke, you had had breaks before, but you really take a long nap here. Each student is individual; so, there is no general instruction how to study such a subject. I recommend writing down the individual equations after studying the particular chapter.

1.12. Main stress schemes

Each one of the main stresses might be tractive + negative compressive – or its value might be zero. It means that we have 9 options of the main stress schemes in total and these are as follows

- 4 three-dimensional schemes (+++), (++)+, (+--) a finally (---)
- 3 two-dimensional schemes (++), (+-) a (--)  
- 2 line schemes (+), (-) 

Each stress might have the value of (+), (-) or (0). Let’s take the statement of the equation quantity ratio (4) and all stresses with some values.

We can arrange all stresses and all conditions into a certain row from overbalance tracton (on the left side) up to overbalance compression stress (on the right side of the row). You can make the row from these known values. Simultaneously, this scheme might be drawn by means of Mohr’s circles. The
interpretation of these schemes relies on the knowledge of the maximal tangible stress, which might cause a formation of plastic deformation. The significant fact is that the set up schemes of the main stresses, which are in the left part mostly traction ones, refer to a limited capability of plastic deformation while the stresses set in the right side of these schemes are the compressive ones which support larger and large plastic deformations.

The tractive stresses are mostly unfavorable; they support the initiation of material defects, especially cracks. The compressive stresses contribute to the improved elasticity, the better material forming; they allow larger plastic deformation without a cohesion rupture.

Such a fact will be cleared by the means of hypothesis of the plastic deformation in the subject of Physical foundations of the plastic deformation.

1.13. Stress deviator scheme

In the preview chapters we learnt about a fact that the tensor of main stresses is possible to split into two main parts, a spherical tensor and a stress deviator. The spherical tensor refers to the resistance towards the body size; the deviator stress refers how the body behaves under the stress application. We have preliminary found the conclusion that the body deformation might be compression, tractive or shear. We are able to differ qualitatively and judge the tensity by means of the deviator stress.

Small values \( d_1, d_2 \) or \( d_3 \) might have different marks and sizes. Theoretically and practically, 3 cases exist (the advantage is that the summary of deviator stress equals 0). Let’s imagine that we can compose the indefinite schemes of main stress tensor, but only 3 schemes of deviator stress – Fig. 21.

- The first case is the one, when we search for the largest component of deviator stress under the condition when we assume that \( d_3 > 0 \). Such a case characterizes the stress deviator scheme.
- In the second case, let’s have the value of \( d_3 < 0 \) and we reach to, so called, the compression deviator scheme. The second option is described by the following formula, mathematically it looks the same as the formula mentioned above at the first case; therefore there are the absolute values.
- Finally, in the third case \( d_3 = 0 \) we choose based on this relationship, there are left only two deviators and we reach the shear deviator scheme.
The similar way as during the examination of the normal components we can try to reach the stress deviator being invariant towards the selected axes. To make it simple, in this case we make only a matrix with the main stress deviators and the normal stress and resultant equation for the invariant of first, second and third-orders of the stress deviator. The coefficients are invariant to changes of coordinates.

\[
\begin{pmatrix}
a_1 - \delta_n & 0 & 0 \\
0 & a_2 - \delta_n & 0 \\
0 & 0 & a_3 - \delta_n
\end{pmatrix}
\]

(58)

\[-\delta_n^3 + I_{1(D6)} \cdot \delta_n^2 - I_{2(D6)} \cdot \delta_n + I_{3(D6)} = 0\]

(59)

\[
I_{1(D6)} = a_1 + a_2 + a_3 = 0
\]

(60)

\[
I_{2(D6)} = a_1 a_2 + a_1 a_3 + a_2 a_3
\]

\[
I_{3(D6)} = a_1 \cdot a_2 \cdot a_3
\]

1.15. Stress intensity

In this chapter we reach the top of our deliberations considering the stress and tension in the body under forming. The stress intensity is the summarizing application of the normal and tangible stresses. It is a kind of mathematic formulation of tensity condition. We can reach the results and the value of stress by means of the mathematical and graphical solutions. The stress intensity expresses the quantity of deformation resistance towards the change of body shape. The total potential energy, which might be indicated as $W_p$ and a potential energy of shape change $W_{tv}$ in the following equation

\[
W_p = W_o + W_{tv}
\]

(61)
The mathematical expression in the form of the following equation, similar to the tensor invariant of second-order of the main stresses allows expressing the stress intensity either by the normal stresses or by the tangible stresses.

\[
S_\delta = \delta i = \frac{1}{\sqrt{2}} \left[ (\delta_1 - \delta_2)^2 + (\delta_2 - \delta_3)^2 + (\delta_3 - \delta_1)^2 \right]^{\frac{1}{2}}
\]

(62)

\[
S_\delta = \delta i = \sqrt{2} \left[ \tau_1^2 + \tau_2^2 + \tau_3^2 \right]^{\frac{1}{2}}
\]

\[
S_\tau = \tau i = (\tau_2 + \tau_3) \left[ (\delta_1 - \delta_2)^2 + (\delta_2 - \delta_3)^2 + (\delta_3 - \delta_1)^2 \right]^{\frac{1}{2}}
\]

(63)

There is the following relationship between the quadratic invariant deviator stress and the stress intensity (63).

It is also possible to derive the intensity of tangible stress in connection with the quadratic invariant coefficient of the deviator stress.

On the left site of the following Fig. 22 there are shown cubes with the individual values of stresses. At first, it is a simple axial stress from the top to the bottom. The second scheme represents a compression stress, the third scheme is a simple space stress, the forth one is a balanced plane tension stress, the fifth scheme is shear stress, the sixth scheme represents a general option of imbalanced plane tension stress.
What is a meaning, a result of these experiments which represent a simple graphical expression of the stress intensity in the end of each row? From the imagination point of view possibly surprising, but correct from the mathematic and logic point of view is that, for example, at the balanced space tension the stress intensity is zero. As another potential surprising result is that the stress intensity is generally
the largest in the shear stress. If we wrap these schemes up we found out that the stress deviator helps to quantify the tensity relationship, which is reduced in the ordinary tension, compression and shear in spite of having unlimited numbers of combinations.

The graphical solution of the stress intensity uses the same method and the assumption which we used for the determination of the quantity of the tangible stress $\tau$ or rather $\tau_n$. Let’s imagine again the system of three main axes making the angle of 120° with the applied values of stresses $\sigma_1, \sigma_2, \sigma_3$. The individual axes (to make it simple, we assume again our statement of $\sigma_1 > \sigma_2 > \sigma_3$ and the stresses are positive in all cases. We reach the stress intensity by the means of the final point of graphical vectors composition of these three stresses together with the center of coordinates.

Fig. 23 Graphical determination of stress intensity

The angle between the axe 1 and the stress intensity is indicated as $\omega_\sigma$ and represents the location of the stress intensity vector towards the axe 1 and it expresses the tensity condition.
The second option of graphical expression of the stress intensity is, so called Rosenberg design, which results from Mohr’s circles. To make it simple, we are going to illustrate three values of stress arranged based on the quantities from the largest up to the smallest \( \sigma_1 \), \( \sigma_2 \), \( \sigma_3 \) and we are going to make the Mohr’s semicircles. The difference between the largest and the smallest stress, i.e. \( \sigma_1 - \sigma_3 \) represents the site of equilateral triangle, which we draw under Mohr’s circles – Fig. 24. The stress location \( \sigma_2 \) on the abscissa A – B represents the point of the stress intensity.

(A note for the ones who are familiar with the goniometric relationships in rectangular triangle: in Fig. 24 there is illustrated the method of th stress intensity determination). The shape of the stress intensity changes with the angle of \( \omega_\sigma \) and reaches a few extremes. If the angle is \( \omega_\sigma = 0 \), the stress intensity reaches the quantity of triangle site, i.e. \( \sigma_1 - \sigma_3 \). The same situation is when the angle \( \omega_\sigma \) is 60°. We get the minimal value of the stress intensity at the angle of 30°. Another relationship is possible to write down where \( \beta \) is, so called Lode’s coefficient and \( \beta = \frac{2}{\sqrt{3}} \approx 1,155 \).
The quantity of the stress intensity changes based on the mutual ratio of stress $\sigma_1$, $\sigma_2$, $\sigma_3$ and its directional vector also changes. In case that the values of $\sigma_1$, $\sigma_2$, $\sigma_3$ change of the same value, for example, in summary or deduction.
by some value of stress, the difference and the stress intensity do not change, only the quantity of octahedral stress changes.

1.16. **Comparison of some reached values**

In theoretical and finally the technological fields, we often define the tensity condition by means of the stress intensity. In some cases and some literature references we see also the value of tangible stress intensity, occasionally only the value of tangible stress. The following relationships are only mathematical expression of mutual dependences.

\[
\frac{S_\tau}{S_\delta} = \frac{\sqrt{2}}{\sqrt{6}} = \frac{1}{\sqrt{3}} \quad \text{and} \quad S_\delta = \sqrt{3} \cdot S_\tau \\
\frac{S_\sigma}{\tau_8} = \frac{3}{\sqrt{2}} \quad \text{and} \quad S_\sigma = \frac{3}{\sqrt{2}} \cdot \tau_8
\] (67)
1.17. Indicator of tensile condition

From the graphical expression of the stress intensity as the vector C-D based on the Fig. 24 we can see that it dependence on the marginal locations of \( \sigma_1, \sigma_3 \) but significantly the value of stress \( \sigma_2 \) is applied. The point D is movable between the points A and B and some cases might occur which we have already explained. The point D can be moved to the position of the point A, then the stress \( \sigma_1 \) is zero, the point D can be moved to the point B, \( \sigma_1 \) is the real maximal stress. In both cases the stress intensity is maximal. The ratio of ND towards AN is defined as the indicator of tensile condition based on the following relationships.

\[
\psi_\delta = \frac{ND}{AN} = \frac{\sigma_2 - \sigma_1 - \sigma_3}{\sigma_1 - \sigma_3} \tag{68}
\]

\[
ND = \delta_2 - \delta_3 - \frac{\delta_1 - \delta_3}{2} \quad \text{and} \quad AN = \frac{\delta_1 - \delta_3}{2} \tag{69}
\]

The value of the tensile condition indicator might be positive or negative. The positive one is in case of moving the point D to the right; the negative one is in case of moving the point D moving to the left. The value of tensile condition in numbers is as follows:

\[
\delta_2 > \frac{\delta_1 + \delta_3}{2} \quad \Rightarrow \quad \psi_\delta > 0 \quad (+) \tag{70}
\]

\[
\delta_2 = \frac{\delta_1 + \delta_3}{2} \quad \Rightarrow \quad \psi_\delta = 0 \tag{71}
\]

\[
\delta_2 < \frac{\delta_1 + \delta_3}{2} \quad \Rightarrow \quad \psi_\delta < 0 \quad (-)
\]

\[ -1 < \psi_\delta < 1 \]

We can express The indicator of tensile condition by these values \( d_1, d_2, d_3 \) with the knowledge of stress deviators.
1.18. Tensity conditions

1.18.1. Linear

We discussed the linear tensity condition in the preview chapters; let’s just repeat that in one-axis the stress tension on cylindrical body, the main stress is perpendicular to the prism bar $\sigma$ to flat $S_0$, and because we have already known that the deformation causes the tangible stress, we seek for its maximal value.

![Linear tension scheme](image)

Fig. 25 Linear tension scheme

The stress $\tau_a$ = one half of main stress and lies in the flat which is inclined towards the axe of applied force of 45°. We have already introduced the formation of the plastic deformation during the examination at one-axis force application.
\[ \sigma_1 = \frac{F}{S_0} \cos \alpha = \frac{S_0}{S_0 \cos \alpha} \Rightarrow S_\alpha = \frac{S_0}{\cos \alpha} \]  

(74)

\[ \sigma = \frac{F}{S_\alpha} = \frac{F}{S_0} \cdot \cos \alpha = \sigma_1 \cdot \cos \alpha \]  

(75)

\[ \tau_{\alpha\text{max}} = \sigma_1 \cdot \cos \alpha \cdot \sin \alpha = \frac{1}{2} \sigma_1 \cdot \sin 2\alpha \]  

\[ \tau_{\alpha\text{max}} = \frac{\sigma_1}{2} \]  

(76)

1.18.2. Surface

The situation at surface stress is quite difficult because of the present of two stresses. It divides into the surface stress, when the normal stresses are applied on the flats; then from Fig. 26, which follows, it is possible to form the mathematic conclusion.

\[ \tau_{\alpha_1} = \frac{1}{2} \sigma_1 \cdot \sin 2\alpha \quad \tau_{\alpha_2} = -\frac{1}{2} \sigma_2 \cdot \sin 2\alpha \]  

(77)

\[ \sigma_{\alpha_1} = \sigma_1 \cdot \cos \alpha \]  

\[ \sigma_{\alpha_2} = \sigma_2 \cdot \sin \alpha \]  

\[ \tau_{\alpha} = \frac{1}{2} \sigma_1 \cdot \sin 2\alpha - \frac{1}{2} \sigma_2 \cdot \sin 2\alpha = \frac{1}{2} (\sigma_1 - \sigma_2) \]  

\[ \tau_{\alpha\text{max}} = \frac{\sigma_1 - \sigma_2}{2} \]  

(78)

\[ \tau_{\alpha} = \frac{1}{2} \sigma_1 \cdot \sin 2\alpha - \frac{1}{2} \sigma_2 \cdot \sin 2\alpha = \frac{1}{2} (\sigma_1 - \sigma_2) \]  

\[ \tau_{\alpha\text{max}} = \frac{\sigma_1 - \sigma_2}{2} \]  

(79)

The stress \( \tau \) necessary for the formulation of the plastic deformation equals to the half of difference of two stresses. In this case, indicated as \( \sigma_1 \) and \( \sigma_2 \).
In case when the normal and shear stresses operate on flats, the situation is mathematically complicated and resulted tangible stress $\tau_{\alpha}$ in its elementary form rather consist of the half of the difference of two tensions, but also the addition of the dependency on the angle of the outer force application.

\[
\delta_{\alpha} = \frac{\delta_1 + \delta_2}{2} + \frac{\delta_1 - \delta_2}{2} \cdot \cos 2\alpha - \tau \cdot \sin 2\alpha
\]

\[
\tau_{\alpha} = \frac{1}{2} (\delta_1 - \delta_2) \cdot \sin 2\alpha + \tau \cdot \cos 2\alpha
\]

Fig. 26 Surface scheme of stress without tangible stresses
1.18.3. Space

And finally, the surface stress, which is apparently more complicated, comes up with the idea that the plastic deformation forms when the stress intensity reaches a certain size given by the mathematic formula given for its quantity.

\[ S_\delta = \delta_i = \frac{1}{\sqrt{2}} \left[ (\delta_1 - \delta_2)^2 + (\delta_2 - \delta_3)^2 + (\delta_3 - \delta_4)^2 \right]^{\frac{1}{2}} \]  \hspace{1cm} (81)
1.19. Differential equation of stress equilibrium

1.19.1. Orthogonal coordinates

For the implementation of theoretical knowledge and following calculations, especially for the computer simulation, it is necessary to implement the differential equations, which allow a calculation of stress changes including the deformation of angle points which are used in the finite element method. The following picture shows the change of stress between the point M, the center and M’ with a center on body diagonal.

The movement of $x+dx$; $y+dy$ values, or rather $z+dz$, occurs. The stress in point M’ differs of small increases. At first, we express the value of tensor stress value in point M.

$$T_{6m} = \begin{bmatrix} \delta_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \delta_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \delta_z \end{bmatrix}$$

(82)
Fig. 29 Differential tension in axe $x$ in orthogonal system

Generally, we operate in the planes of the coordinate system $x, y, z$. The following expression of stress condition tensor in point $M'$ comes to the equilibrium equations determination and they include the differential increases.

\[
\tau_{\delta m'} = \begin{bmatrix}
\delta_x + \frac{\partial \delta_x}{\partial x} \cdot dx \\
\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y} \cdot dy \\
\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z} \cdot dz \\
\delta_y + \frac{\partial \delta_y}{\partial y} \cdot dy \\
\tau_{zy} + \frac{\partial \tau_{zy}}{\partial z} \cdot dz \\
\delta_z + \frac{\partial \delta_z}{\partial z} \cdot dz
\end{bmatrix}
\]  

(83)

The set must be in equilibrium; so, if we select a direction, for example axe $x$, then we write the summary of all stresses down appearing in axe $x$ and this equals 0.
We reach the set of partial differential equations which represents the general condition of space stress.

\[
\begin{align*}
\left( \delta_x + \frac{\partial \delta_x}{\partial x} \cdot dx \right) \cdot dy \cdot dz - \delta_x \cdot dy \cdot dz + \\
\left( \tau_{xy} + \frac{\partial \tau_{xy}}{\partial y} \cdot dy \right) \cdot dx \cdot dz - \tau_{xy} \cdot dx \cdot dz + \\
\left( \tau_{xz} + \frac{\partial \tau_{xz}}{\partial z} \cdot dz \right) \cdot dx \cdot dy - \tau_{xz} \cdot dx \cdot dy = 0
\end{align*}
\]  

(84)

General equations must comply with the marginal conditions on outer surface of forming body; it is a statically indefinite condition. In total, we have 6 unknown quantities, if we consider the tangible stress as pairing. In some cases we can use the equations roughly, simplified, for example, all derivations of \( y \) equal zero and all derivations with stress of \( \tau \), in which we are, equals zero too. Subsequently, the entire set of equations becomes simple; for example in the figure for the case of solution in equation \( y, z \), i.e. from the front view.

\[
\begin{align*}
\frac{\partial \delta_x}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} &= 0 \\
\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} &= 0 \\
\frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \delta_z}{\partial z} &= 0
\end{align*}
\]  

(85)

1.19.2. Cylindrical coordinates

The differential equations of equilibrium in cylindrical coordinates
Real forming processes cannot always run the most common way, i.e. the plane deformation which is typical for the rolling of flat roll products. Some forming processes, for example, bending, punching and forging and drawing would be very difficult to put into the orthogonal coordinates. That is why we use the cylindrical coordinates sometimes. The elementary point determination in space is the set of three magnitudes indicated as $z, \varphi, r$.

![Diagram](image)

Fig. 30 Differential stress in axe $r$ of cylindrical system

The figure shows the general stress conditions including the partial differential equations which might serve, as well as in orthogonal coordinates, as a background of the simulation solution of forming processes by methods of definite elements. There is occurring an issue of flats and directions determinations, whereas we accept mostly simply solutions.
1.19.3. Spherical coordinate systems

![Spherical coordinate system diagram](image)

Fig. 31 Differential stress in axe $r$ of spherical system

We must solve some of real forming problems in the spherical coordinate system, where a point is given by, so called, a vector with two angles, i.e. $\theta, \varphi, r$. In the Fig. 31 there is only indicated the statement of one from the stress values in the direction of the vector radius.

In the conclusion of the chapter, we show the elementary coordinates for the description in various coordinate systems.

$$\begin{bmatrix} x, y, z \end{bmatrix} \quad \begin{bmatrix} z, \varphi, r \end{bmatrix} \quad [r, \varphi, \theta]$$  \hspace{1cm} (87)

1.20. Differential equations of equilibrium in gap between two rolls

(longitudinal rolling of flat rolled products)

In the theoretical study the equilibrium of forces method is used for the deformation resistance investigation in the deformation zone and consequently the size of forces and moments up to necessary performance of the operation execution might be deduced. The rolling is the most
Metal forming theory

widespread method of forming. From the worldwide data we can say that (intelligible based on the region and tradition) with 80 up to 90% of all steel production are processed primarily by rolling and from this survey the 60 up to 70 % processed to the longitudinal flat rolled products, i.e. metal sheet and bends. From that reason, in the chapter, we approach to an analysis of the theoretical deliberations with the aim to determine a process of the deformation resistance values longitudinal the deformation zone. This way is primarily determined for the cold rolling by its mathematic derivation.

We define the zone of deformation by its length, i.e. the difference between an input plane and output plane and we determine what is a pressure angle $\alpha$.

![Fig. 32 Deformation zone in rolling and indefinable element](image)

Geometrical deformation zone starts by a lagging zone (in the figure on the left area of input height $h_0$), it passes to a sticking zone (in the area of indefinable element) and in the output there is a leading zone (in the right at output height $h_1$). Out of geometric area of deformation zone before the material input to the rolls there is a bend of the initiating deformation and after the metal output from rolls there is a zone of decay. For the mathematic derivation we object an element which is far from zero axes from the center and determined by O in the distance of $x$ and the thickness of given element is $d_x$. Step by step, we define the stress value in the rolling axe or axe $x$, which are determined by stress $\sigma_x$, or rather $\sigma_x + d\sigma$. On the surface touching the rolls the wipe force might be defined as $\pm \mu \sigma$ and perpendicular to the value of deformation resistance of stress $\sigma$. For the calculation it is necessary to know the flats too, on which the stress applies. We define the terms and conditions based on which we can perform the mathematical analysis:
Our indefinable element is from the top to the bottom bounded by the surfaces of two same rolls of radius $R$ (it is not the asymmetrical rolling where the rolls might have the different radius, so rather semi-diameter).

The main stress $\sigma$ in direction of axe $x$ is split in the general height $h$ balanced on the surface. Such a term and condition is not met in practice, but we are going to use it due to the mathematic derivation.

The angle $\varphi$ is the zone between value of $0^\circ$ and the value $\alpha$ (by the engagement angle)

The deformation resistance, i.e. stress $\sigma$ is perpendicular to the rolls surface.

The friction forces indicated by the value of $\pm \mu \sigma$ direct to the output, from the angle $\alpha_n$ ($\alpha_n$ is the neutral angle, which represents the zone of sticking or also the neutral plane.

The density of $\pm \mu \sigma$ directs to the output into the neutral plane towards the material travel.

The element weight $b = 1$.

Based on these simplifications we can approach to the mathematical analysis. First we make the equilibrium equation in the point $x$ if form as follows

$$\frac{d h}{h + dh} \left( \delta_x \cdot h + 2 \delta \sin \varphi \cdot \frac{dx}{\cos \varphi} - 2 \mu \delta \cos \varphi \cdot \frac{dx}{\cos \varphi} \right) + \delta_x h + 2 \delta dx(tg \varphi \pm \mu) = 0$$

and we modify it into the following formula

$$S\delta = \frac{A}{12} \left[ (\delta_1 - \delta_2)^2 + (\delta_2 - \delta_3)^2 + (\delta_3 - \delta_1)^2 \right]^{\frac{1}{2}} \approx \delta_K$$

Let’s take the intensity knowledge which defines the resistance towards the body shape and basically, it shows the quantity of stress necessary to initiate the plastic deformation. We can compare this value to the value of the glide limits. It is very approximate comparison, but it is enough for our purposes.

Further we propose that the middle stress $\sigma_2$ is half to the stress $\sigma_1$ and $\sigma_3$ from the smallest to the largest normal stress.
Another derivation results from these equations

\[ \delta_2 = \frac{\delta_1 + \delta_3}{2} \]  

(90)

\[ \delta_1 = \frac{2}{\sqrt{3}} \delta_2 + \delta_3 \]  

(91)

\[ \delta_1 = \delta_x = \frac{2}{\sqrt{3}} \delta_2 - \delta \]

\[ \delta \delta_x = -\delta_2 \]

After derivation we can apply some of the obtained magnitudes into the originally derivate equation of equilibrium.

\[ \left( \frac{2}{\sqrt{3}} \delta_2 - \delta \right) dh - \delta_2 \cdot \delta_x + 2 \delta dx (\tan \varphi + \mu) = 0 \]  

(92)

The problem with the investigation of the general height \( h \) as the function of some real detectable magnitude occurs. The value \( h \) finally equals to the following formula

\[ \varphi = \sqrt{\frac{\Delta h}{R}} \Rightarrow \varphi^2 = \frac{\Delta h}{R} = \Delta h = \varphi^2 \cdot R \]

\[ h - h_1 = \varphi^2 \cdot R \]

\[ h = h_1 + \varphi^2 \cdot R \]

\[ \tan \varphi = \frac{x}{R} \]

\[ h = h_1 + R \cdot \tan^2 \varphi \]  

(93)

(94)

For another calculation it is quite beneficial to introduced a parameter \( T \), in which the output height occurs including the general height \( h \) of indefinable element.
Finally, we can determine the value of differential weight of element $dx$

$$dx = \frac{dh}{2\tan \gamma}$$

(96)

We use a new constant $A$ with known operational value for other calculations

$$A = 2\mu \sqrt{\frac{R}{h_1}}$$

(97)

The basic rolling parameters, i.e. the roll radius $R$, output height $h_1$ and the coefficient of friction $\mu$ we apply into the new constant $A$.

Now we can apply the obtained magnitudes into the equation of equilibrium which we multiply by the value of $\exp(\pm AT)$.

$$\left( \frac{2}{\sqrt{3}} 6\kappa \delta \right) \cdot dh - d\delta \cdot h + 2 \delta \cdot dx (\tan \gamma \pm \mu) = 0$$

$$dh = 2T h_1 (T^2 + 1) dT$$

$$h = h_1 \cdot (T^2 + 1)$$

$$M = \frac{A}{2} \sqrt{\frac{h_1}{R}}$$

$$d\delta - \left( 2 \frac{2}{\sqrt{3}} 6\kappa \cdot T \pm \delta A \right) dT = 0$$

$$\left( \exp(\pm AT) \right) \cdot \delta = 2 \frac{2}{\sqrt{3}} 6\kappa \left[ \frac{\exp(\pm AT) \cdot (1 \pm AT)}{A^2} - C \right]$$

$$C = \frac{1}{2} \left[ 1 + \frac{2}{A^2} - \frac{T^3}{\sqrt{3} 6\kappa} \delta \right]$$

(98)

(99)

(100)
The following mathematical modification lies in the integration and our problem is the integration of the constant \( C \), for which we must determine the marginal terms and conditions. In the beginning we apply the relationship from the equation (100).

The last necessary introduced constant is \( B \)

\[
B = \frac{2m}{h_1}
\]  \hspace{1cm} (101)

We get the right complex equation (103) for the lead area which applies between the angles determined by this relationship (on the left of the equation (102))

\[
\alpha_n < \gamma < 0 \quad \alpha > \gamma > \alpha_n
\]  \hspace{1cm} (102)

\[
\frac{\delta}{\frac{2}{\sqrt{3}} \delta_k} = \exp(B \cdot x) \cdot \left(1 + \frac{2}{A^2} \cdot \frac{\sigma_1}{\frac{2}{\sqrt{3}} \delta_k} \right) - \frac{2}{A^2} \cdot (1 + Bx)
\]  \hspace{1cm} (103)

On the left side of the equation (103) the value of deformation material resistance in the deformation zone, especially in this equation in a numerator position and in a denominator, expresses the relationship for the plane deformation where the stress \( \sigma_2 \) is the middle stress, a half-value of the summary \( \sigma_1 + \sigma_3 \).

In the equation, besides the constants \( A \) and \( B \), which are given by the input terms and conditions of the own rolling, there is also a value \( \sigma_1 \), which represents, i.e. front traction used for the cold rolling.

The same way, we can derive an equation for the lagging zone, again given in the angle range (on the right side in the equation (102) in form of the equation

\[
\frac{\delta}{\frac{2}{\sqrt{3}} \delta_k} = \exp \left[B \cdot (l_d - x) \right] \cdot \left[1 + \frac{2}{A^2} \cdot (1 - B \cdot l_d) - \frac{\delta_0}{\frac{2}{\sqrt{3}} \delta_k} \right] - \frac{2}{A^2} \cdot (1 - Bx)
\]  \hspace{1cm} (104)

There are also two constants \( A \) and \( B \) plus another one \( \sigma_0 \), which represents the value of, so called, breaking, rear traction. It is also used in the real rolling processes. In addition, in the equation there is the value of deformation zone of the length \( l_d \), which is the magnitude easily derive from the
geometrical terms and conditions of rolling and it is the length of deformation zone as \( l_d = \sqrt{R \Delta h} \).

The simplification of these equations describing the common way of rolling without any traction (both front and rear) is in a form of the equation of the advance

\[
\frac{\delta}{\sqrt{\frac{2}{\pi^2}}} \delta_K = \exp(B \cdot x) + \frac{2}{A^2} \left[ \exp(B \cdot x) - 1 - B \cdot x \right]
\]

(105)

and the equation for lagging

\[
\frac{\delta}{\sqrt{\frac{2}{\pi^2}}} \delta_K = \exp[B(\Delta_d - x)] \cdot \left[ 1 + \frac{2}{A^2} \cdot (1 - B \cdot x) \right] - \frac{2}{A^2} (1 - B \cdot x)
\]

(106)

The advantage of these 4 equations (103 up to 106) is a fact that the value of deformation resistance in the zone of deformation is described by the rolling dependency on the geometrical terms and conditions, i.e. on the radius of the roll \( R \), on the input and output height, which are added by the value of the friction coefficient.

Terms' Summary of Application of stress while rolling


Questions to discussed subject matters

1. Which forces might be applied to a body in space?
2. What is the isotropic body?
3. In which units are the stress and the force express?
4. What is the difference between the ideally elastic and ideally plastic bodies?
5. Express in your own words what the real rigid body is.
6. How many and what are the stresses in coordinate planes?
7. Express mathematically the force equilibrium in some axe.
8. How do you reach the expression of the normal stress?
9. Determine mathematically the main normal stress and what the stress expresses?
10. What kind of combination of stress must be reached the space ellipsoid goes through the other bodies up to the abscissa?
11. Does any statement for the main stress quantity determination and their ratio exist?
12. Is it possible to illustrate the space stress condition in plane?
13. Select any combination of stress which should have at least one negative and draw the Mohr’s circles and describe the stress.
14. How big is a tangible stress and where does it have to apply in order to form the plastic deformation in one axe application of force?
15. What is a tensor and it is possible to express a stress like that?
16. Is it possible to divide the tensor to more parts?
17. What is the mathematical expression between stress deviators?
18. How all three stress invariants are determined and what do they express?
19. Derive $\cos \alpha = \ldots$ and you reach the expression for octahedral normal stress.
20. How is the tangible stress determined graphically?
21. Form 9 schemes of main stresses in line.
22. How many and what kind of deviator schemes can we determine?
23. What is the stress intensity and how is it determined mathematically and graphically (more ways)?
24. Form any scheme of three main stresses and determine the normal stress and the deviators.
25. What quantity does the stress intensity reach in the angle of 30° in Rosenberg design of stress intensity?
26. What is the indicator of stress condition and Rhode’s coefficient?
27. How is the maximal tangible stress in plane tensity determined without present tangible stresses?
28. Can you write the simple differential equation of equilibrium for derivation based on $x$ equals zero in octahedral system?

29. How is the point in space determined in various coordinate systems?

30. Is it possible to solve the process of deformation resistance longitudinal deformation zone in rolling of flat rolled products only as the independency on the distance of $x$ from the out plane?

2. Deformation during forming process

**Time to study:** 6 hours

**Aim** You will be introduced with the deformation of a forming body. You will be able to define the basic ways of mathematical description of changes during the forming process. The same way as under stress there is the deformation of tensor. You graphically determine deformation intensity. We will clarify a term of deformation speed. You will generally learn about a tensile test.

**Interpretation**

A relocation of metal particles is done by a complicated space curve. We will explain only a principle of relocation of particles from point $M$ to point $M'$, while point $M$ is defined in time 0 and point $M'$ in time $t$. 

---64---
During the deformation there is pending the relocation of particles. Most of the time, it happens through the complicated space cure and not so simply as follows

\[ \begin{align*}
    x &= x + u \\
    y &= y + v \\
    z &= z + w
\end{align*} \]  

(108)

Let’s introduce briefly the basic ways of the deformation expression. Let’s have one dimension of the forming body, for example height \( h \). This way, there is the expression of, so called, absolute deformation \( \Delta h \) in the formula, which is added by the measurement of length, mostly in mm

\[ \begin{align*}
    \Delta h &= h_0 - h (mm) \\
    \Delta b &= b_0 - b (mm) \\
    \Delta \ell &= \ell_0 - \ell (mm) \\
    \varepsilon_h &= \frac{\Delta h}{h_0} \\
    \varepsilon_b &= \frac{\Delta b}{b_0} \\
    \varepsilon_\ell &= \frac{\Delta \ell}{\ell_0}
\end{align*} \]  

(109)

Another way is a relative change; we can call it here as a proportional draft \( \varepsilon_h \) in form of having the original height (Lagrange) in the denominator or final height (Euler)
It is possible to express relative perceptual deformation in form of

\[
\varepsilon_h \cdot 100(\%) \quad \varepsilon_b \cdot 100(\%) \quad \varepsilon_\ell \cdot 100(\%)
\]

Theoretically, the most correct is, so called, a real, logarithmical deformation \( e_h \), accordingly a real deformation of height in form of following equation (112)

\[
e_h = \frac{h - h_0}{h_0} = \ln \frac{h}{h_0} \quad e_b = \ln \frac{b}{b_0} \quad e_\ell = \ln \frac{\ell}{\ell_0}
\]

And finally the last way, it is a possibility to express it, so called, a coefficient of deformation defined by ratio of two values and indicated by Greek letter

\[
\gamma = \frac{h}{h_0} \quad \beta = \frac{b}{b_0} \quad \lambda = \frac{\ell}{\ell_0}
\]

We will discuss this problem in details and it will be explain in exercises.

2.1. Tensor expression

The similar way as in case of tension and following stress tensor, we can determine the tensor expression of deformation. In general form the formula is as follows

\[
T_\varepsilon = \begin{bmatrix}
\varepsilon_x & \frac{\gamma_{xy}}{2} & \frac{\gamma_{xz}}{2} \\
\frac{\gamma_{yx}}{2} & \varepsilon_y & \frac{\gamma_{yz}}{2} \\
\frac{\gamma_{zx}}{2} & \frac{\gamma_{zy}}{2} & \varepsilon_z
\end{bmatrix}
\]

At the existence of main axes, the tensor of deformation reaches the values based on the following matrix and, the same way as in case of stress tensor, we can divide it into the special tensor and deformation deviator.
From the point of the basic ways of expression’s view we will find out that at keeping the material value the addition of real deformations equals 0; it follows that the spherical tensor of deformation equals zero as well and the deformation deviator equals directly to the quantity of the real deformation.

\[
\begin{bmatrix}
e_1 & 0 & 0 \\
0 & e_2 & 0 \\
0 & 0 & e_3
\end{bmatrix} = K_e + D_e = D_e
\]

\( e_1 + e_2 + e_3 = 0 \Rightarrow K_e = 0 \)

\( e_1 = S_e \cdot \cos \omega_e \quad e_2 = S_e \cdot \cos(\omega_e - 120^\circ) \quad e_3 = S_e \cdot \cos(\omega_e - 240^\circ) \)

The resultant effect of the deformation is the deformation intensity which might be theoretically determine as the intensity of real deformation. The individual components \( e_1, e_2, e_3 \) reach the stated shapes (again in the way of expression of 3 main axes under the angles of 120°).

\[
S_e = \left[ \frac{2}{3} \left( e_1^2 + e_2^2 + e_3^2 \right) \right]^{\frac{1}{2}}
\]

\[
S_e = \frac{\sqrt{2}}{3} \left[ (e_1 - e_2)^2 + (e_2 - e_3)^2 + (e_3 - e_1)^2 \right]^{\frac{1}{2}}
\]

and we can determine the invariant first-order up to the third-order of deformation tensor.

\[
I_1(Te) = e_1 + e_2 + e_3 = 0
\]

\[
I_2(Te) = e_1 \cdot e_2 + e_2 \cdot e_3 + e_3 \cdot e_1
\]

\[
I_3(Te) = e_1 \cdot e_2 \cdot e_3
\]
Based on the mathematical expression besides this form, it is possible to determine the deformation intensity graphically based on the Fig. 34.

The deformation is hardly ever uniform; it is valid only in case that an increase of values $e_1, e_2, e_3$ keeps its size, so the deformation intensity in graphical expression continues in the direction of original angle, which is between the intensity deformation vector and axe 1.

Quite often the changes of body occur while the increases of deformation values $e_1, e_2, e_3$ do not have the same ratio and therefore a new value of deformation intensity indicated $S_e'$ starts to depart from the originally founded angle.
In main axes 1, 2, 3 system there is the deformation determined by the vector, as has been already said, which keeps the same space position and orientation and changes its size while the deformation increases or decreases. That is why, also in front view of octahedral axes, which make a projection of beams of three rays making the angle of 120°, the deformation intensity is determined by the permanent vector inclination. We call this deformation a homogenous deformation. In case of inhomogeneous deformation vector changes its position and comprehensibly in space of main coordinates as well as in plane of their projection to rays of 120°. The deformation intensity cannot be illustrated by a line, but the curve which comes from the common center. The orientation of inhomogeneous deformation corresponds to the small increase of deformation and it is then determined by the center of tangent considered in point \( p \) of the intensity curve and its size in infinity small size of tangent arc \( p' \), which corresponds to the infinite small segment of tangent.

We can reach a law that in point \( p \), or rather \( p' \) the perpendicular lines to rays 1, 2, 3 restrain, or rather, final points of deformation increases \( de_1, \ de_2, \ de_3 \). The similar way, as have already illustrated deformation \( e_1, \ e_2, \ e_3 \) we might mathematically determine these increases which consequently give the change increase of inhomogeneous deformation of intensity.
2.2. State of deformation

Under the condition of the constant quantity the conclusion that there are only 3 real schemes of deformation is reached. Two schemes are space ones and 1 is plane. The following Fig. 36 illustrates these 3 schemes of deformation.

2.3. Deformation speed

Among the metal particles there comes to a relative particle transition at various speed. We come simply from the change of deformation increase in time. From the following derivation we have the unit of the deformation speed $s^{-1}$.

$$\dot{\varepsilon} = \frac{\Delta \varepsilon}{\Delta t} = \frac{\Delta h}{h_0} \cdot \frac{1}{\Delta t} = \frac{\Delta h}{\Delta t} \cdot \frac{1}{h_0} = \frac{V}{h_0} \left( \frac{1}{s} \cdot \frac{1}{s} \right) \left( s^{-1} \right)$$

$$\dot{\varepsilon} = \frac{d e}{d t} = \frac{d h}{h} \cdot \frac{1}{d t} = \frac{d h}{d t} \cdot \frac{1}{h} = \frac{V}{h} \left( s^{-1} \right)$$
Fig. 36 Swaging, drawing, plane deformation
We strongly warn that this deformation speed is not possible to exchange with a tool speed, for example, the speed of rolls rotation or the swage movement while forging, thus some advancing speed, which has the unit m/s. Analogically, we can determine also the tensor of deformation speed.

\[
T_\dot{\varepsilon} = \begin{bmatrix}
\dot{\varepsilon}_1 & 0 & 0 \\
0 & \dot{\varepsilon}_2 & 0 \\
0 & 0 & \dot{\varepsilon}_3 \\
\end{bmatrix} = D\dot{\varepsilon}
\]

(122)

### Speed of deformation in tension test (simplified)

The tensile test is one of the elementary tests of material characteristic features, i.e. strength and plastic feature. In the tensile test it is necessary to consider the two areas of deformation. The first one is divided steady along the entire length of testing rod and it is the continuous homogeneous. The second one is the unsteady inhomogeneous, which is focused on the section of rod length, where a local reduction, or rather neck.

![Fig. 37 Tensile test in area of uniform deformation](image)

In the first phase, when the yield point is exceeded, but there is still not reach the local deformation concentration speed of particle speed of transmission in the individual lateral section uniformly increases from zero value in the point of rod fastening in rigid part up to the value \(v_m\), which is the speed of fastening jaw on the other end of the rod. Such a speed is a linear dependency on the length \(l\).
of the lateral section from fastening head. The speed is the relative one. It is possible to calculate the deformation speed from the simple relationship based on the following formula

\[ \dot{\epsilon} = \frac{v}{L} = \frac{V_m}{L} = \tan \alpha = k \text{const, } \left( s^{-1} \right) \]

\[ \dot{\epsilon} = \frac{V_m}{h} \left( s^{-1} \right) \]

In the second phase of test the deformation is centralized only in the section identified as \( l_2 \). The length of rod is divided in 3 sections, where the speed is zero from the fasten part, so rather the origin of length \( l_1 \), because the absolute speed is in the section also zero. The lateral sections do not move in the given length cross section \( l_1 \).

In the section \( l_3 \) all sections are transmit by the same speed \( v_m \) because in this section the rod is not prolonged anymore. In the section \( l_2 \) the relative speed changes continuously from zero value up to the value \( v_m \). The curve has an inflexion point in the point of the smallest section. The deformation speed in this section would be possible to determine by derivation of the relative speed curve \( v \). The approximate value of maximal deformation speed is possible to calculate also from the speed \( v_m \) of movable parts fastening the jaws and length \( l_2 \) in the neck which we measure after the splitting of testing rod in the form of the equation.
The calculation is approximate, since the curve process of relative speed in $v$ in section $l_2$ we replace by the curve process based on the oblique connecting line of point $v = 0$ and $v_m = v$. We can apply this knowledge later, because we will learn that a nature deformation resistance of metal is principally depended on three basic parameters, a temperature, deformation volume and deformation speed.

**Deformation speed in tensile test (simplified)**

These are the tests where between two jaws, upper and lower, the testing body is kept, and mostly a roll and/or flat cuboid sample (Plane Strain Compression Testing). In consequence of the existence of abrasive forces there occurs the non-uniform deformation, which is increasing based on the deformation degree. The relative speed in the front area, which is in touch with the movable upper jaw, so rather the upper moving swage, we indicate the same way as in tensile test by the value relationship $v_m$. To simplify it we calculate the average value of deformation speed $\dot{\varepsilon}$ from known height of pěchovaného body and the immediate speed of swage $v_m$. By that the task comes to the solution of the deformation speed at homogenous deformation and then $\dot{\varepsilon} = \frac{v_m}{l_2}$, where the length $l_2$ is replace by the height $h$.

**Terms' summary of the Deformation during forming process**

The points transmission in space. The basic way of mathematical description of changes at deformation. The deformation tensor. The graphical determination of deformation intensity. Uniform and non-uniform deformation. The deformation states. The deformation speed.
1. How is it possible to describe the change of the point location in space?
2. What is the difference between the absolute and relative deformations?
3. Make mathematical comparison of the absolute and relative deformations as the functional relationship and illustrate it graphically.
4. What is it the mathematically real deformation?
5. How are the deformation coefficients determined?
6. What is the relationship between the spherical tensor and deformation deviator?
7. Do the deformation invariants exist and what do they express?
8. Describe the difference between the uniform and non-uniform deformations.
9. What kind of deformations do you know and how is the deformation intensity determined?
10. How the deformation speed derived and what is its unit?

3. **Relationships between stress and strain**

**Time to study:** 3 hours

**Aim**  
We will introduce with the basic theoretical deliberations about the relationship stress-strain. We will be able to assess the differences between the flexible, flexible-plastic and plastic ones.

**Interpretation**

Both terms, the stress and the strain, were described in the preview chapters. The foundation of relationship between stress and strain understanding is the distribution into 3 basic options. In the technological practice one of the most important tasks determines the strain resistance under real forming conditions, i.e. the stress-strain curve.
The deformation is only flexible, thus elastic, if it changes a shape and a volume of a body.

The second deformation is elastically – plastic; the parts of the volume are variously deformed and the shape change is permanent after the force application.

The big plastic deformation, generally these deformations are inhomogeneous, which significantly changes the body shape and the volume remains constant.

(As an example, which is practical and does not belong to the forming theory, try to imagine a fluently cast steel block with the dimensions of 150x150x12000 mm, which is being hot rolled into the wire with the diameter of 5.5 mm, which length is several kilometers and final shape is a wire coil from which a wire of diameter 0.2 mm in length of several hundred kilometers is being cold drawn). At big plastic deformation, theoretically and partially practically, there occurs always some initial flexible deformation with regards to its size, which we mostly neglect. The big plastic deformation might be done in hot and also in cold. This problem is a part of Physical theory of plasticity subject, but for better understanding of following mathematical relationships we assume it is necessary to clarify it briefly. The plastic characteristic features of formed substances resist while forming and the resistance changes together with the temperature, which is essential thermo mechanical magnitude. Predominantly, it is known that the deformation resistance decreases (is lower):
- With the increasing temperature
- In single-phase structure
- In material which is easily and quickly renewed.

The metals have the crystalline composition that is why the physical processes inside the crystal grains and their ages are very important for studying of big plastic deformations. It is related to the structural changes. These processes affect physical and also the mechanical characteristic features of forming metal. Because of a fact that the material renovation is predominantly caused by recrystallization and its full process depends on the temperature based on the relationship \( T_r > 0.4 T_t \), where \( T_r \) is the temperature in K, above which occurs the recrystallization and \( T_t \) is the temperature of melting in K.

The transitions under \( T_r \) are indicated as the cold melting and transitions above \( T_t \), as the hot melting. The cold forming is characterized by the deformation, or rather a texture origin and a change of the mechanical characteristics features and significant phenomenon which is a metal strengthening. The plastic characteristic features, or rather a plasticity supply, are exhausted gradually.

The hot forming is characterized by currently pending strengthening and renewal processes which change the mechanical and physical characteristic features. The metal keeps its big plasticity supply.
being able to reach big inhomogeneous deformations. These simple glances do not theoretically compete exactly; we can wrap it up that it is better to divide the forming processes to those in which the strengthening happens and those, predominantly cold transmissions where no strengthening occurs.

Let’s come back to the 3 basic ways of relationships between stress and strain. The deformation is flexible is mathematically determined by Hook’s Law, where $E$ is the module of elasticity in tension and equals to the value of 210 000 MPa in steel (If there are no drawings in this and following chapters, it is because they are going to be explain in lectures and/or consultations). The value $\mu$ is the Poisson number in the equation.

\[
\sigma = E \cdot \varepsilon
\]

Otherwise, the equation for the deformation size it is possible to write down by, so called, constant $m$, which is the Poisson constant. For big plastic deformation is valid that the value $\mu = 0.5$ and for the flexible deformation the value $\mu = 0.3$.

\[
e_1 = \frac{1}{E} \left[ \sigma_1 - \mu (\sigma_2 + \sigma_3) \right]
\]

\[
e_1 = \frac{1}{E} \left[ \sigma_1 - \frac{\sigma_2 + \sigma_3}{m} \right]
\]

Besides the module of elasticity in tension there exists also the shear elasticity module based on the equation

\[
\gamma = \frac{1}{2(1+\mu)} \cdot E = \frac{1}{3} E
\]

We can divide the second elastically plastic group into three groups. A variant $a$, which tells us that the directional tensors of stress and deformations are similar. A variant $b$ explains us the relationship between the stress and strain where the linear invariant of stress tensor is straight proportional to a
linear invariant of strain tensor. And finally the third variant says that the quadratic invariant of tension deviator of stress is the function of quadratic invariant of deformation deviator.

We will discuss all three elastic – plastic variants.

**Variant a**

Based on the following equation

\[
\frac{D_e}{\sqrt{I_2 D_e}} = \frac{D_0}{\sqrt{I_2 D_0}}
\]

we can reach a gradual derivation and replacing up to a constant \( \psi \) and after the mutual subtraction of equations and comparison we reach the following formulas in all three examined real deformations

\[
\frac{\sqrt{3}}{S_6} \cdot D_6 = \frac{2}{\sqrt{3} \cdot S_e} \cdot D_e
\]

\[
D_e = \frac{\sqrt{3} \cdot \sqrt{3} \cdot S_e \cdot D_6}{2 \cdot S_6} = \frac{3}{2} \cdot \frac{S_e}{S_6} \cdot D_6 = \psi \cdot D_6
\]

\[
e_1 - e_8 = \psi (\delta_1 - \delta_8)
\]

\[
e_2 - e_8 = \psi (\delta_2 - \delta_8)
\]

\[
e_3 - e_8 = \psi (\delta_3 - \delta_8)
\]

\[
e_1 - e_2 = \psi (\delta_1 - \delta_2)
\]

\[
e_2 - e_3 = \psi (\delta_2 - \delta_3)
\]

\[
e_3 - e_1 = \psi (\delta_3 - \delta_1)
\]

\[
\frac{e_1 - e_2}{\delta_1 - \delta_2} = \frac{e_2 - e_3}{\delta_2 - \delta_3} = \frac{e_3 - e_1}{\delta_3 - \delta_1} = \frac{3}{2} \cdot \frac{S_e}{S_6} = \psi
\]

In words it is possible to express the equation that the directions of maximal real stresses merge with the directions of main shear deformation.

**Variant b**
Variant $b$ comes from the following equation and expresses the relationship between middle linear deformation and middle linear stress, i.e. between $e_8$ and $\sigma_8$.

\[
I_{47e} = \alpha \cdot I_{47s}
\]

\[
e_8 = \frac{1-2\mu}{E} \cdot \delta_8
\]

(134)

(135)

Such a value we can apply into the equations with the used constant $\psi$

\[
e_1 = \psi (\delta_1 - \delta_8) + \frac{1-2\mu}{E} \cdot \delta_8
\]

(136)

Hencky replaced the constant $\psi$ by dependence on the elasticity module in shear and new constant $\varphi$

\[
\psi = \frac{1}{2G} + \varphi
\]

(137)

\[
e_1 = \left(\frac{1}{2G} + \varphi\right) \cdot \delta_1 - \left[\left(\frac{1}{2G} + \varphi\right) - \frac{1-2\mu}{E}\right] \cdot \delta_8
\]

(138)

and reached to the equations for $e_1$, $e_2$, $e_3$ in the following formula

\[
e_1 = \frac{1}{E} \left[\delta_1 - \mu (\delta_2 + \delta_3)\right] + \frac{2}{3} \varphi \left[\delta_2 - \frac{1}{2} (\delta_2 + \delta_3)\right]
\]

(139)

\[
e_2 = \ldots...
\]

\[
e_3 = \ldots...
\]

The first element of the summation equation represents the elastic deformation, thus we come back to the Hook’s Law and the second member represents the plastic deformations (let us warn you that $\varphi$ is not a constant, it is a function of previously derivate value $\psi$ and also the module of shear elasticity $G$).

These three resultant equations are general (for example, if $\varphi = 0$ it results with no plastic deformation and the second element in the equation disappears and there is left only the elastic deformation).

**Variant c**
Variant \( c \) is represented by following equations

\[
\sqrt{I_{2D}} = \frac{S\delta}{\sqrt{3}} \quad \sqrt{I_{2De}} = \frac{V}{2} \cdot S_e
\]  \hspace{1cm} (140)

and generally the stress intensity is the deformation intensity function and conversa. It is possible to express their mutual relationship by known equations.

\[
\frac{S_e}{S\delta} = \frac{\sqrt{2}}{3} \left( \frac{(e_1-e_2)^2+(e_2-e_3)^2+(e_3-e_1)^2}{(\delta_1-\delta_2)^2+(\delta_2-\delta_3)^2+(\delta_3-\delta_1)^2} \right)
\]  \hspace{1cm} (141)

Big plastic deformations are quite complicated from the point of description of relationship between stress and deformation. There is coming the inhomogeneous deformation. There is pending the strengthening in the material. It is or might be conducted by mollification or as we call it renewal. The renewal runs by the means of treatment or recrystallizing and it might be static or dynamic. It comes to the structural changes of phases and the structure changes. The precipitation phenomenon occurs in the material. The values of modules \( E \) and \( G \) are not the constant together with temperature. …

It results with enormous complication for determination of real stress-strain curves. The angle of inclination \( \alpha \) of growing stress curve, i.e. the resistance deformation, changes to \( \alpha' \), the stress is then the function of variable module of elasticity \( E' \) and in its final formula

\[
tg \alpha = E
\]  \hspace{1cm} (142)

\[
tg \alpha' = E'
\]  \hspace{1cm} (143)

\[
\delta = f(E')
\]  \hspace{1cm} (144)

The inclination angle \( \alpha \) of growing stress curve, i.e. deformation resistance, changes to \( \alpha' \), the stress is then the function of variable module of elasticity \( E' \) and in its final formula
Elastic deformation, elastic-plastic deformation and big plastic deformation.

Questions to the subject matters

1. What is the difference between the cold deformation and hot deformation?
2. What are the module of elasticity $E$ and the module of elasticity in shear $G$?
3. What is the relationship between these modules?
4. What is the elastic deformation?
5. What is the elastic-plastic deformation?
6. What is the big plastic deformation?
7. Which magnitudes, previously derivate in the chapter of stress and deformation get to the relationship of stress-strain?
8. What happens in the material in big plastic deformations?

Solved tasks

Below please find the task resulting from the theoretical sub-chapter 1.16.

4.4. Rolling with stresses

4.4.1 Theoretical introduction

The calculation methods of deformation resistance in deformation zone in longitudinal rolling result usually from the analysis of deformation zone and their consequences direct to the establishment of theoretical equation of forces equilibrium in the rolling gap. It should be considered gradually the length of connection metal with rolls and more generally the shape of constant curve, or rather, the
plane, the weight of rolling stocks and its changes along the length of zone deformation, but especially
the quantity and stress distribution on the surface of deformed material with rollers. Besides these few
named and many other geometrical parameters there exist many agents which influence the resultant
value of deformation resistance and consequently the forces calculation, a moment (of labor) and
necessary performance. There belong especially chemical composition and metallurgical
classifications features, temperature, friction, strengthening and renewal mechanisms, the deformation
speed etc.

The quantity and process of the value of deformation resistance in the length of deformation zone is
possible to express by the formula

$$\sigma = \sigma_k \left[ (\delta - 1) \left( \frac{h_0}{h} \right)^\delta + 1 \right]$$

(4.4.1)

where the equations is for the zone of lagging zone (to the entry plane to the neutral one), analogous
equation is for the advance zone; while the tensions are accepted, i.e. front and rear axes, when

$$\zeta_1 = 1 - \frac{\sigma_1}{\sigma_k} \quad \text{so rather} \quad \zeta_2 = 1 - \frac{\sigma_a}{\sigma_k}$$

(4.4.2)

where $\sigma$ is the deformation resistance in MPa
$\sigma_k$ is the glide end in MPa
$\sigma_1$ is the front traction in MPa
$\sigma_0$ is the rear traction in MPa
$h_0, h_1, h$ are input, output and general height of rolling stock in mm

$\delta$ is the coefficient, usually $\delta = 2 \mu \frac{R}{\Delta h}$
$\zeta_1, \zeta_2$ are coefficient of front and rear traction

then the equation for the lagging zone might be as follows

$$\sigma = \frac{\sigma_k}{\sigma} \left[ \left( \frac{h_0}{h} \right)^\delta \left( \zeta_1 \delta - 1 \right) + 1 \right]$$

(4.4.3)

The similar way the task is solved by Bland and Ford, who propose the lagging equation based on the
Orovan’s theory
Practically all published equations consist of the variable general height $h$, which calculation is possible based on the knowledge of geometrical terms and conditions of rolling, but the total utility and simplicity of calculation is becoming complicated. Therefore the aim of the thesis is to provide with equations for both areas as is the lagging and the advance, where the only variable is the distance $x$ as coordinate from the output plane back to the input plane for the value of length of deformation zone $l_d$, i.e.

$$0 \leq x \leq l_d$$  \hspace{1cm} (4.4.5)

These new equations must result from the equilibrium equation for the expressed element. Resultant equations are as follows:

$$\frac{\sigma}{2\sigma_h} = \exp \left(B\cdot x \right) \left[ 1 + \frac{2}{A^2} \left( 1 - \frac{\sigma_0}{\sigma_k} \right) \right] - \frac{2}{A^2} \left( 1 + B\cdot x \right)$$ \hspace{1cm} (equation 6)  \hspace{1cm} (4.4.6)

$$\frac{\sigma}{2\sigma_h} = \exp \left(B\left(l_d - x\right) \right) \left[ 1 + \frac{2}{A^2} \left( 1 - B\cdot l_d \right) \right] - \frac{2}{A^2} \left( 1 - B\cdot x \right)$$ \hspace{1cm} (equation 7)  \hspace{1cm} (4.4.7)

$$\frac{\sigma}{2\sigma_h} = \exp \left( B\cdot x \right) + \frac{2}{A^2} \left[ \exp \left( B\cdot x \right) - 1 - \left( B\cdot x \right) \right]$$ \hspace{1cm} (equation 8)  \hspace{1cm} (4.4.8)

$$\frac{\sigma}{2\sigma_h} = \exp \left( B\left(l_d - x\right) \right) \left[ 1 + \frac{2}{A^2} \left( 1 - B\cdot l_d \right) \right] - \frac{2}{A^2} \left( 1 - B\cdot x \right)$$ \hspace{1cm} (equation 9)  \hspace{1cm} (4.4.9)

$$A = 2\mu \frac{R}{h_i} \hspace{1cm} B = \frac{2\mu}{h_i}$$  \hspace{1cm} (4.4.10)
so, the process of variable stress is really \( \frac{\sigma}{2\sigma_k} \) for given geometrical terms and conditions, i.e. the roll radius \( R \), input and output heights \( h_0 \) and \( h_1 \) and the coefficient of friction \( \mu \) only the function of distance \( x \). The validity of the individual equations is based on the following Fig. 4.4.1.

### 4.4.2 Rubric

**4.4.2.1** Illustrate graphically the stress process on rolls from the input plane up to output plane under these terms and conditions:

\[ h_0 = 2,5 \text{ mm}; \ h_1 = 2,02 \text{ mm}; \ R = 75 \text{ mm}; \]

for the rubric 4.4.2.1.a is \( \mu_1 = 0,11; \)  
for the rubric 4.4.2.1.b is \( \mu_2 = 0,13; \)  
for the rubric 4.4.2.1.c is \( \mu_3 = 0,18; \)

determine the values of stress peaks with the accuracy of two decimal places and x axis coordinate to neutral plane (graphically and analytically).

**4.4.2.2** Investigate how big front \( (c_2) \) traction or rear \( (c_1) \) traction you have to use in the used rear, or rather front traction was the stress peak of 20 % lower than in rolling without tractions. Determine the x-axis coordinate of neutral plane (graphically). The option of front, or rather rear traction is in % quantity of \( \frac{\sigma}{2\sigma_k} \). Given values are in Table 4.4.1.

**4.4.2.3** Assume the necessity of neutral plane transmission towards the output of ...% of length of deformation zone. Result from the results from the rubric 4.4.2.1. Determine at least one combination of front and rear traction to comply with the assumed necessity. Try to express mathematically all combinations of front and rear traction in formulas

\[ c_1 = k_1 Y + q_1 \]
\[ c_2 = k_2 Y + q_2 \]  

(4.4.11)

where \( c_1 \) is the front traction in % of value \( \frac{\sigma}{2\sigma_k} \), \( c_2 \) is the rear traction in % of value \( \frac{\sigma}{2\sigma_k} \)

\( k_1, k_2 \) are coordinates for given assumption of constant, \( q_1, q_2 \) are points of intersection
$Y$ is the value of stress peak ($y$–axe coordinate)

4.4.2.4 Assume the option of change of stress peak including the change of neutral plane position without utilization of traction by changed fraction coefficient. Result from the elementary rubric of 4.4.2.1. The stress peak must be decreased to ......% of elementary volume (indicate it by $Y$). Find the required coefficient of friction and the position of neutral plane (indicate by $X$).

**Tab. 4.4.1** Values for rubrics

<table>
<thead>
<tr>
<th>1.4.2.1</th>
<th>$\mu = 0,11$</th>
<th>$\mu = 0,13$</th>
<th>$\mu = 0,18$</th>
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<tr>
<td>1.4.2.2</td>
<td>$c_1$ 5</td>
<td>$c_1$ 30</td>
<td>$c_1$ 5</td>
</tr>
<tr>
<td></td>
<td>$c_1$ 10</td>
<td>$c_2$ 10</td>
<td>$c_1$ 10</td>
</tr>
<tr>
<td></td>
<td>$c_1$ 15</td>
<td>$c_1$ 15</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$c_1$ 20</td>
<td>$c_1$ 20</td>
<td></td>
</tr>
<tr>
<td></td>
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<tr>
<td></td>
<td>$c_2$ 20</td>
<td></td>
<td>$c_2$ 20</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>1.4.2.3</th>
<th>$\mu = 0,11$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Movement of the peak in direction of o output</td>
<td>$+10%$</td>
</tr>
<tr>
<td></td>
<td>Movement of the peak in direction of o output</td>
<td>$+20%$</td>
</tr>
<tr>
<td></td>
<td>Movement of the peak in direction of o output</td>
<td>$+30%$</td>
</tr>
<tr>
<td></td>
<td>Movement of the peak in direction of o output</td>
<td>$-10%$</td>
</tr>
<tr>
<td></td>
<td>Movement of the peak in direction of o output</td>
<td>$-20%$</td>
</tr>
<tr>
<td></td>
<td>Movement of the peak in direction of o output</td>
<td>$-30%$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1.4.2.4</th>
<th>$\mu = 0,18$</th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>to</td>
<td>70% $Y$</td>
</tr>
<tr>
<td></td>
<td>to</td>
<td>80% $Y$</td>
</tr>
<tr>
<td></td>
<td>To</td>
<td>90% $Y$</td>
</tr>
</tbody>
</table>
4.4.3 Instruction

ad 4.4.2.1 Incorporate into the equations which express the process without traction utilization. Choose the range of $x$ at least per 0.5 mm. Make a table of values, draw a graph and subtract the values of the neutral plane position and stress peak.

---

<table>
<thead>
<tr>
<th></th>
<th>100% Y</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>110% Y</td>
</tr>
<tr>
<td></td>
<td>120% Y</td>
</tr>
<tr>
<td></td>
<td>130% Y</td>
</tr>
<tr>
<td></td>
<td>140% Y</td>
</tr>
</tbody>
</table>

---

**Fig. 4.4.1 Equations validity**

[Graph showing equations validity]

**Fig. 4.4.2 Clear graph for the solution 4.4.2.**

---86---
ad 4.4.2.2 First clarify based on specific (individual) rubric which traction you use. Incorporate into the equation with given traction, (let’s say the rear one), where instead of the value of \( \frac{\sigma_0}{2\sigma_k} \), use the given quantity and solve the equation for the variable \( x \). Calculate \[
\begin{array}{c}
\sigma_0 \\
2\sigma_k \\
\sqrt{3}
\end{array}
\]

**Fig. 4.4.3** Determination of point of intersection \( x \) while using the rear traction \( c_0 = 0.05 \) and the depreciation of stress peak of 20%. Determine \( x \) – axis coordinate. Modify the second equation for determination of calculations with tractions (let’s say here with the front one) for the calculation into the formula \( \frac{\sigma_1}{2\sigma_k} = \ldots \) and incorporate found value of \( x \) – axis coordinate and determine the value (here \( c_1 \)). Draw the gradual graphs.

ad. 4.4.2.3 Firstly determine the value of \( x_1 \), where should be the neutral plane. From the equations, where traction is used, calculate \( c_1 \) and \( c_2 \) for given \( x_1 \) and determine the constants \( k_1 \), \( k_2 \), \( q_1 \) and \( q_2 \). Draw a graph.
Fig. 4.4.4 Combination of tractions at given quantity of stress peaks in movable arm of rolling force in output of the length of 10 % of the deformation zone length

ad. 4.4.2.4 Apparently, you reach two algebra equations with two unknowns which solution is the best by means of iteration in the given case. Estimate $x$ and solve.

The example of rubric 4.4.2.1.

<table>
<thead>
<tr>
<th>$\mu$</th>
<th>A</th>
<th>B</th>
<th>R [mm]</th>
<th>X [mm]</th>
<th>h0 [mm]</th>
<th>h1 [mm]</th>
</tr>
</thead>
<tbody>
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<td>0.11</td>
<td>1,340533</td>
<td>0.108911</td>
<td>75</td>
<td>6</td>
<td>2,5</td>
<td>2,02</td>
</tr>
<tr>
<td>0.13</td>
<td>1,584267</td>
<td>0.128713</td>
<td>75</td>
<td>6</td>
<td>2,5</td>
<td>2,02</td>
</tr>
<tr>
<td>0.18</td>
<td>2,1936</td>
<td>0.178218</td>
<td>75</td>
<td>6</td>
<td>2,5</td>
<td>2,02</td>
</tr>
</tbody>
</table>

♦ Solution
It results from the mathematical relationships, mentioned in the chapter 4.4.1. As a result there is stated a figure with three various coefficients of friction. The point 0 on the x-axe coordinate represents the plane of metal output from rolls, the point indicated 6 on the left on the same axe is the
distance x from the plane of output towards to input plane; it is a length of deformation zone. The curves represents the equations on the Figure (4.4.6) a (4.4.8).

Used literature which might be used for further study.