Darja Noskievičová

Special Statistical Methods for Quality Management

2012

INTRODUCTION

This student learning support is intended for the subject „Special Statistical Methods„ in English language which is offered to the foreign students in the frame of the Master study branch „Quality Management“ at the Faculty of Metallurgy and Material Engineering, VŠB-Technical University of Ostrava. It can be also used as an additional study material for the subject „Basic Statistical Methods for Quality Management“ (Bachelor studies of the Quality Management branch in Czech language) and the subject “Special Statistical Methods“ (Master studies of the Quality Management branch in Czech language) to increase the students´ expert foreign language competencies .

This textbook is focused on the complex approach to statistical process control (SPC) from the theoretical basis, classical and unconventional SPC methods to the practical aspects of the SPC implementation including case studies.

Processing this textbook I have capitalized on my many years´ experience with teaching SPC (at the university or in the industrial companies) and with practical implementation of SPC in the companies of the various industrial branches. The case studies are based on the real practical implementations solved in the frame of the diploma theses which I supervised. Partial methodical questions have been included in the paper which I published on the 10th International symposium SHMD 2012, held in Šibenik, Croatia from 16. 6. 2012 till 23.6. 2012. Many themes were also discussed and consulted with experts from the foreign universities (Croatian, Polish, Russian, German, Hungarian) and from various foreign companies during the conference meetings.

This textbook is divided into 12 chapters. First four chapters are devoted to the theoretical basis for the following chapters. Chapters 5 - 7 are focused on the practical aspects of the SPC implementation. Starting with the chapter 8 the textbook is focused on the practical problems of the SPC implementation. In chapters 10-12 the real applications of SPC can be found. They show the implementation of SPC in condition of the heat production, aluminium processing and rolling of plates.

To support the process of learning the chapters have standardized structure. Every chapter contains these parts: „Chapter structure“, „Time for learning“, „Goal“, „Lecture“ or „Case study“, „Questions“, „Additional study resources“. The parts „Examples“ and „Solution to examples“ can be found in chapters 1, 2, 3, 4, 5, 6 and 8. All chapters excluding chapters with case studies also contain the part „Summary of terms“. Chapters 2, 3, 4, 5, 6, 7 and 9 have in addition the part „Topics for the supplementary self-study“. Into chapters 2, 4, 5, 7, 8, 9, 10, 11 and 12 the part „Topics for review“ is incorporated.

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1 VARIABILITY, STATISTICAL STABILITY AND CAPABILITY OF THE PROCESS

Chapter structure

Time for learning

Goal

Lecture

- **Definition of the process variability**
- **Causes of the process variability**
- **Definition of the process statistical stability**
- **Basic tool for the analysis of the process statistical stability**
- **Definition of the process capability**
- **Basic tools for the process capability evaluation**
- **Relation between the process statistical stability and capability**
- **Definition of the process improvement**

Summary of terms

Questions

Example

Additional study resources

Solution to example

Time for learning: 180 min

Goal: After studying this chapter,

- **you will understand fundamentals of the process variability;**
- **you will recognize basic types of the process variability causes and their differences;**
- **you will understand difference between the process statistical stability and the process capability;**
- **you will have an understanding of the basics of the process improvement;**

you will be familiar with basic tools for the analysis of the process statistical stability and capability.

Lecture

Definition of the process variability

Variability is a common attribute of every process which causes lack of its repeatability.

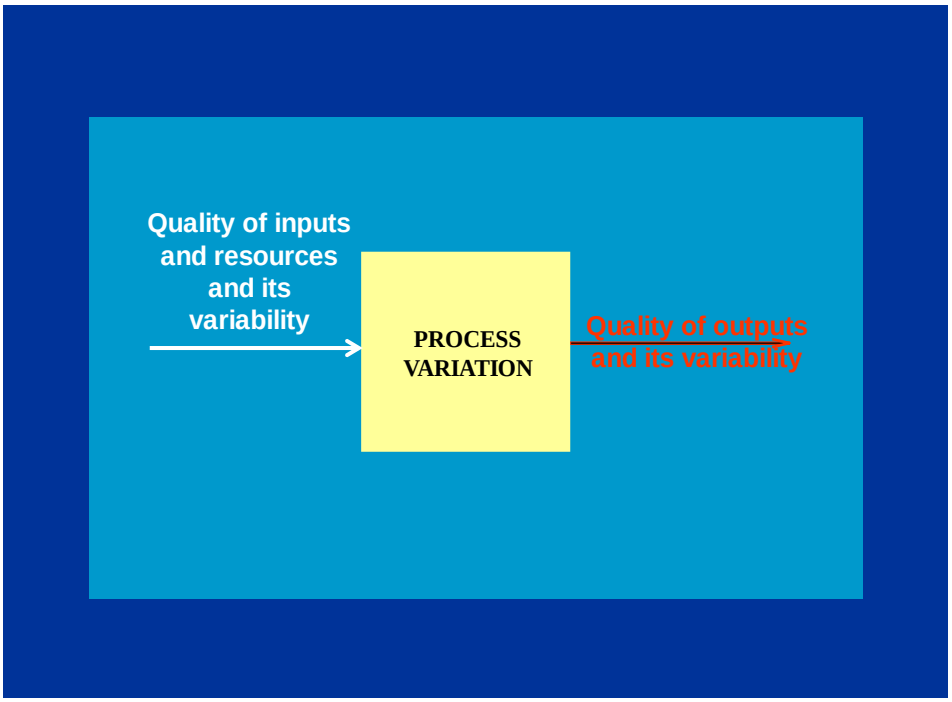


Fig. 1.1 Variability

Even if the process conditions are relatively stable, processes and their inputs and outputs are objectively influenced by various impacts (see Fig.1.1) that cause impossibility to produce two wholly identical products. In spite of this fact it is possible to study these impacts and to create conditions for variability being inside its own natural limits and stable in time. Knowledge of these process limits enables forecasting the future process behaviour although individual measurements cannot be predicted.

Reduction of the process variability then can lead to:

- more consistent production process;
- lower probability of the production of nonconforming units;
- lesser rate of the inspection and lower costs for the inspection and testing;
- lower costs of the process failures, waste and reworking;
- more satisfied customers.

Restriction of the variability of the process and its inputs should result into reduction of the variability of the characteristics of the process outputs.

Causes of the process variability

The process variability can be divided into two types: the variability caused by common causes and the variability caused by assignable causes [1], [3].

1. **Common causes** (random causes) form a wide complex of causes that individually cannot be identified. Each of these causes contributes to the overall variability only at a small rate. In case that only these variability causes have an influence, the process can be described as follows:
   - process is reproducable and the quality of its outputs is predictable;
   - process is statistically stable, i.e. the process is „in control“, or the process is “in statistical control“ ([1]). It means that the type and parameters of the distribution of the quality characteristic or of the process parameter used for the evaluation of the process variability are known and stable.

   Elimination of effects of these causes needs systemic long-term managerial decisions. Examples of common causes can be seen in Fig 1.2.

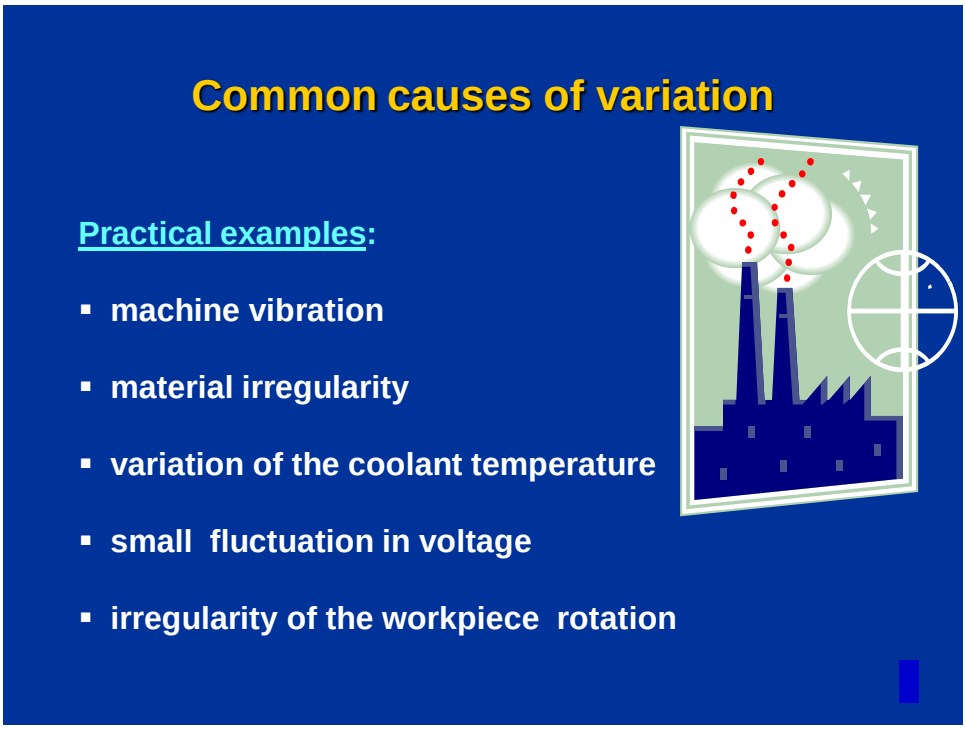


Fig. 1.2 Examples of common causes of variability

2. **Assignable causes** represent the effect of variability sources which do not have an influence on the process in common conditions. They evoke real changes of the process manifested by unnatural variation of the data used for the process variability analysis. When these causes have an influence, too, the process behaviour can be described as follows:

- process is not reproducable and the quality of its outputs is un-predictable;
- process is not statistically stable, i.e. the process is "out of control",or the process is "out of statistical control" ([1]). It means that the type and parameters of the distribution of the quality characteristic or the process parameter used for the evaluation of the process variability change in time.

Elimination of effects of these causes usually requires only local intervention of the person directly responsible for actions realized in the frame of the given process. Assignable causes can be divided into two groups:

1. Occasional (sporadic) causes;
2. Persisting causes.

Occasional causes arise suddenly. They cause short-term changes of the process. Then they vanish and they can come back again sometimes in the future. The process changes evoked by these causes used to be larger. *Persisting causes* call on longer-lasting deviations of the quality characteristic or the process parameter. Examples of assignable causes can be seen in Fig. 1.3.

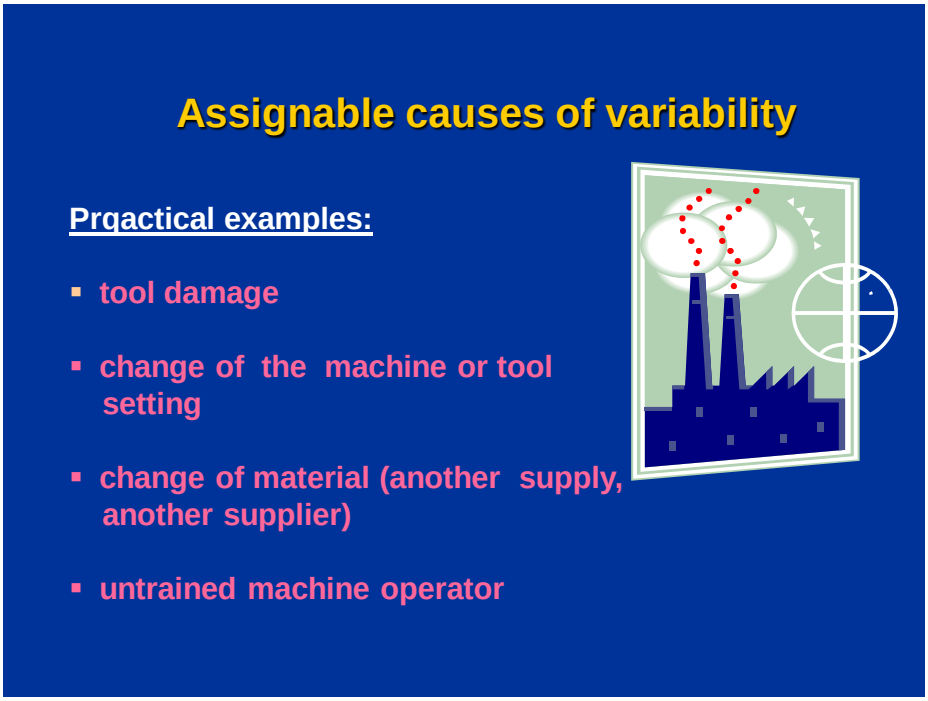


Fig.1. 3 Examples of assignable causes of variability

Definition of the process statistical stability

Statistically stable process is a process which is affected only by common causes of variability.

Statistically stable process is depicted in Fig. 1.4 a).

Statistically unstable process is a process which is also affected by assignable causes of variability.

Statistically unstable process is depicted in Fig. 1.4 b).

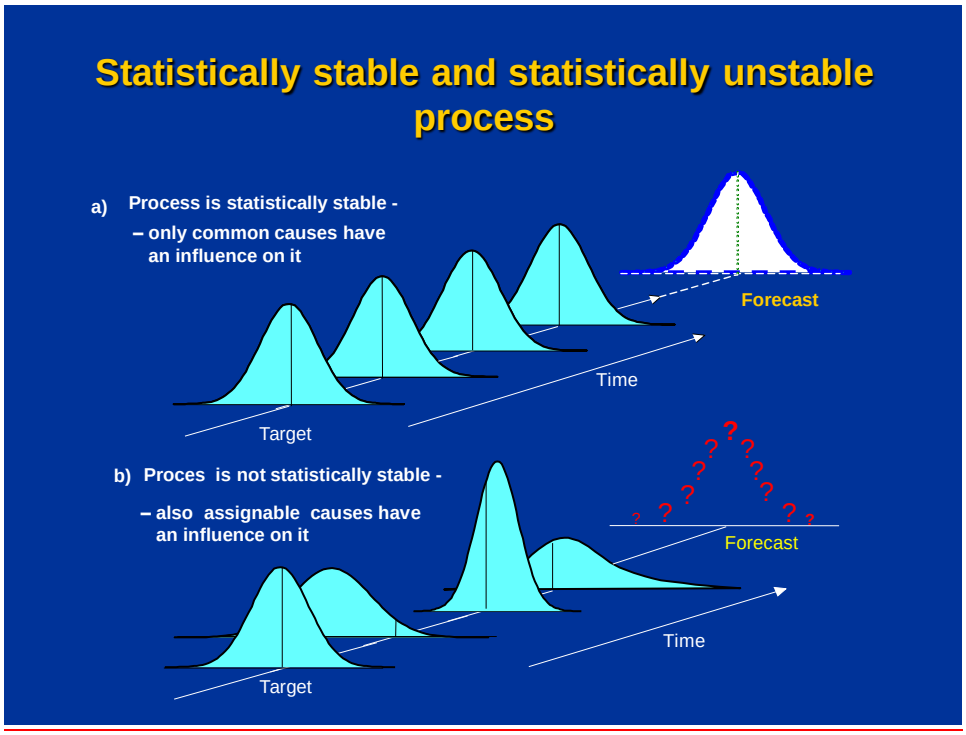


Fig. 1.4 Statistically stable and unstable process

Basic tool for the analysis of the process statistical stability

Basic instrument for the analysis of the process statistical stability is a **control chart**. Detail information about this basic instrument of the SPC is given in Chapter 3 of this textbook. An example of a control chart for the statistically stable process is in Fig. 1.5 – all points lie inside the control limits and no unusual patterns are present. A control chart for the statistically unstable process is in Fig. 1.6 – two points lie outside the control limits; it signals that the process is out of control with high probability.

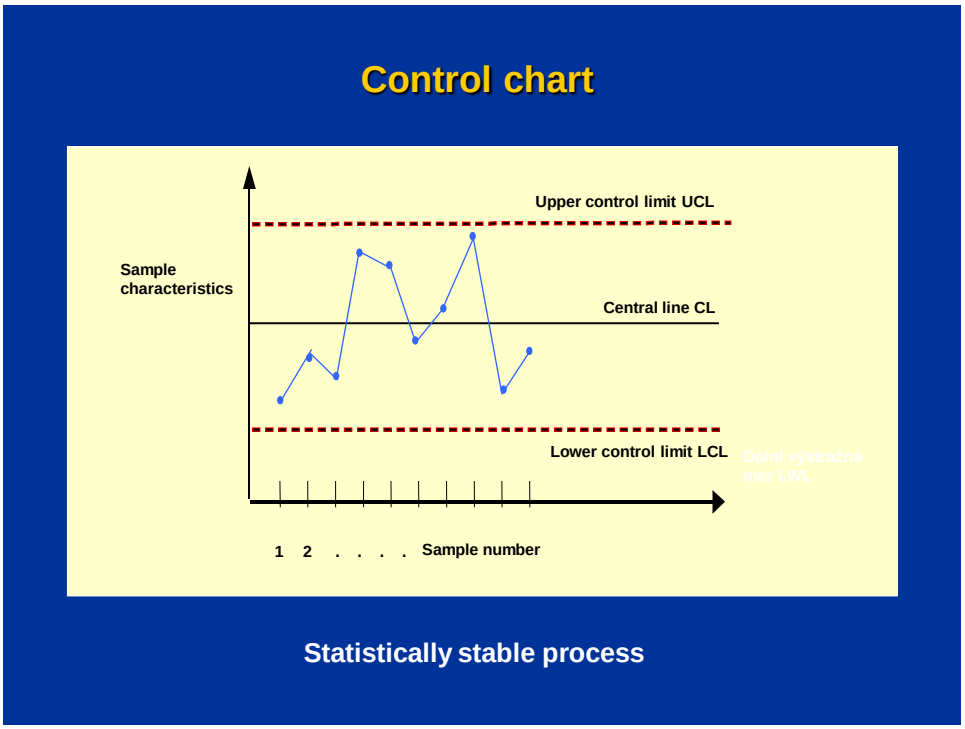


Fig. 1.5 Example of a control chart for statistically stable process

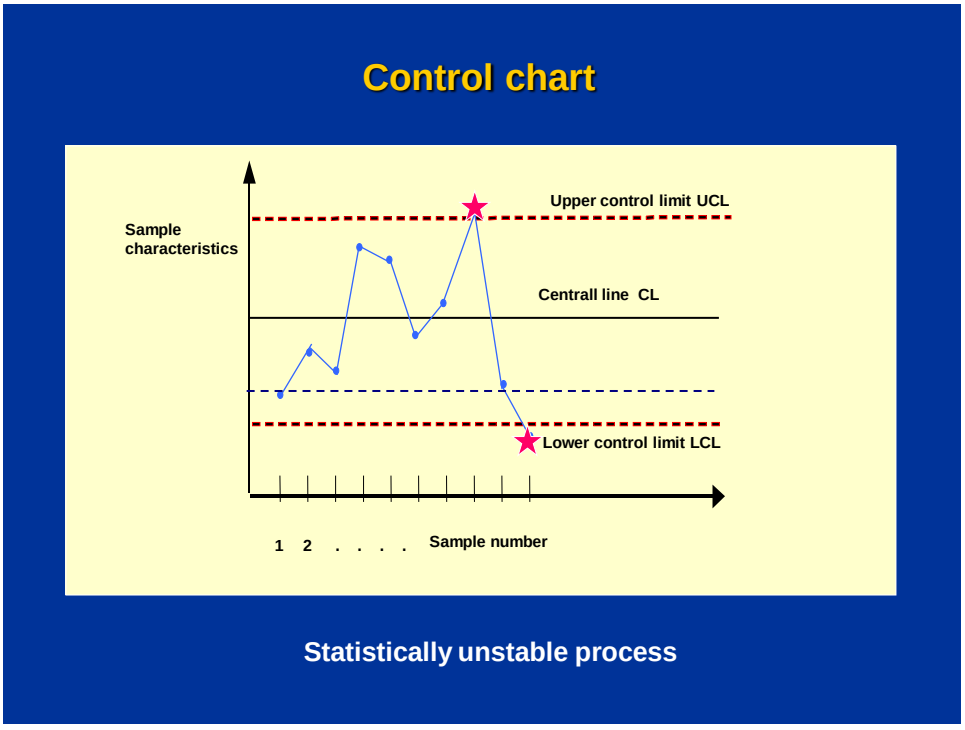


Fig. 1.6 Example of a control chart for statistically unstable process

Definition of the process capability

Capable process is a process which is able to meet customer requirements on a long term and stable basis.

In the frame of the capability analysis we explore if the statistically stable process is able to meet customer requirements defined for instance as tolerance limits (LSL - lower specification limit, USL - upper specification limit). The schematic depiction of the capable process (part A) and non-capable process (part B) can be seen in Fig. 1.7.

Statistical stability is one of the main assumptions for correct evaluation of the process capability.

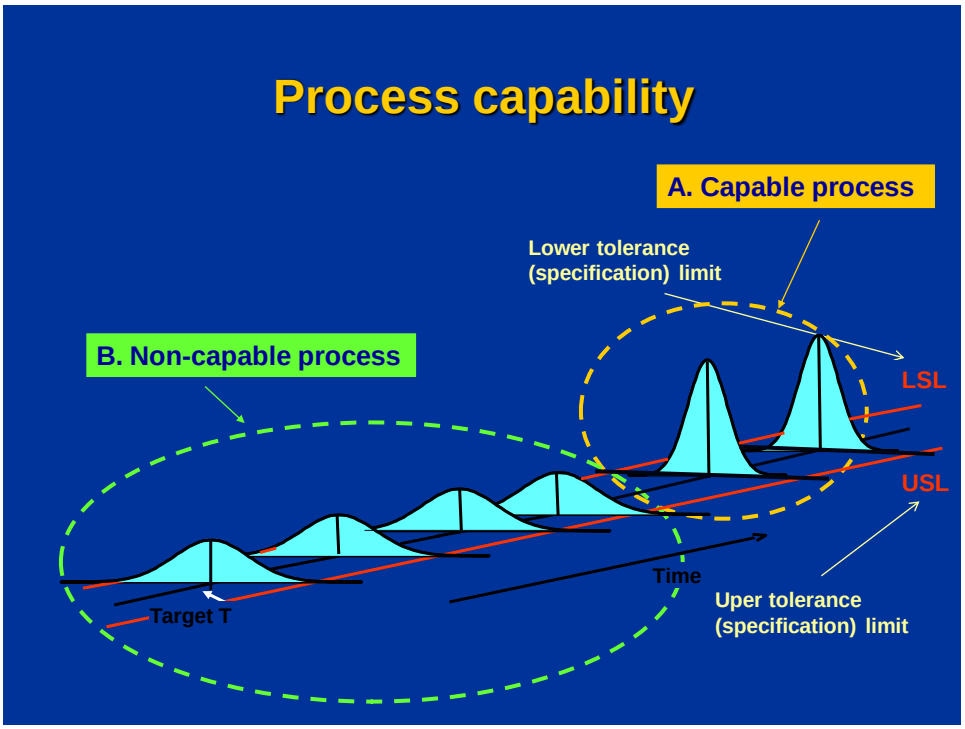


Fig. 1.7 Capable and non-capable process

In part A of Fig. 1.7 we can see that the process is statistically stable and only a small part of the process is outside the tolerance limits. For that reason the process can be considered capable. The capability rate can be quantified using capability indices (for more see Chapter 3 of this textbook). Part B of Fig. 1.7 shows that the process is statistically stable but a significant part of it goes outside the tolerance. The process cannot be considered capable.

Basic tools for the process capability evaluation

Histogram and capability indices are the main tools for the process capability evaluation and analysis (for more see Chapter 3 of this textbook).

Relation between the process statistical stability and capability

Combinations of two basic properties of every process are shown in the table below (Fig. 1.8). The table also contains short instructions of what to do in a particular situation (combination).

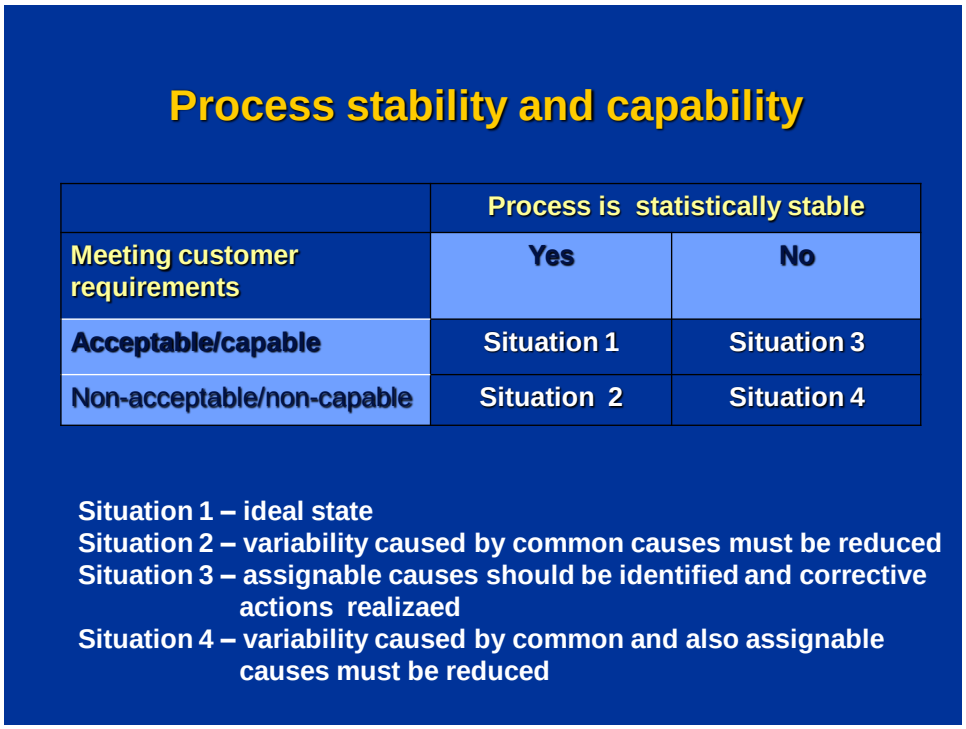


Fig. 1.8 Combinations of basic process properties

In situations 3 and 4 the terms „capable“ and „non-capable“ process cannot be used because capability presumes a statistically stable process. For that reason the terms acceptable/non-acceptable process are used. Every situation is depicted in a particular figure (see Fig. 1.9 – 1.12).

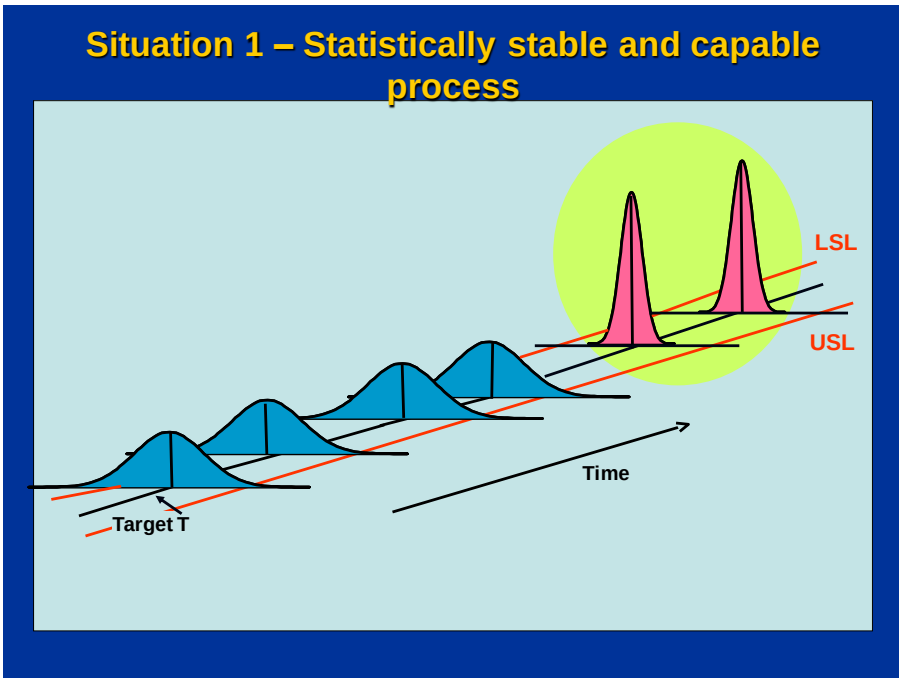


Fig. 1.9 Schematic depiction of situation 1

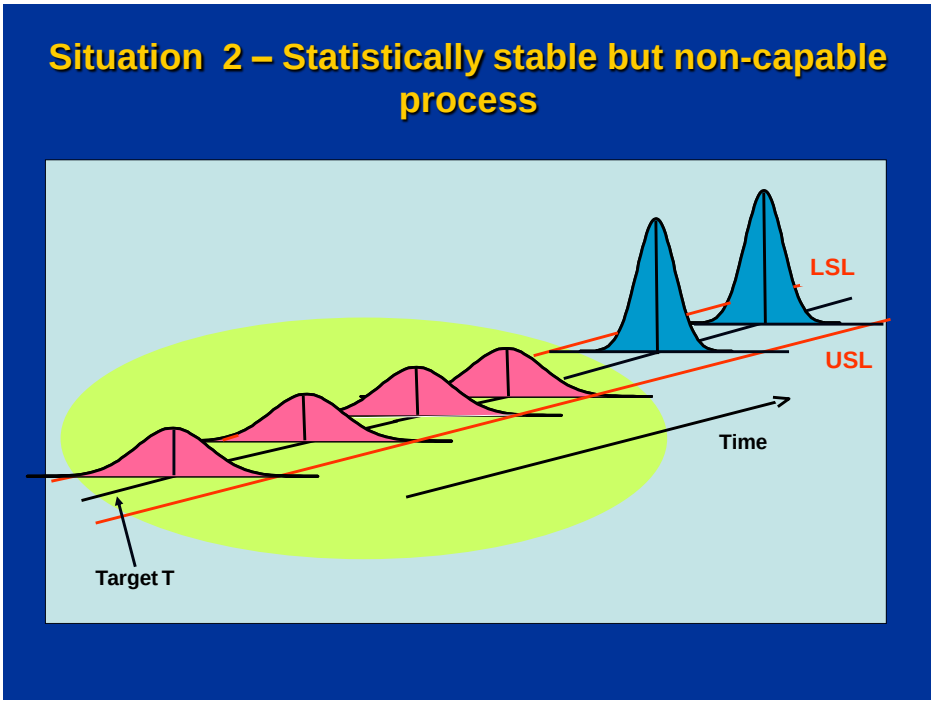


Fig. 1.10 Schematic depiction of situation 2

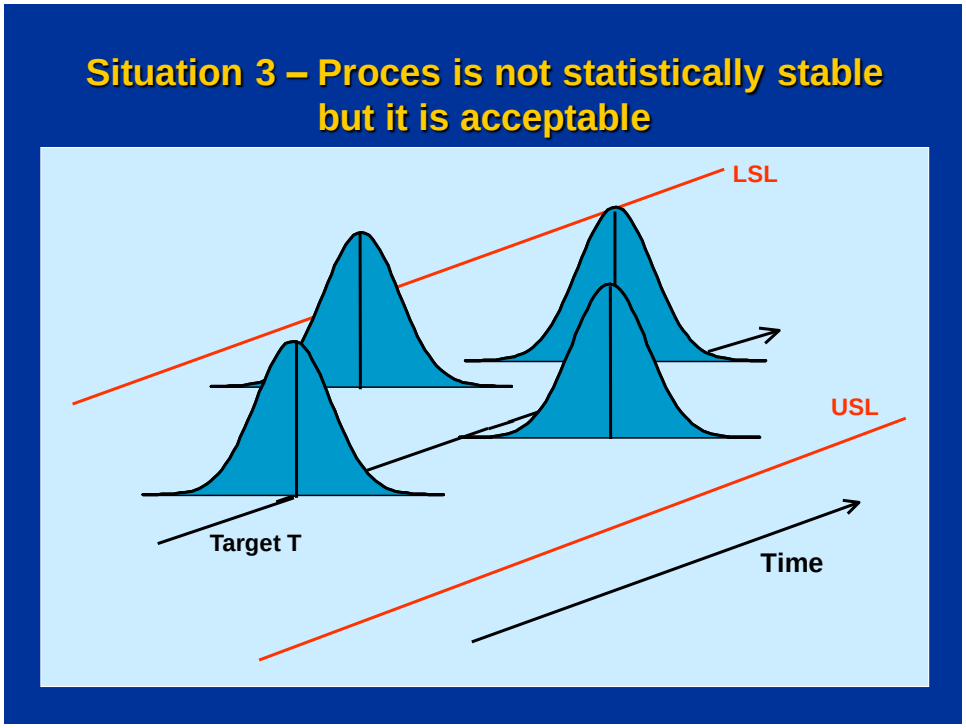


Fig. 1.11 Schematic depiction of situation 3

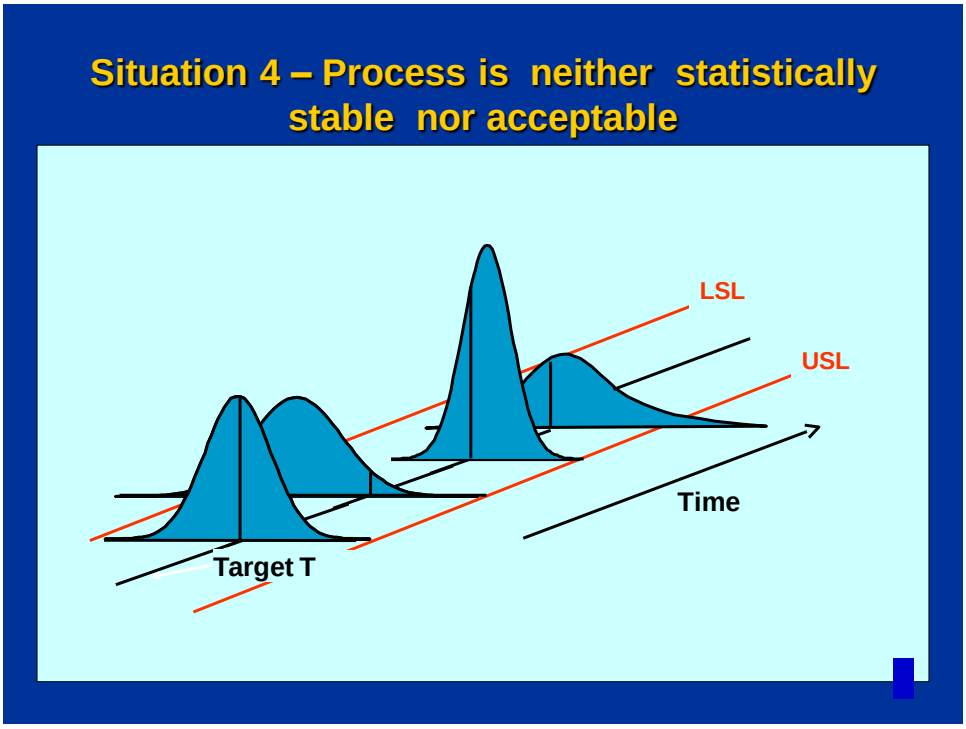


Fig. 1.12 Schematic depiction of situation 4

Definition of the process improvement

Concept of the process improvement, being in relation to the concept of the process variability applied in this textbook, can be expressed as follows:

If we want to implement a **continual process improvement**, then it is necessary to permanently monitor the process behaviour with the aim to meet and maintain statistically stable state through identification and partial or total elimination of effects of assignable causes. The process behaviour and quality of its outputs are then predictable and it is possible to objectively evaluate its capability, i.e. its ability to meet customer requirements in the long-term aspect. After that the natural variability caused by common causes of the process can be reduced. It means that the process capability will rise and it represents the real **process improvement.** (see Fig. 1.13).

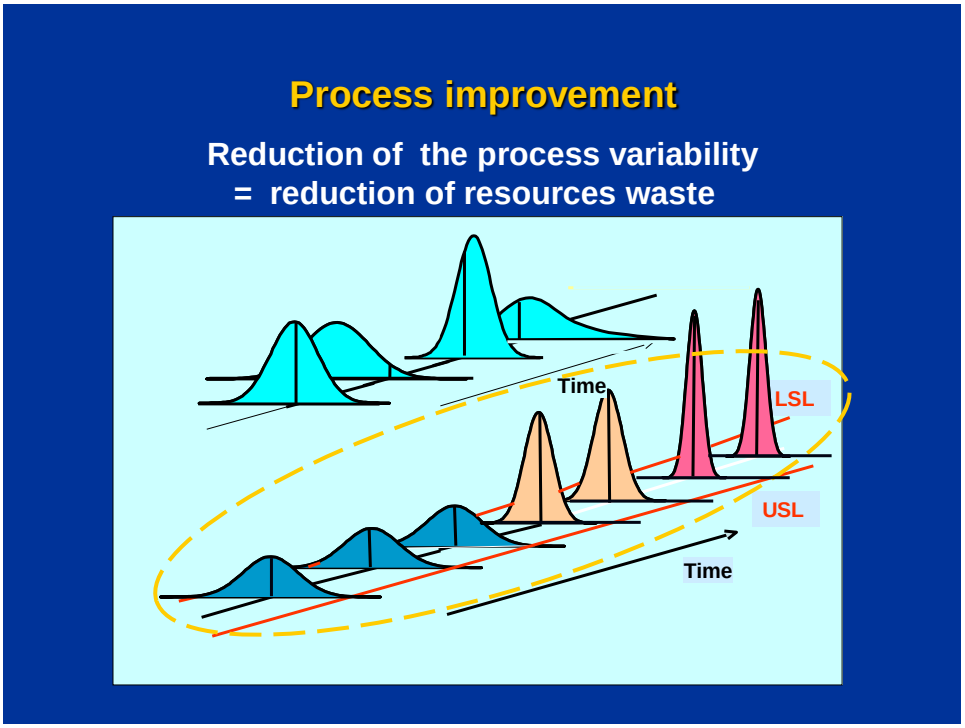


Fig. 1.13 Schematic depiction of the process improvement

Summary of terms

After studying this chapter you should understand these terms:

- **process variability**
- **common causes**
- **assignable causes**
- **statistically stable process**
- **statistically non-stable process**
- **control chart**
- **capable process**
- **non-capable process**
- **acceptable process**
- **non-acceptable process**

process improvement

Questions

1. **What is the reason for the process variability?**
2. **Is it possible to eliminate the process variability totally?**
3. **What groups of causes generate the process variability?**
4. **Why are we interested in the process variability in relation to meeting the customer requirements?**
5. **Define common causes of the process variability and give some practical examples.**
6. **Define assignable causes of the process variability and give some practical examples.**
7. **What basic tools for the analysis of the process statistical stability do you know?**
8. **What causes of the variability affect the process when it is statistically stable?**
9. **What causes of the variability affect the process when it cannot be considered statistically stable?**
10. **How is it possible to achieve the statistically stable process?**
11. **What does a „capable process" mean?**
12. **What is the relation between the process statistical stability and the**

process capability?

13. **What is the relation between a "capable" and an "acceptable" process?**
14. **What is the concept of the process improvement applied in this textbook?**
15. **What is the relation between improvement, statistical stability and capability of the process?**

Example

Let the process "solid target shooting" be the object of our studies (see Fig. 1.14).

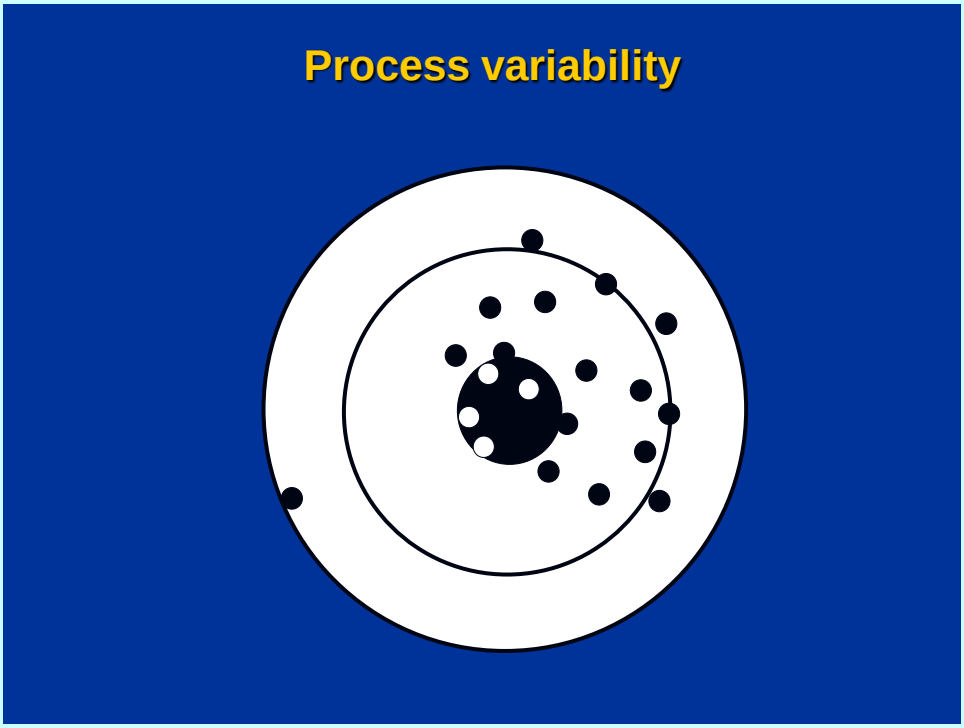


Fig. 1.14 Output of the process "solid target shooting"

Your tasks are as follow:

- find out from the picture if the process has been affected only by common causes or by some assignable causes, too;
- define some probable assignable cause;
- define various common causes of the variability of this process.

Additional study resources

[1] MONTGOMERY, D. C.: *Statistical Quality Control. A Modern Introduction*. New York : J. Wiley & Sons, 2012. 768 p. ISBN: 978-1-1183-2257-4.
[2] TOŠENOVSKÝ, J. – NOSKIEVIČOVÁ, D.: *Statistické metody pro zlepšování jakosti*. Ostrava: Montanex, 2000. 362 p. ISBN 80-7225-040-X.
[3] WHEELER, D.: *Understanding Variation. The Key to Managing Chaos*. 2nd Ed. Knoxville, Tennessee: SPC Press Inc., 2000. 158 p. ISBN 978-0945320531.

Solution to example

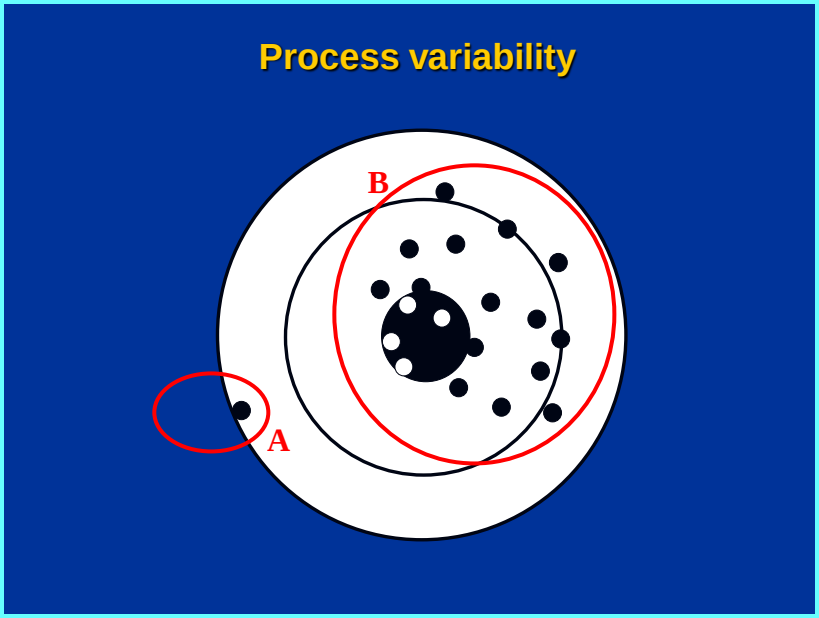


Fig. 1.15 Solution to the task

Analysing Fig. 1.14 we can state that the process was effected by an assignable cause which caused that one hit (point A, Fig. 1.15) lies so far from the other ones (field B, Fig. 1.15). This large departure could be caused for instance by a blast, sudden quiver of a shooter's hand, disturbing the shooter by some sound, movement, flash, etc.

The rest of hits were affected by common causes. These causes can be divided into several groups such as:

Human factor (shooter)

age, physical disposition, talent, experience, training, eyes, ears, psychical resistance, etc.

Environment

temperature, pressure, lifting, etc.

Methods

frequency, length, quality of training, etc.

Gun

quality, rate of wear and tear, maintenance method, etc.

Projectiles

weight, shape, quality, etc.

2 COMPLEX STATISTICAL ANALYSIS OF ONE-DIMENSIONAL DATA

Chapter structure

Topics for review:

- general procedure of testing a statistical hypothesis

- exploratory graphs

Time for learning: 480 minutes

Goal: after studying this chapter

- **you will become aware of the necessity to know and verify the data prerequisites for appropriate statistical methods selection;**
- **you will be able to apply suitable instruments for verifying particular prerequisites and apply them in a complex way;**
- **you will learn to select the most suitable test of statistical hypothesis in**

relation to a specific situation;

- **you will achieve the ability to correctly interpret selected exploratory graphs and tests of statistical hypothesis.**

Lecture

Basic procedures for verifying data prerequisites

There exist two basic ways of how to verify data prerequisites:

- exploratory graphs,

- testing a statistical hypothesis.

Basic verified prerequisites

Basic data preconditions which should be verified as a basis for correct analysis of other data are as follows:

- data normality,
- data homogeneity,
- constancy of the mean and dispersion,
- data independence.

Basic graphical tools for verifying data prerequisites

The table below presents the most applied exploratory graphs related to the data prerequisites.

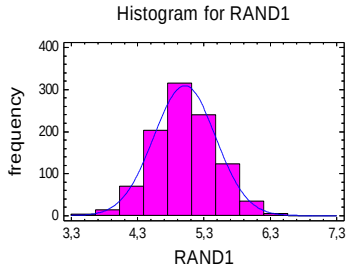
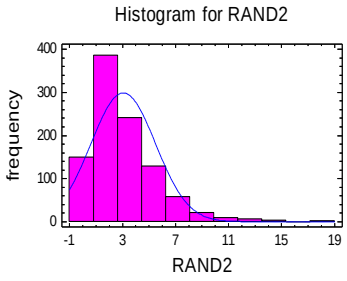
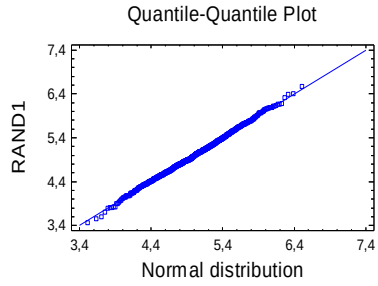
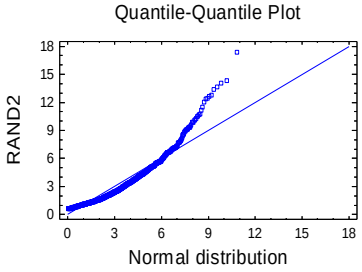
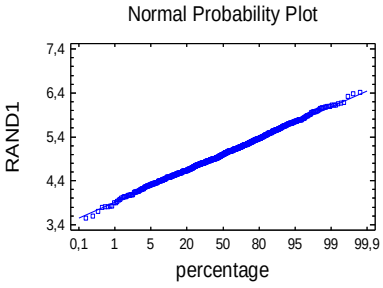
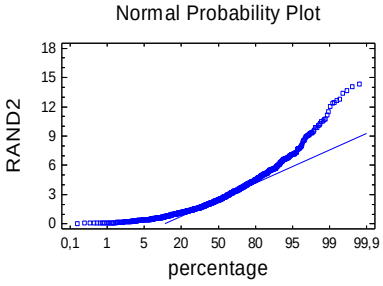
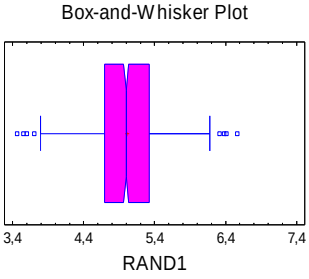
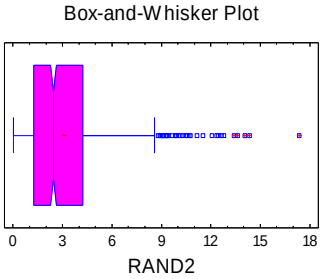
Tab. 2.1 Graphical tools for data prerequisites verification

Verified prerequisite	***Tool***
Data correlation	Scatter plot (correlation graph)
Data autocorrelation	Scatter plot (autocorrelation graph) Sample autocorrelation function ACF Sample partial autocorrelation function PACF
Data normality	Q-Q plot (quantile-quantile graph) Probability plot Box and Whisker plot Histogram
Constant dispersion	Scatter plot $\overline{x}$, s
Constant mean	Graph of the original time-series Graph of the time-series of differences Sample autocorrelation function ACF
Data homogeneity	Q-Q plot Probability plot Box and Whisker plot Histogram

Examples of graphs

In the following table examples of the exploratory graphs – for normally distributed data on the left and for asymmetric distribution (skewed to the right) on the right - can be seen.

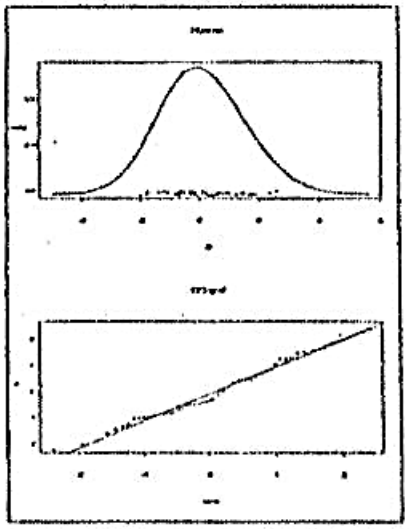
Tab. 2.2 Examples of exploratory graphs

Graphs for normally distributed data	Graphs depicting right skewed data
 Histogram	 Histogram
 Q-Q plot	 Q-Q plot
 Probability plot	 Probability plot
 Box and Whisker plot	 Box and Whisker plot

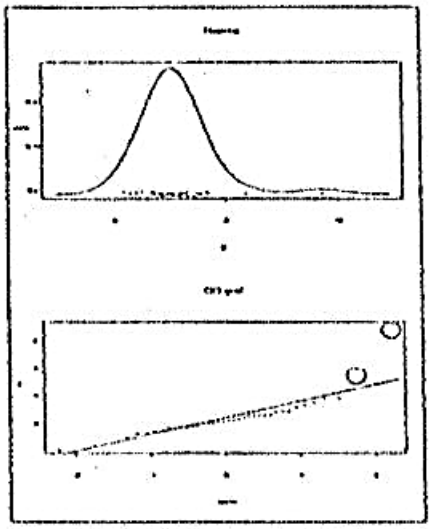
Interpretation of Q-Q plots

In the following table Q-Q plots for the data from various distributions are depicted.

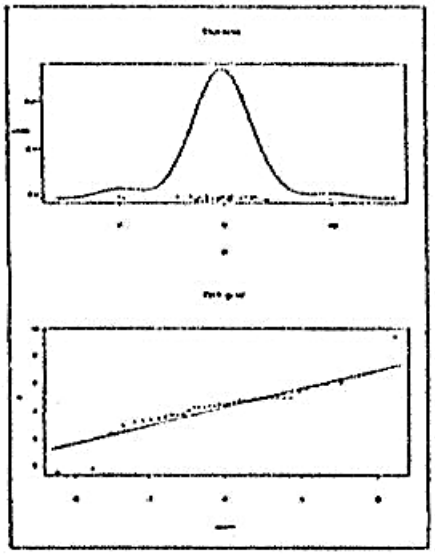
Tab. 2.3 Q-Q plots for data from various distributions



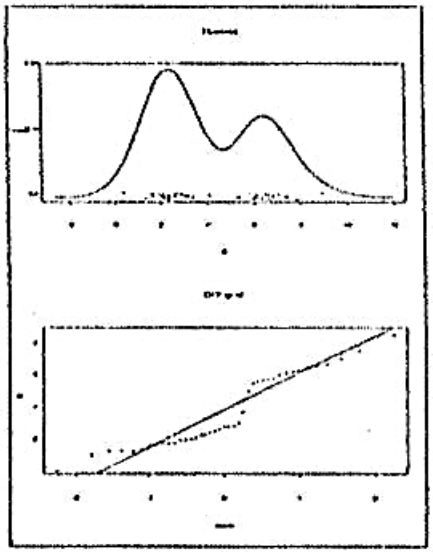
A Approximately normal distribution



B Approximately normal distribution, two outliers

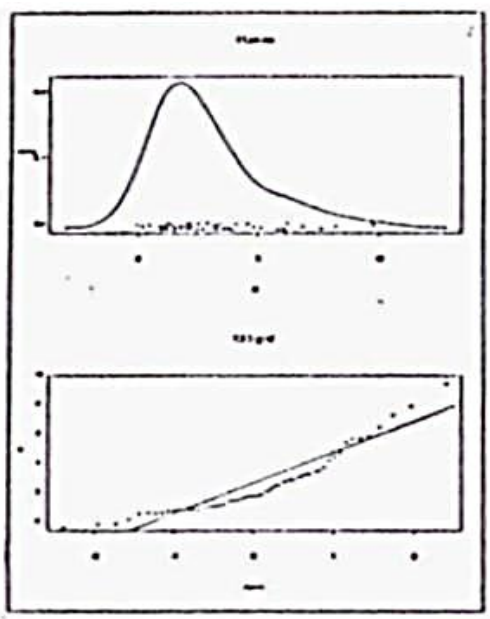


C Approximately normal distribution, two outliers on the left, one on the right

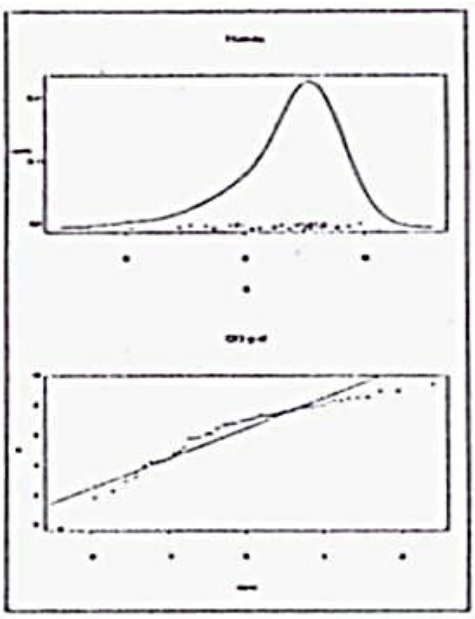


D Bimodal distribution

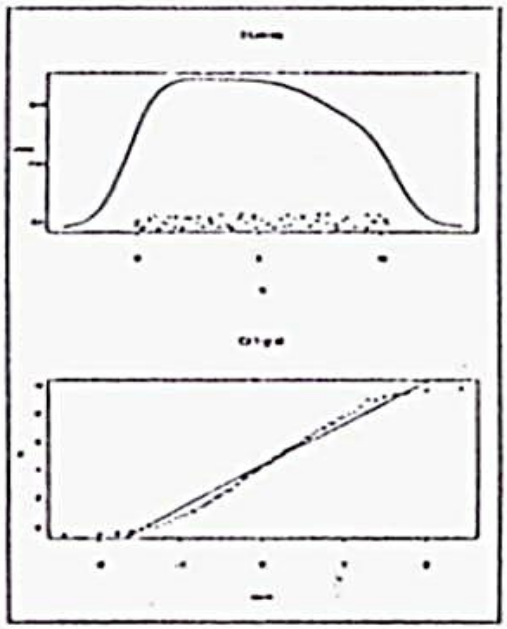
Tab 2.3 – cont.



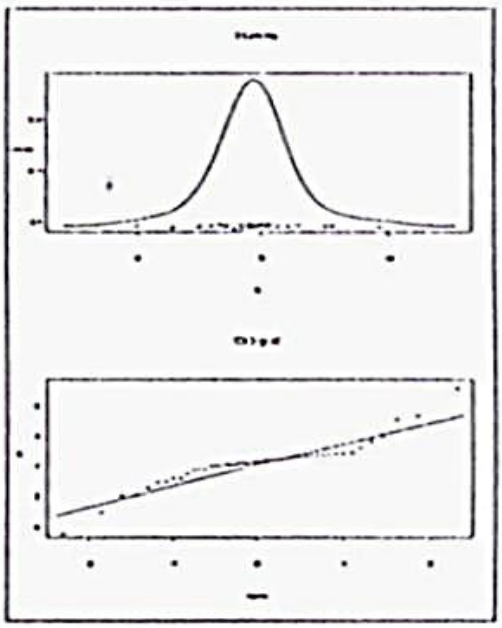
E Right skewed distribution



F Left skewed distribution



G Distribution with kurtosis smaller than normal distribution



H Distribution with kurtosis bigger than normal distribution

The following table contains a brief interpretation of the previous situations A- H.

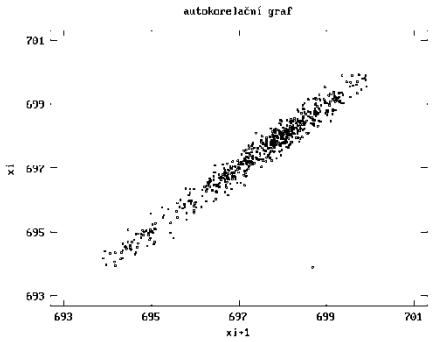
Tab. 2.4 Interpretation of Q-Q plots

Situation	***Interpretation***
A	- it is expected that data come from normal distribution - verification using some normality test and suitable exploratory graph
B,C	- existence of outliers - it can be assumed that the tests for outliers correctly identified these points - outliers shall be verified, explained and alternatively eliminated from the data set
D	- mixture of two samples from different distributions
E, F	- data with high probability do not come from normal distribution - using arithmetic average as an estimator of the mean should be incorrect - look for and eliminate outliers based on common tests should be wholly incorrect (misrepresentation of the significance intervals, quantiles estimation, control limits, etc.) - effective solution – data transformation
G	- excessive rounding - making use of the whole tolerance area instead of maintaining the process close to the centre of it - strong autocorrelation or trends - higher dispersion - worse C_p and higher losses
H	- non-constant dispersion

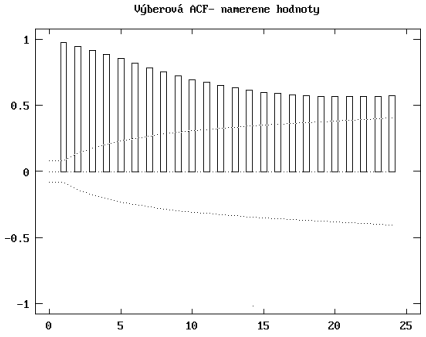
Graphical verification of data independence

In the following table some graphical tools which can offer information about data independence are summarised.

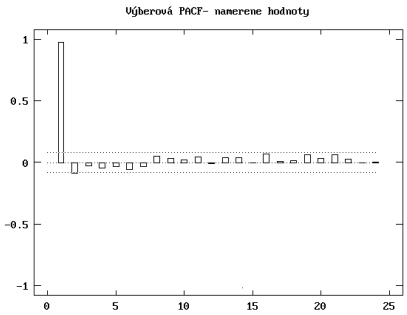
Tab. 2.5 Graphs for verification of data independence



Scatter plot of autocorrelation



Sample autocorrelation function ACF



Sample partial autocorrelation function PACF

Verification of data prerequisites using testing of statistical hypotheses – summary of basic tests

The list of the tests of statistical hypotheses in relation to a particular data prerequisite is given in Tab. 2.6. Selection of the right test is also crucial for a correct data analysis and for correct conclusions.

Tab. 2.6 Selected tests of statistical hypothesis

Verified prerequisite	***Tool***
Data independence	• Test of significance of the correlation coefficient • Test of significance of the Spearman coefficient
Data non-autocorrelation	• Nonparametric tests for randomness • Test of significance of the autocorrelation coefficient
Data normality	• X^2 test (for large data sets sized $n > 100$ • Shapiro-Wilk test (for data sets sized $8 \leq n \leq 50$) • Test of sample skewness (for data sets sized $n \geq 8$) • Test of sample kurtosis (for data sets sized $n \geq 8$) • Combined test of kurtosis and skewness (for the data sets sized $20 \leq n \leq 1000$)
Constant dispersion in time	• Bartlett test
Constant mean in time	• Test of significance of the straight line slope • Brown and Forsyth F-test
Data homogeneity	• Dixon test, Grubbs test

Normality tests based on the sample skewness and kurtosis

For all three tests mentioned below the following definitions are common:

Null hypothesis H_0:

$x_1, x_2, \ldots, x_n$ is a random sample from the normal distribution $N(\mu,\sigma^2)$.

Alternative hypothesis H_1:

$x_1, x_2, \ldots, x_n$ is not a random sample from the normal distribution $N(\mu,\sigma^2)$.

Different features of every test are described below.

1. Test based on the sample skewness

Properties of the test statistic:

Suppose that H_0 is true, then the expected value and dispersion of the random variable a_3 (sample skewness) can be computed as follows:

$$E(a_3) = 0 \qquad D(a_3) = 6(n-2)/(n+1)(n+3), \tag{2.1}$$

Standardized random variable U_a can be then computed:

$$U_a = \frac{a_3}{[D(a_3)]^{1/2}}, \tag{2.2}$$

This test statistic has asymptotically (for $n \to \infty$) standardized normal distribution N(0,1).

Defining the critical region:

Critical region of this test can be formulated as follows:

$$W = \{x_1,..x_n : |U_a| \rangle u_{1-\alpha/2}\}, \tag{2.3}$$

where

$u_{1-\alpha/2}$ - is a $100[1-(\alpha/2)]\%$ quantile of N(0,1).

Formulation of the test result:

a) if $|U_a| \rangle u_{1-\alpha/2}$, then the test rejects H_0 for H_1 on the significance level approximately equal to α,

b) if $|U_a| \leq u_{1-\alpha/2}$, then the test does not reject H_0.

2. Test based on the sample kurtosis

Properties of the test statistic:

Suppose that H_0 is true, the expected value and dispersion of the random variable e_4 (sample kurtosis) can be computed as follows:

$$E(e_4) = 3 - \frac{6}{n+1} \qquad D(e_4) = \frac{24n(n-2)(n-3)}{(n+1)^2(n+3)(n+5)}, \tag{2.4}$$

Standardized random variable U_e can be then computed:

$$Ue = \frac{e_4 - E(e_4)}{[D(e_4)]^{1/2}}. \tag{2.5}$$

This test statistic has asymptotically (for $n \to \infty$) standardized normal distribution N(0,1).

Defining the critical region:

Critical region of this test can be formulated as follows:

$$W = \{x_1, \ldots x_n : |U_a| \rangle u_{1-\alpha/2}\}, \tag{2.6}$$

where

$u_{1-\alpha/2}$ - is a tabelated $100[1-(\alpha/2)]\%$ quantile of N(0,1).

Formulation of the test result:

a) if $|U_e| \rangle u_{1-\alpha/2}$, then the test rejects H_0 for H_1 on the significance level approximately equal to α,

b) if $|U_e| \leq u_{1-\alpha/2}$, then the test does not reject H_0.

3. Combined test based on the sample skewness and kurtosis

- Test statistic: $C_1 = U_a^2 + U_e^2$ (2.7)
- Supposing that data are normally distributed this test statistic has asymptotically $\chi^2(2)$ distribution.
- Critical value: $\chi^2_{1-\alpha}(2)$.
- Formulation of the test result
  a) if $C_1 > \chi^2_{1-\alpha}(2)$, then the test rejects H_0 for H_1 on the significance level approximately equal to α;
  b) if $C_1 < \chi^2_{1-\alpha}(2)$, then the test does not reject H_0.

Outlier tests

1. Dixon test

Correct application of this test supposes meeting the folowing prerequisites:

- data are normally distributed,
- data are sorted.

Procedure of the Dixon test application is as follows:

1. Defining H_0: the deviation is random; the tested value is not outlier.
   Defining H_1: the deviation is not random; the tested value is outlier.
2. Computation of the test statistic:

$$Q_1 = \frac{x_2 - x_1}{x_n - x_1}, \text{ resp. } Q_n = \frac{x_n - x_{n-1}}{x_n - x_1}. \tag{2.8}$$

3. Choice of the significance level α and critical value Q_α (it depends on the sample size n and α).
4. Decision about rejection of H_0:
   a) if $Q_1 > Q_\alpha$, resp. $Q_n > Q_\alpha$, then H_0 is rejected and the tested value is considered an outlier;
   a) if $Q_1 \leq Q_\alpha$, resp. $Q_n \leq Q_\alpha$, then H_0 is not rejected and the tested value is not considered an outlier.

2. Grubbs test

Correct application of this test supposes meeting the prerequisite that data are normally distributed.

Procedure of the Grubbs test application is as follows:

1. Defining H_0: the deviation is random, the tested value is not outlier.
2. Defining H_1: the deviation is not random, the tested value is outlier.

3. Computation of the test statistic:

$$T_1 = \frac{\bar{x} - x_1}{s}, \text{ resp. } T_n = \frac{x_n - \bar{x}}{s} \quad . \tag{2.9}$$

4. Choice of the significance level α and setting the critical value T_α (it depends on the sample size n and α).

5. Decision about rejection of H_0:

   a) if $T_1 > T_\alpha$, resp. $T_n > T_\alpha$, then H_0 is rejected and the tested value is considered an outlier;

   a) if $T_1 \leq T_\alpha$, resp. $T_n \leq T_\alpha$, then H_0 is not rejected and the tested value is not considered an outlier.

Solution of outliers

When some outlier is identified, it is necessary to analyse the cause of its existence. According to the nature of this cause it must be decided if this outlier is to be removed from the data set or not. Some recommendations related to this decision are summarized in the following table.

Tab. 2.7 Types of outliers and their solution

Type of outlier	*Impacts/Solution*
- gross errors - typing errors - verifiable failure of people or equipment	- correction - elimination
- impact of faults, incorrect gauging, technological errors	- elimination of an outlier only on condition that the outlier will be identified, its cause will be explained and some corrective action will be taken
- data are from the distribution with non-constant variance	- outliers shall not be eliminated (elimination should lead to underestimation of variance, increase of the false risk in SPC, increase of capability indices *Solution*: - robust estimators
- data are from skewed distribution	- outliers shall not be eliminated (all estimates of parameters, significance intervals or control limits in control charts etc. could be incorrect) *Solution:* - data transformation, robust estimators.

Tests for randomness

In this paragraph two different tests for randomness are described.

The definition of hypotheses is the same for the both tests:

H_0: data are independent (they are not auto-correlated);

H_1: data are not independent (they are auto-correlated).

1. test

This is based on the number of runs (changes in the trend of values).

n_1 = the number of runs upwards; n_2 = the number of runs downwards.

2. test

This test is more sensitive to global tendencies in the data as compared to the previous test. It comes out of the number of exceedings of the median Me (i.e. cases when the value x_i > Me is followed by the value x_{i+1}< Me, or the value x_i < Me follows the value x_{i+1}> Me)

n_1 = number of values above Me,

n_2 = number of values below Me.

Further on, the following can be applied to both tests:

The expected value and dispersion of the number of runs (changes) *R* can be computed as follows:

$$E(R) = \frac{2.n_1.n_2}{n_1 + n_2} + 1, \tag{2.10}$$

$$D(R) = \frac{2n_1n_2(2n_1n_2 - n_1 - n_2)}{(n_1 + n_2)^2(n_1 + n_2 - 1)}, \tag{2.11}$$

The formulae for the computation of the test statistics are:

$$Z = \frac{|R - E(R)| - 0{,}5}{\sqrt{D(R)}} \quad \text{for } R \geq E(R), \tag{2.12}$$

$$Z = -\frac{|R - E(R)| - 0{,}5}{\sqrt{D(R)}} \quad \text{for } R < E(R). \tag{2.13}$$

For *Z* variable *P*- values which are compared to the value of significance level α are computed ([4], [5]). The decision about the data independence for $\alpha = 0.05$ is formulated as follows:

a) if P- value ≤ 0.05 , then H_0 is rejected (data are not considered independent - they are auto-correlated);

b) if P- value > 0.05 , then H_0 is not rejected (data are considered independent - they are not auto-correlated).

Summary of terms

After studying this chapter you should understand the following terms:

- **exploratory graph**
- **run**
- **test of the sample skewness**
- **test of the sample kurtosis**
- **combined test of the sample skewness and kurtosis**
- **tests for randomness**
- **Dixon test**
- **Grubbs test**
- **A-D test**
- **Shapiro-Wilk test**
- **correlation graph**
- **autocorrelation graph**
- **autocorrelation function**
- **partial autocorrelation function**
- **Q-Q graph**
- **outlier**

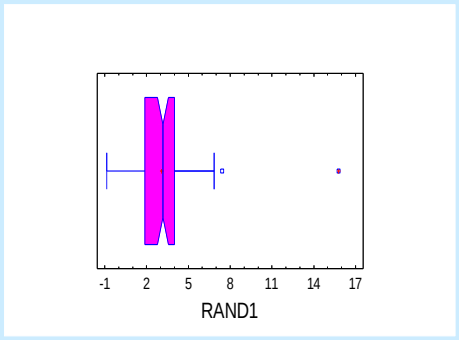
Questions

1. **Is it sufficient to verify the data prerequisites using only graphs or only tests of statistical hypotheses? If not, explain why.**
2. **Which graphs are helpful for the verification of the data homogeneity?**

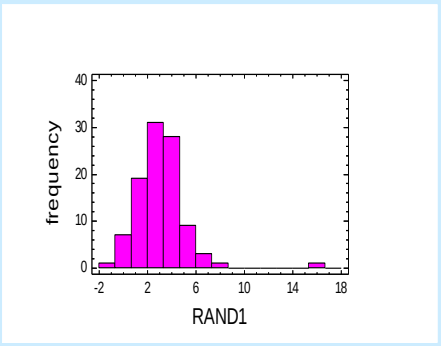
3. **What can be read from a Q-Q plot?**
4. **Which plots can be used for verifying the data independence?**
5. **How can the data normality be graphically verified?**
6. **Describe the general procedure of testing a statistical hypothesis.**
7. **What is the criterion for correct choice of the test for verifying the data normality?**
8. **What represent columns in autocorrelation function graph?**
9. **What is the distribution of the test statistic in the test based on the sample skewness?**
10. **What is the distribution of the test statistic in the combined test based on the sample skewness and kurtosis?**
11. **How is the alternative hypothesis formulated in the combined test based on the sample skewness and kurtosis?**
12. **How is the null hypothesis formulated in the X^2 test?**
13. **Which hypothesis is verified in the tests for randomness?**
14. **Which statement is opposed to the null hypothesis in the Grubbs test?**
15. **Which prerequisites shall be met for a correct application of the Grubbs test?**
16. **Is it correct to remove the identified outlier from the data set every time? If not, why?**
17. **What is the Box-Pierce test?**

Example

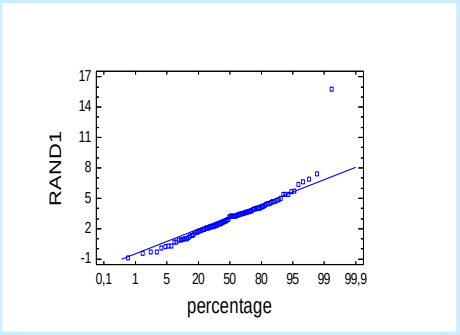
Let us have the data set of 100 values of the shaft diameter (the data set RAND1). Verify that the data are normally distributed, homogeneous and independent. Verify the validity of these hypotheses on the base of the interpretation of the following graphs and test results (see Fig. 2.1 - 2.5 and Tab. 2.8 - 2.11, tasks a) – i)).



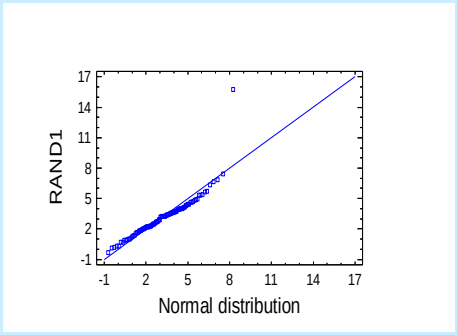
a) Fig. 2.1



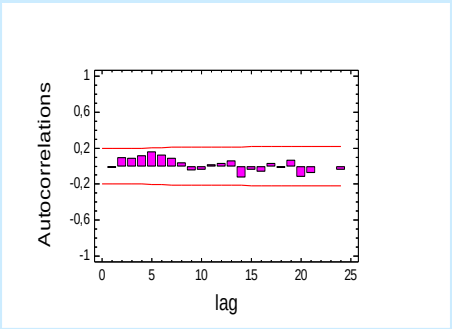
b) Fig. 2.2



c) Fig. 2.3



d) Fig. 2.4



e) Fig. 2.5

At every figure and table explain what a particular graph or test represents.

f) Tab. 2.8

Goodness-of-Fit Tests for RAND1

Chi-Square Test

	Lower Limit	Upper Limit	Observed Frecuency	Expected Frequency	Chi-Square
	at or below	0,639463	8	12,50	1,62
	0,639463	1,64123	12	12,50	0,02
	1,64123	2,39035	18	12,50	2,42
	2,39035	3,06114	11	12,50	0,18
	3,06114	3,73193	20	12,50	4,50
	3,73193	4,48106	15	12,50	0,50
	4,48106	5,48282	9	12,50	0,98
above	5,48282		7	12,50	2,42

Chi-Square = 12,6396 with 5 d.f. P-Value = 0,0270009

Estimated Kolmogorov statistic DPLUS = 0,0992692
Estimated Kolmogorov statistic DMINUS = 0,0545804
Estimated overall statistic DN = 0,0992692
Approximate P-Value = 0,279046

EDF Statistic	Value	Modified Form	P-Value
Kolmogorov-Smirnov D	0,0992692	1,00014	<0.05*
Anderson-Darling A^2	1,48735	1,49884	0,0007*

g) Tab. 2.9

Tests for Randomness of RAND1

Runs above and below median

Median = 3,1763
Number of runs above and below median = 50
Expected number of runs = 51,0
Large sample test statistic z = -0,100509
P-value = 0,919935

Runs up and down

Number of runs up and down = 65
Expected number of runs = 66,3333
Large sample test statistic z = -0,199458
P-value = 0,8419

Box-Pierce Test

Test based on first 24 autocorrelations
Large sample test statistic = 13,7236
P-value = 0,952649

h) Tab. 2.10

Grubbs' Test (assumes normality)
Test statistic = 6,03599
P-Value = 1,66864E-9

i) Apply and interpret results of the tests based on the sample skewness, sample kurtosis and combined test based on the sample skewness and kurtosis (see Tab. 2.11).

Tab. 2.11

Stnd. skewness	9,01163
Kurtosis	12,385
Stnd. kurtosis	25,2808

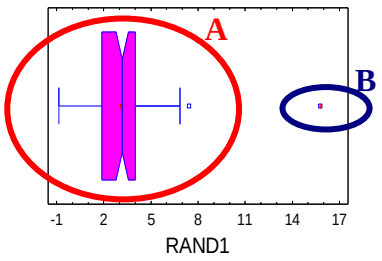
Additional study resources

[1] DIXON, W. J. - MASSEY, F. J. *Introduction to statistical analysis* (4th ed.). New York: McGraw-Hill., 1983.
[2] MONTGOMERY, D. C.: Introduction to Statistical Quality Control.
[3] SNEDECOR, GEORGE W. – COCHRAN, W. G.: *Statistical Methods*.
[4] Statgraphics Centurion XV. *On-line User Manual*. Available from www.statgraphics.com/documents.htm.
[5] Statgraphics Plus Version 5.0.

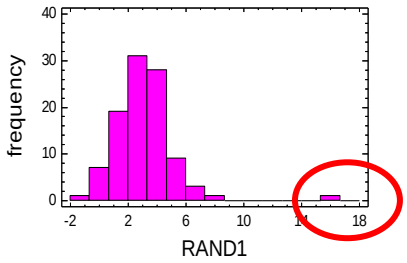
Topics for supplementary self-study

- Bartlett test,
- Brown and Forsyth test,
- Wild test,
- Durbin-Watson test,
- test of significance of the correlation coefficient,
- test of significance of the straight line slope,
- theoretical base of graphs (probability plot, Q-Q plot, Box and Whisker plot, ACF, PACF),
- detailed interpretation of ACF, PACF (Box-Jenkins methodology),
- Box-Pierce test.

Solution to example



a) Fig. 2.1



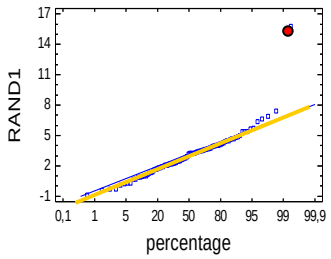
b) Fig. 2.2

Ad a)

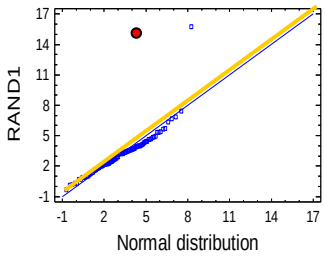
In Fig. 2.1 the Box and Whisker plot can be seen. Position of the mean in relation to the median and approximately the same length of the vertical lines (whiskers) (see ellipse A) signalize that the data apparently have symmetrical distribution. But there is apparently one outlier (see ellipse B).

Ad b)

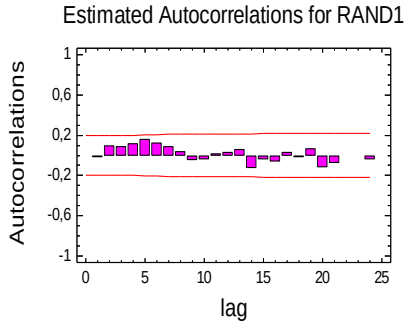
In Fig. 2.2 a histogram can be seen. Based on the analysis of its shape we can come to the similar conclusion that the data apparently have symmetrical (perhaps normal) distribution but there is probably one outlier.



c) Fig. 2.3



d) Fig. 2.4



e) Fig. 2.5

Ad c) and d)

Based on the analysis of figures 2.3 **(**probability plot) and 2.4 (Q-Q plot) it can be arrived at similar but more concrete conclusion that the data are normally distributed – yellow straight lines representing the supposed theoretical distribution are tightly covered with blue points representing the empirical distribution. In addition, there can be seen one point (the red one) that is located significantly far from the rest of points; this point probably represents the outlier.

Ad e)

In Fig. 2.5 we can see a correlogram. It is the graph of the autocorrelation function ACF. The k-th column represents the value of the sample autocorrelation coefficient r_k for the k-th lag. As all columns lie in the confidence interval (given by red lines), it can be considered that the original analysed data are independent (not auto-correlated).

Ad f) – Tab. 2.8

Goodness-of-Fit Tests for RAND1

Chi-Square Test

Lower Limit	Upper Limit	Observed Frecuency	Expected Frequency	Chi-Square
at or below	0,639463	8	12,50	1,62
0,639463	1,64123	12	12,50	0,02
1,64123	2,39035	18	12,50	2,42
2,39035	3,06114	11	12,50	0,18
3,06114	3,73193	20	12,50	4,50
3,73193	4,48106	15	12,50	0,50
4,48106	5,48282	9	12,50	0,98
above 5,48282		7	12,50	2,42

Chi-Square = 12,6396 with 5 d.f. P-Value = 0,0270009

Estimated Kolmogorov statistic DPLUS = 0,0992692
Estimated Kolmogorov statistic DMINUS = 0,0545804
Estimated overall statistic DN = 0,0992692
Approximate P-Value = 0,279046

EDF Statistic	Value	Modified Form	P-Value
Kolmogorov-Smirnov D	0,0992692	1,00014	<0.05*
Anderson-Darling A^2	1,48735	1,49884	0,0007*

In Tab. 2.8 results of the selected goodness-of-fit tests (X^2 test, A-D test) can be found. In our case we verify conformity of the supposed normal distribution and the sample empirical distribution. The null hypothesis the validity of which is verified using all mentioned tests and is formulated as follows: the data have the normal distribution.

As we have 100 values, it can be applied both X^2 test and A-D test. The tests results are evaluated using P-value. If P value is greater than the chosen significance level for a given test (α), then the null hypothesis cannot be rejected. The value of α is predominantly chosen to be equal to 0.05. Tab. 2.8 shows that all P-values (the red boxes) are less than 0.05. It means that the null hypothesis is rejected and the data are not considered to be normally distributed.

But these tests did not identify the outliers as the previous graphs and they lead to the incorrect result.

This example shows that when doing the statistical analyses of the data, it is vital to combine numerical procedures with graphical tools.

Ad g) - Tab. 2.9

```
Tests for Randomness of RAND1

Runs above and below median
---------------------------
     Median = 3,1763
     Number of runs above and below median = 50
     Expected number of runs = 51,0
     Large sample test statistic z = -0,100509
     P-value = 0,919935

Runs up and down
----------------------------
     Number of runs up and down = 65
     Expected number of runs = 66,3333
     Large sample test statistic z = -0,199458
     P-value = 0,8419

Box-Pierce Test
---------------
     Test based on first 24 autocorrelations
     Large sample test statistic = 13,7236
     P-value = 0,952649
```

In Tab. 2.9 results of the tests for randomness are given. Using these tests the validity of the null hypothesis - that the data are random (independent) - is verified.

We evaluate the tests again with the aid of comparison of the P-value with the chosen significance level $\alpha = 0.05$. For all tests the P-value is greater than 0.05. It means that the null hypothesis cannot be rejected and the data can be considered independent (random). The test confirmed the result of the analysis of ACF (see point e), Fig. 2.5).

Ad h) Tab. 2.10

```
Grubbs' Test (assumes normality)
--------------------------------
Test statistic = 6,03599
P-Value = 1,66864E-9
```

Tab. 2.10 shows results of the Grubbs´ test for the verification if the value 15.7679 is an outlier.

Remember that the null hypothesis of this test says that the data set is homogeneous and it does not include any outlier. Once again - for the assessment of the given data - we compare the P-value with the specified significance level $\alpha = 0.05$. If the P-value is greater than α, the null hypothesis cannot be rejected. In our case, the P-value is less than 0.05. It means that we reject

the null hypothesis and the value 15.7679 is considered an outlier. This value had a significant impact on results of the test for verification of the data normality.

Ad i)

In this paragraph the application and interpretation of the tests based on the sample skewness, sample kurtosis and the combined test based on the sample skewness and kurtosis should have been carried out. All the tests can be applied because the range of the data set is 100 values.

Applying the first two tests we will proceed as follows:

1. Remember that we will verify validity of the null hypothesis: "Data are normally distributed".
2. We compute standardized sample skewness U_a (Std. Skewness) and standardized sample kurtosis U_e (Std. Kurtosis). These indices represent values of the test statistic of the first two tests.

   Ua = 9.01163;

   Ue = 25.2808.
3. We set the value of the significance level $\alpha = 0.05$.
4. We set the limits of the region of the null hypothesis acceptance. It means that we will be looking for the quantiles of the standardized normal distribution $u_{\alpha/2}$ and $u_{1-\alpha/2}$ (it results from the fact that the test statistics U_a and U_e have approximately standardized normal distribution). Using SW (for instance [4], [5]) or statistical tables we will find out that the supposing chosen $\alpha = 0.05$, $u_{\alpha/2} = u_{0.025} = -1.96$ and $u_{1-\alpha/2} = u_{0.975} = 1.96$.
5. Based on the comparison of U_a and U_e with the limits of the $u_{\sigma/2}$ and $u_{1-\sigma/2}$ it can be concluded that the data have no normal distribution (both U_a and U_e lie outside the region of the null hypothesis acceptance, defined by the interval $\langle -1.96; 1.96 \rangle$).

Applying the combined test based on the sample skewness and sample kurtosis we can proceed as follows:

1. We compute the test statistics $C_1 = U_a^2 + U_e^{2,}$ i.e.

   $C_1 = 9.01163^2 + 25.2808^2 = 720.3328$.
2. We set the value of the significance level $\alpha = 0.05$.

3. We set the boundary between the region of the null hypothesis acceptance and the region of its rejection. It means that we will be looking for the quantile X^2 distribution with two degrees of freedom, $X^2_{1-\alpha}$ (2) (it results from the fact that the test statistic C_1 has approximately X^2 distribution with 2 degrees of freedom). Using SW (for instance [4], [5]) or statistical tables we will find out that the supposing chosen $\alpha = 0.05$, quantile $X^2_{1-\alpha}$ (2) = 5.99.
4. Based on the comparison of C_1 with the boundary $X^2_{1-\alpha}$ (2) it can be again concluded that the data have no normal distribution (C_1 is greater than α). We cannot accept the null hypothesis. Data are not considered to be normally distributed.

The results of these tests were also affected by the outlier in the data set. That is why we must again remind us of the fact that the statistical data analysis using testing statistical hypotheses shall be completed with the exploratory graphs.

3 INTRODUCTION TO STATISTICAL PROCESS CONTROL

Chapter structure

Time for learning

Goal

Lecture

- **Definition of statistical process control**
- **Goals of SPC**
- **Basic tool of SPC**
- **Construction of control chart**
- **Analysis and interpretation of control chart**
- **Selection of Shewhart control chart**
- **Phases of SPC**
- **Example of control card**

Summary of terms

Questions

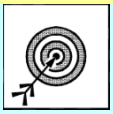
Examples

Additional study resources

Topics for the supplementary self-study

Solution to examples

Time for learning: 720 min



Goal: After studying the chapter

- **you will know what the statistical process control is and what its purpose is;**
- **you will know what Shewhart control charts are;**
- **you will know statistical preconditions of applying Shewhart control charts;**
- **you will know the structure of the basic tool of the statistical process control;**

- **you will be clear about the difference between control and tolerance (specification) limits;**
- **you will be able to derive formulae for the computation of control limits in the Shewhart control charts;**
- **you will be able to choose the correct type of the Shewhart control chart, to construct and interprete it;**
- **you will know what the control card is and what it contains;**
- **you will know what the goal of the statistical process control phases is and what must be done in the frame of every phase;**
- **you will know how to achieve the statistically stable process;**
- **you will be able to utilize information from control charts for the process capability analysis.**

Lecture

Definition of Statistical Process Control

Statistical Process Control (SPC) is direct, regular and ongoing sample quality control based on mathematical-statistical evaluation.

Prevention of the resource waste is the central idea of modern approaches to quality assurance. Prevention can be realized using continual collection and processing of data about the process behaviour to influence the process with the aim to produce outputs with requested properties and functions.

SPC represents the preventive tool of quality cotrol. Based on the timely detection of the significant process deviations from the target it enables realization of interventions into the process and maintaining it on the acceptable level or even its improving.

Principle of the SPC lies in the division of the process variability into two types: variability caused by common causes and variability cased by assignable causes (for more, see Chapter 1 of this textbook).

Goals of SPC

can be defined as follows:

- to enable reaching the process statistical stability;
- to maintain the process on a required level;
- to distinguish common causes from assignable causes;
- to prevent nonconforming units;
- to enable, as fast as possible, to get the process under control when assignable causes are having an influence on it;
- to set up the process only when it is necessary;
- to make conditions for the evaluation of the process capability;
- to make conditions for the continual process improvement;
- to document the process control for a customer;
- to enable reduction of the obvious type of inspection (for instance incoming inspection at the customer).

Basic tool of statistical process control

Control chart is the basic tool of the SPC (see Fig. 3.1). It is a graphical mean for depicting the process variability evolution in time based on the testing of statistical hypothesis. To give a statistical signal as fast as possible when some assignable cause starts to have an influence on the process and to avoid any false signal about an assignable cause when the process is in a statistical control is the main function of the effective application of the SPC.

Decision on the process statistical stability is enabled by three basic lines: CL, LCL and UCL.

CL – *central line* corresponds to so called reference value of the used characteristics. It can be defined as:

a) nominal value;
b) value based on the historical experience with the process;
c) an estimation computed from measurements collected when the process had been statistically stable;

The efficiency of the control chart is also affected by a corrective setting of the control limits UCL and LCL. These limits are called **action limits**, too. They determine the area of the influence only of common causes of the process variability. For that reason, they are the basic criterion for the decision to do or not to do some control action.

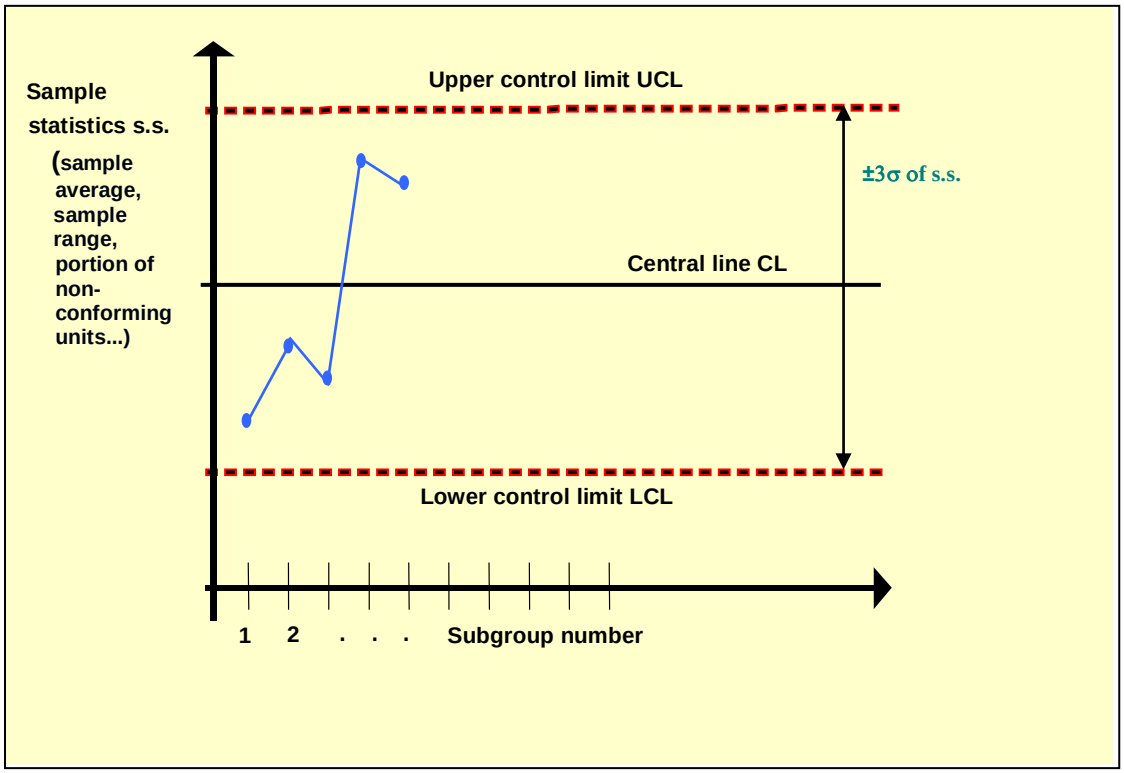


Fig. 3.1 Basic structure of control chart

Control limits are determined statistically. They must not be confused with specification limits (tolerances). They are most frequently determined at the distance of ± 3σ of the applied statistic from the CL.

In some applications additional limits called warning limits - UWL (Upper Warning Limit) and LWL (Lower Warning Limit) - are used. They are obviously set to ±2σ far from CL.

Formulae for computing the CL, UCL and LCL at the Shewhart control charts can be found in Chapter 6 of this textbook (for the derivation of these formulae see [2], [4], [5]).

Construction of control chart

Construction of the control chart will be demonstrated on the mostly applied control charts, i.e. average and range charts ($\bar{x}$, R). Procedure of the control chart construction can be divided into the following steps:

1. Collection of 20-25 subgroups (k = 20 – 25) of n measurements (4 or 5 units per one subgroup) in regular and in advance set control intervals – for instance one hour (see Fig. 3.2).
2. Recording of the measurements into the prepared SPC card (see Fig. 3.2).

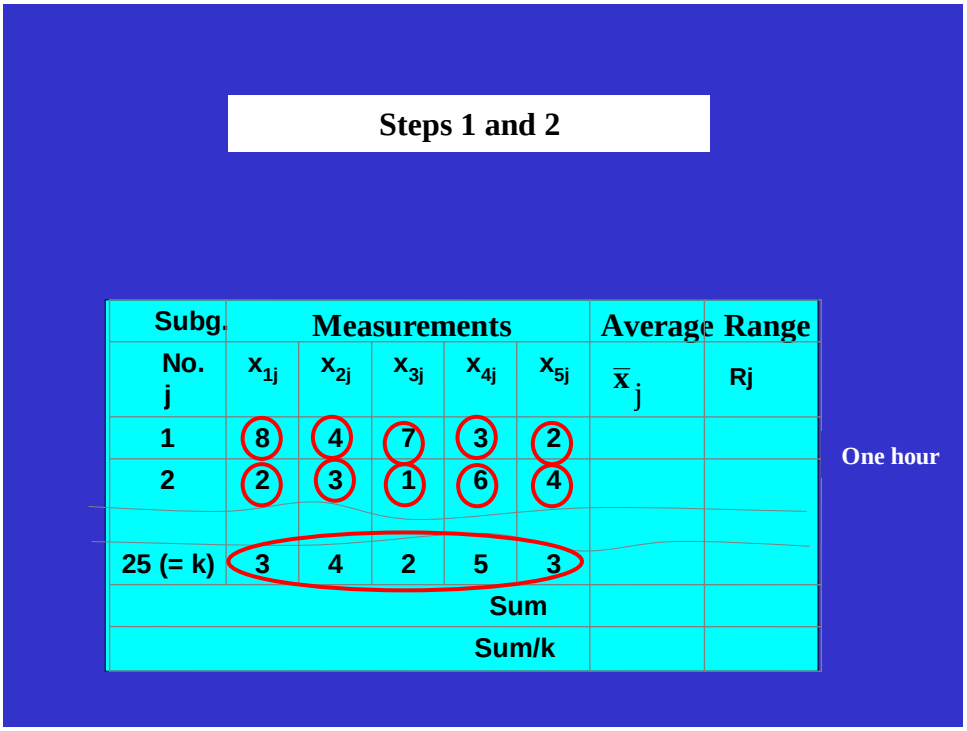


Fig.. 3.2 Steps 1 and 2

3. Computation of the sample average and sample range for each subgroup (see Fig. 3.3).
4. Computation of CL, UCL, LCL for the range chart (see Fig. 3.4).
5. Construction of the range chart (see Fig. 3.4).
6. Computation of CL, LCL and UCL for the average chart (see Fig. 3.5).
7. Construction of the average chart (see Fig. 3.5).

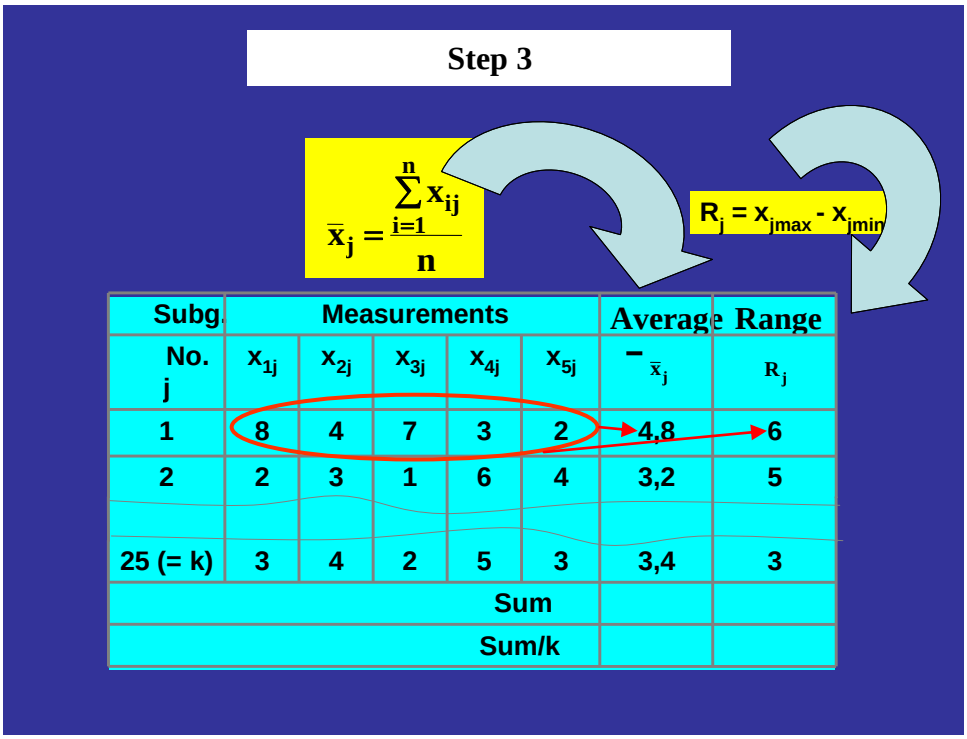


Fig. 3.3 Step 3

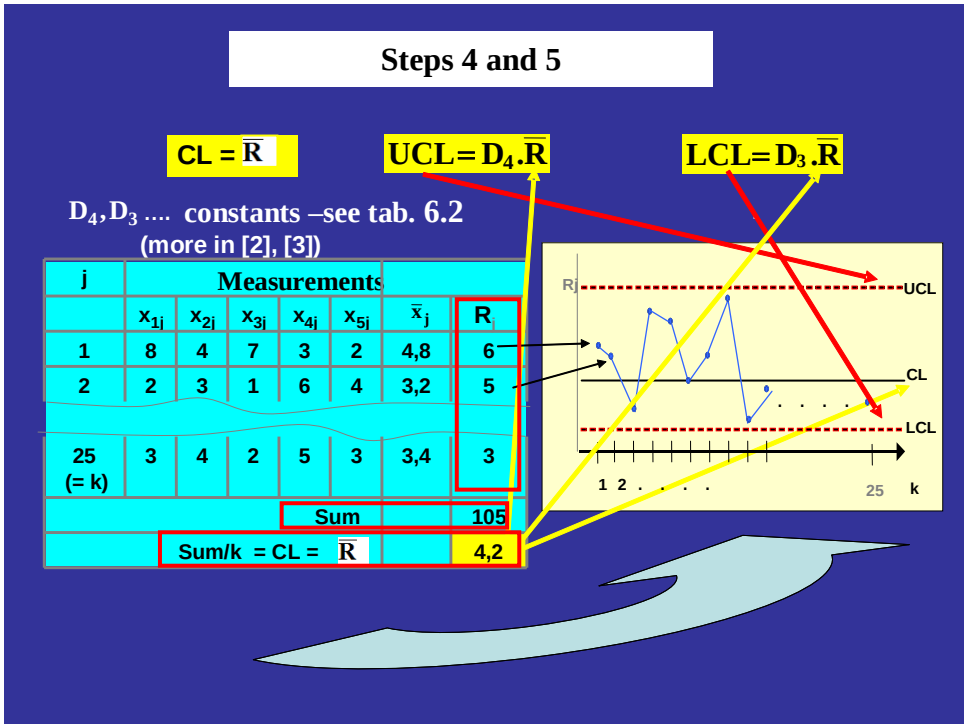


Fig. 3.4 Steps 4 a 5

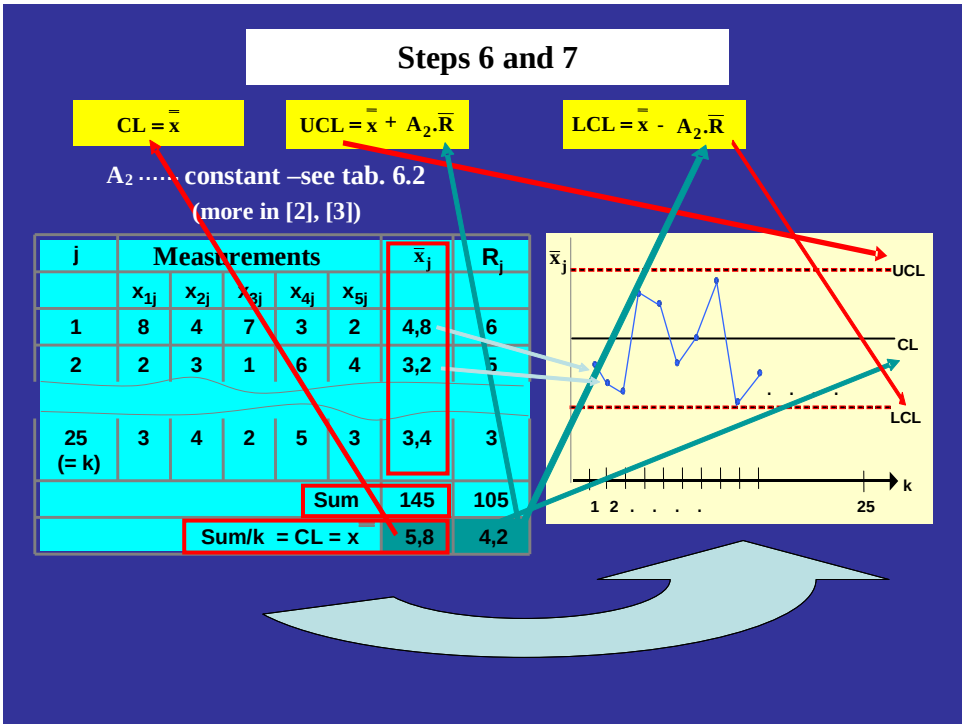


Fig.. 3.5 Steps 6 and 7

Analysis and interpretation of control chart

For interpretation of the control chart the following rules can be applied:

a) if all points lie inside the LCL and UCL, the process can be considered statistically stable and no control action is required.

b) if any point lies outside either the LCL or UCL, the process cannot be considered statistically stable. In such a situation, identification of the assignable cause of this deviation must be realized. Then the implementation of a suitable corrective or preventive control action with the aim to eliminate or at least to reduce effect of this assignable cause must be made.

When applying warning limits, the following two rules can be used:

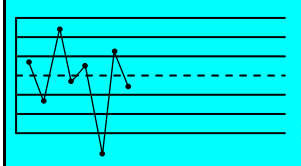
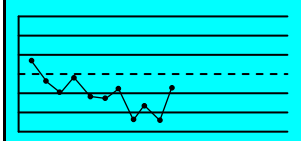
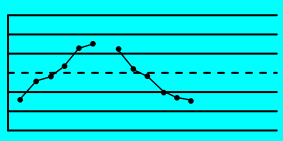
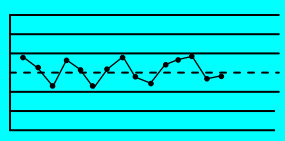
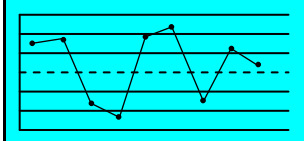
a) if the points lie between the warning limits, it can be supposed that the process is statistically stable and there is no need to make any control action.

b) if any point lies between the UWL and UCL, or LWL and LCL, the following procedure is recommended: irrespective of the control interval, the additional sample must be done immediately. If the point in the control chart corresponding to this new sample lies between warning limits, there is no need to make the control action. But when this new point lies outside the warning limits, it means that the process is not statistically stable with high

probability and than the assignable cause must be defined and a suitable control action must be realized.

For conventional Shewhart control charts, additional criteria for supporting the decision about the process statistical stability were designed. They are known as **tests of non-random patterns.** When points in a control chart form a special pattern, it is also a signal of the assignable cause influence. Such cause must be identified and a suitable control action must be realized. Summary of basic tests of non-random patterns for the Shewhart range and average control charts ($\bar{x}$, R) is in Tab. 3.1 including possible assignable causes (for more, see [1], [2], [3], [5]).

Tab. 3.1 The most used tests of non-random patterns

Situation in control chart	Description	Possible assignable causes
	**Points outside the control limits**	***Range control chart (R)*** **- increase of dispersion caused by a current change in the process elements** **- change of the gauge or inspector** **- data misrepresentation** ***Average control chart ($\bar{X}$)*** **- power surge** **- broken tool** **- gage jumped setting**
	**9 consecutive points lying beyond or below CL**	***Range control chart (R)*** **- long term increase (decrease) in process variability)** **- gage drift** **- new operator** ***Average control chart ($\bar{X}$)*** **- measuring equipment setting** **- different operator, material, method** **- fixture change**
	**6 consecutive points going up or down (trend)**	***Range control chart (R)*** **- decrease or improvement in an operator skill** **- gradual improvement or deterioration in incoming material** **- changes in maintenance** ***Average control chart ($\bar{X}$)*** **- tool wear** **- seasonal effects**
	**15 consecutive points lying in the central third of the zone between control limits**	***Both control charts*** **- incorrectly computed control limits** **- incorrectly adjusted gage** **- subgroups containing data from different sources (different machines, operators, tools …)** **- process improvement**
	**8 consecutive points lying outside the central third of the zone between control limits**	***Bothcontrol charts*** **- incorrectly computed control limits** **- incorrectly plotted points** **- incorrectly adjusted gage** **- different subgroups containing data from different resources (in one subgroup data from one source)** **- large difference in the method of measurement**

This table also includes the main symptom of the statistical instability – point outside the control limit (the first situation).

Phases of SPC

The whole SPC process can be divided into four main phases:

I. Preparatory phase;

II. Phase of verification and ensuring the process statistical stability;

III. Phase of analysis and ensuring the process capability;

IV. Phase of ongoing process control.

I. Preparatory phase

In the frame of this first phase of the SPC many activities must be planned and realized to create preconditions for the effective SPC implementation. The following list contains the main above mentioned activities:

1. Creation of suitable conditions for the effective SPC implementation;
2. Process definition and its analysis:
   - process speed, length of the process cycle repeatability, character of technology;
   - type and area of the effect of the causes that can evoke abnormalities in the process;
   - type and form of control and inspection;
   - requests on the process precision;
   - relations between different kinds of nonconformities and their causes;
   - determination of the most suitable part of the process for the SPC implementation;
3. Selection of the controlled quality characteristics or the process parameter with respect to:
   - customer requirements;
   - the area of existing or potential problems (claims …);
   - correlation between quality characteristics and process parameters;
5. Definition of the process data collection:
   - what, where, when, who, how;
   - determination of the sample size;
   - determination of the control interval;
   - determination of the control places;

- determination of the sample number;
- selection of a suitable method for the sample (subgroup) creation;
- selection of a suitable method for the data collection and processing;

6. Measurement system analysis (for instance the R&R method);
7. Selection of a suitable control chart;
8. Preparation of the data collection and their recording;

II. Phase of verification and ensuring the process statistical stability

Correct setting of control limits - that should delimit the zone of influence of only common causes - is the main goal of the second phase.

Algorithm of reaching this goal is clarified using the most applied pair of control charts ($\bar{x}, R$). The steps of this algorithm are as follows:

1. Construct the range control chart (R chart) and analyze it from the point of view of the process statistical stability (see Fig. 3.6).
2. When there are some points outside the control limits or when there are some non-random patterns, identify assignable causes, remove them, accept and realize the corrective or preventive control actions;
3. Remove the subgroups where the influence of assignable causes had been signalized from the chart;
4. Recompute CL, LCL and UCL for the R chart.
5. Construct and analyze the R chart with the recomputed CL, LCL, UCL.
6. a) When the effect of assignable causes is again identified in the R chart with the recomputed CL, LCL and UCL, return to the step 2 of the block A in Fig. 3.6.

6. b) When no assignable cause is signalized in the R chart, compute and analyze the average control chart ($\bar{\bar{x}}$ chart) (see Fig. 3.7): procedure is the same as the previous one for the R chart, only step 6 is slightly different:

   6. a) When the effect of assignable causes is again identified in the $\bar{\bar{x}}$ chart with the recomputed CL, LCL and UCL – return to the step 2 of the block B in Fig. 3.7.

   6. b) When no assignable cause is signalized in the $\bar{\bar{x}}$ chart - switch to phase III..

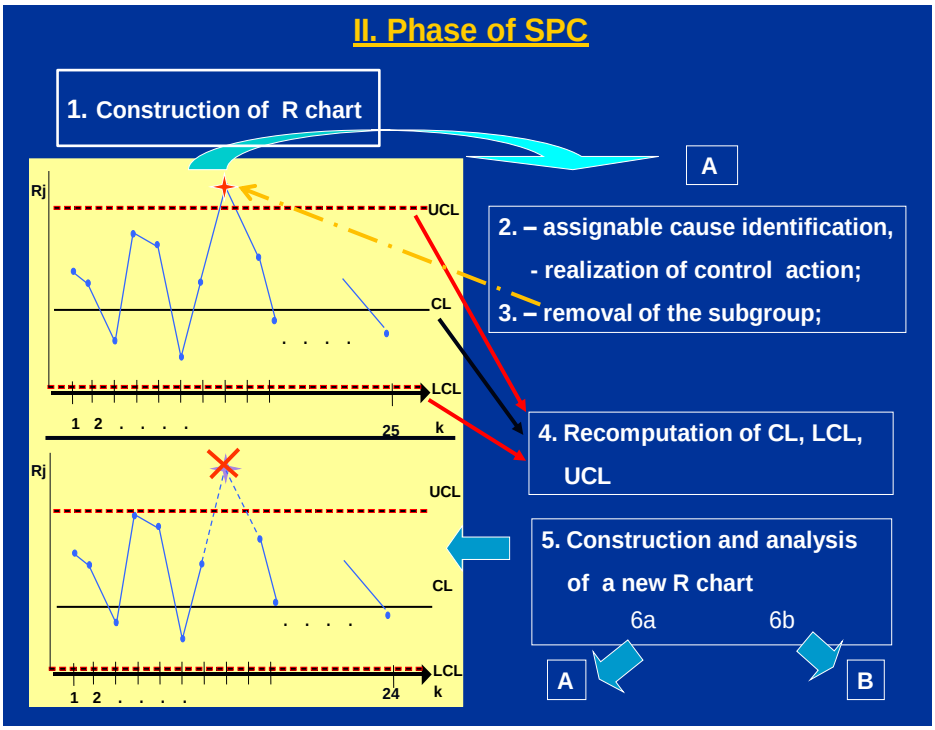


Fig. 3.6 II. phase of SPC – construction and analysis of control chart (R)

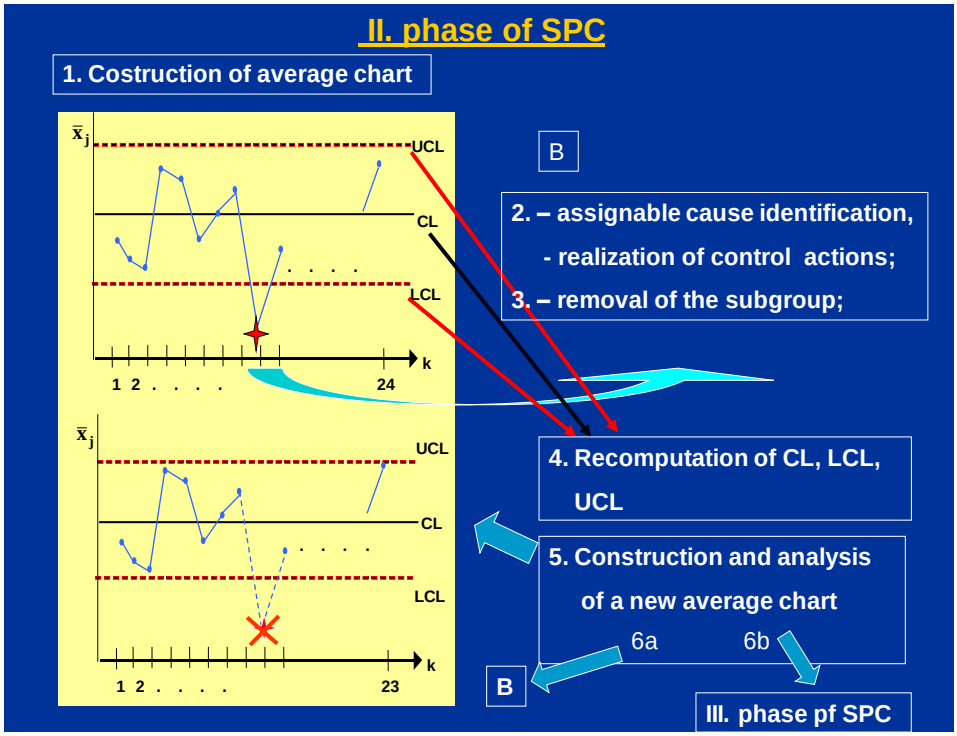


Fig. 3.7 II. phase of SPC – construction and analysis of control chart ($\bar{x}$)

III. Phase of verification and ensuring the process capability

Analysis of the process capability is the third phase in a wider interpretation of the SPC. In this phase we analyze whether the process, which is statistically stable after the previous phase, is also able to continuously meet consumers' requirements given as specification (tolerance) limits.

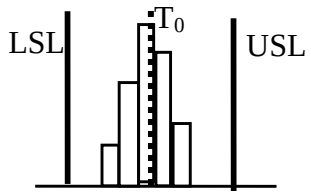
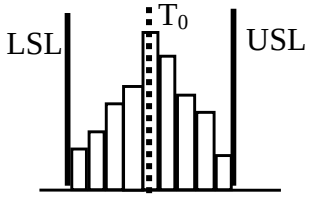
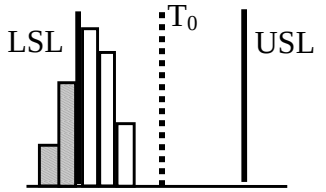
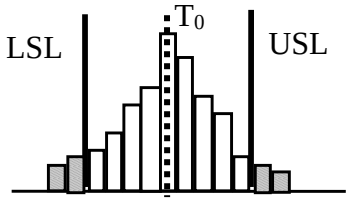
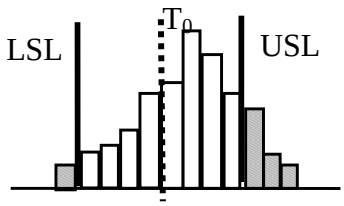
Increase in the rate of the process capability can be reached through

- reduction of the process variability caused by common causes;
- implementation of the larger system measures in management authority (new technology, new machines, different materials …).

The process capability can be analysed using graphical and numerical tools. Histogram is one of the graphical tools. If specification limits USL and LSL and the centre of the specification zone T_0 are supplemented to the histogram, it can offer primary information about the process capability and effects having influence on it. When the histogram signalizes non-capable process, possible measures for the increasing capability can be considered. Possible situations and their solutions can be seen in the following table.

Note:
Histogram is a graphical representation of the interval distribution of frequencies. In contrast to statistical indicators the histogram provides a global picture of the population.

Tab. 3.2 Histogram and capability analysis

Situation	Measures
	Process is capable. No actions are necessary.
	Process is close to being capable, in the short term no actions are necessary. From the long-term view it is necessary to analyze the process with the aim to improve it and to increase the rate of its capability.
	The process produces non conforming units. It is not capable. It is necessary to set the process on the middle of the specification zone.
	The process is centered on the middle of the specification zone. But it produces nonconforming units. It is not capable because of too large variation. It is necessary to accept some measures to reduce this variation: transfer of the production to more precise machine, acquisition of the new machine, ,change of specification limits.
	Process is not on the middle of the specification zone plus its variation is too large. It is not capable. Suitable measures: acquisition of the new machine, reduction of the width of the specification zone.

Rate of the capability is commonly quantified using capability indices, mostly the indices Cp and Cpk.

Application of these traditional and old indices is limited by several conditions:

1. Process must be statistically stable.
2. Measurements must be normally distributed.
3. Specifications (tolerances) must express customer requirements.

4. The process is centered in the middle of the zone between the specifications (tolerances) (precondition for Cp).
5. The measurement system variability is low.

Formulae for computation of indices Cp and Cpk are depicted in Fig. 3.8.

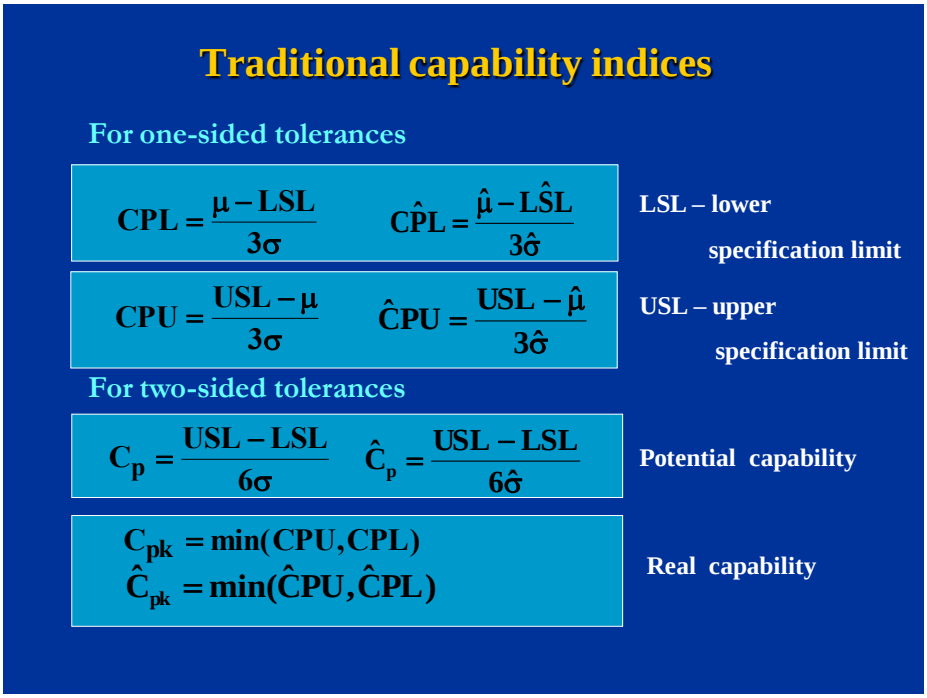


Fig. 3.8 Formulae for computation of traditional capability indices

One way of how to estimate parameters σ and μ in these formulae uses information from the control charts. It is explained in the solution of the example 3 in the Chapter 6 of this textbook.

Structure of these indices is really simple. The indices are based on the ratio of the acceptable process variation to the real process variation. This principle is clarified using the following figures 3.9 and 3.10.

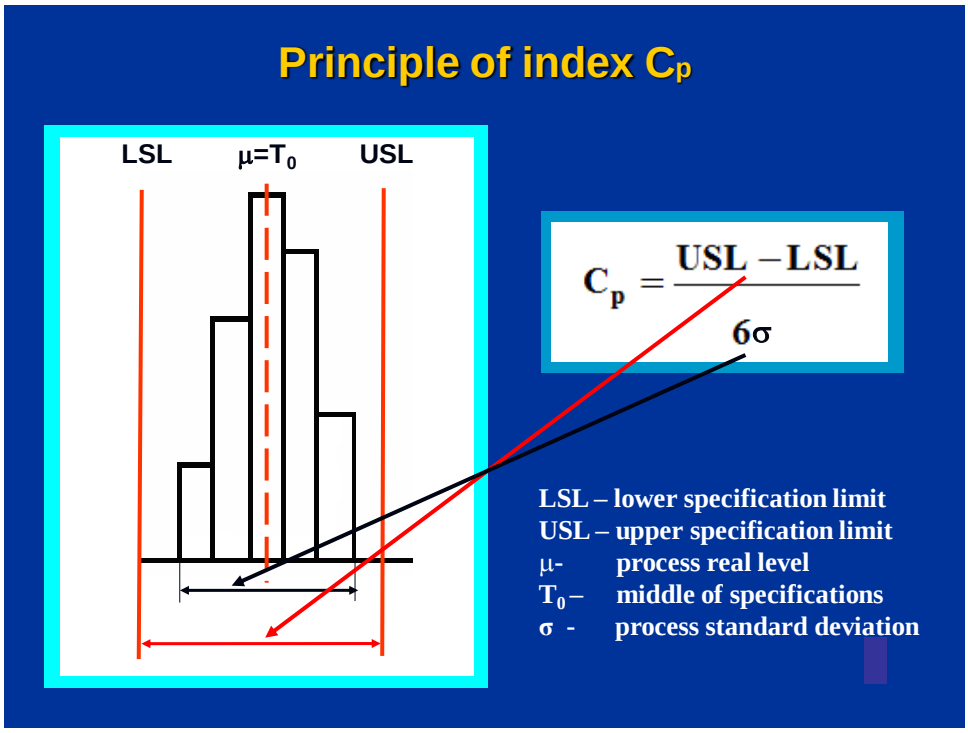


Fig. 3.9 Principle of index C_p

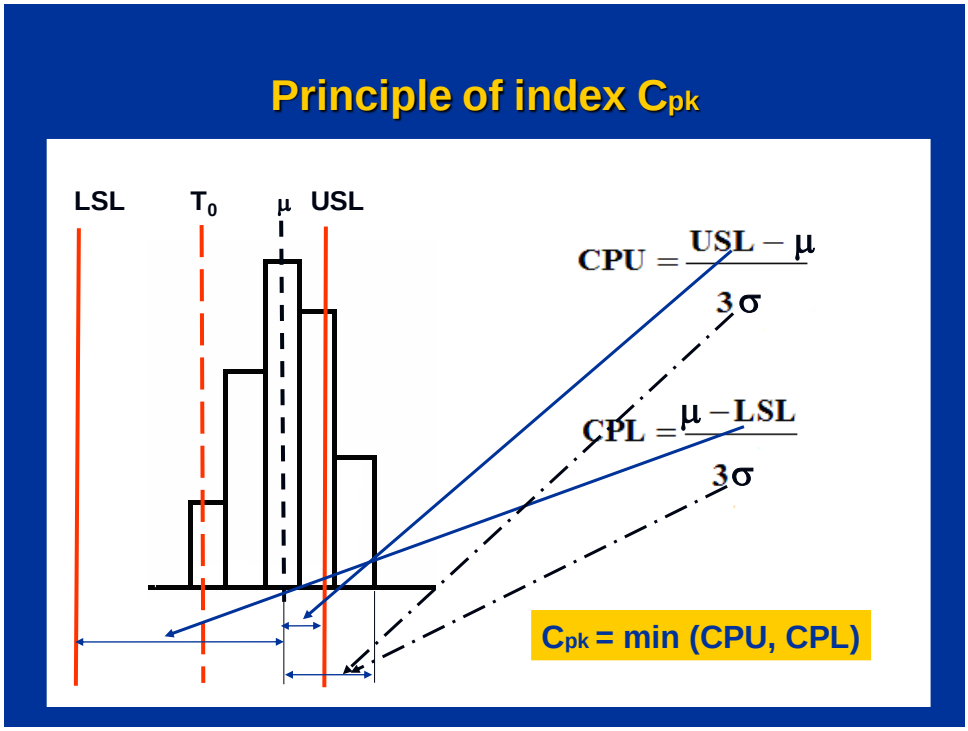


Fig. 3.10 Principle of index C_{pk}

Relations between the indices C_p and C_{pk} can be generalized as follows: $\mathbf{C_p \geq C_{pk}}$.

Compared to the situation depicted in Fig. 3.11 on the left (process is centered in the middle of the specification zone), the process on the right of this figure was drifted to the lower specification limit.

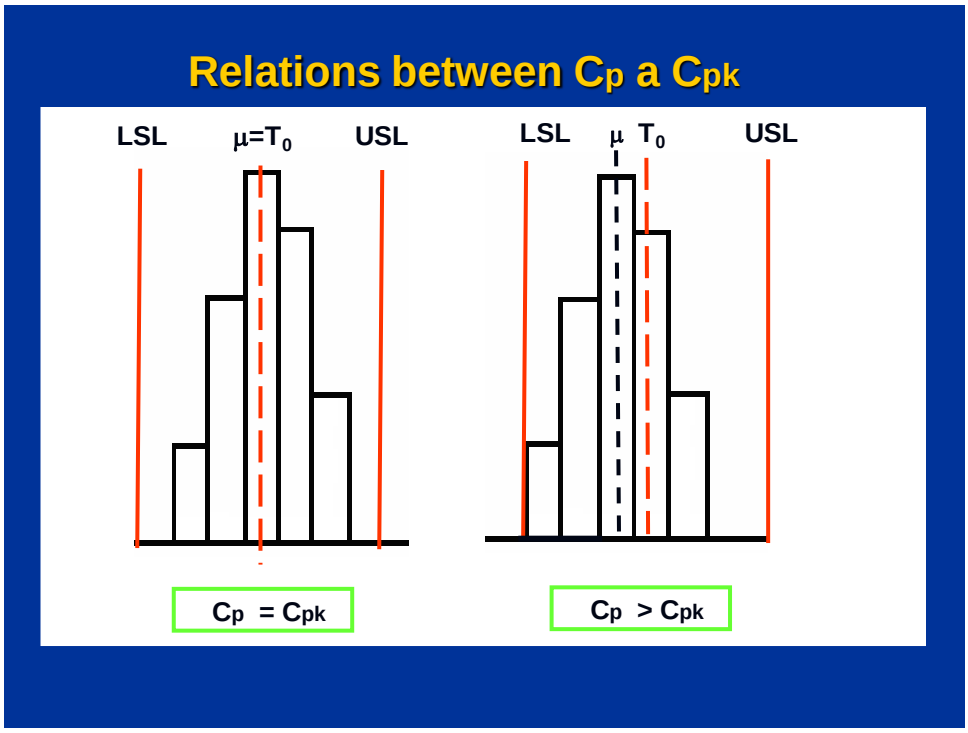


Fig. 3.11 Relations between indices C_p and C_{pk} (drift to LSL)

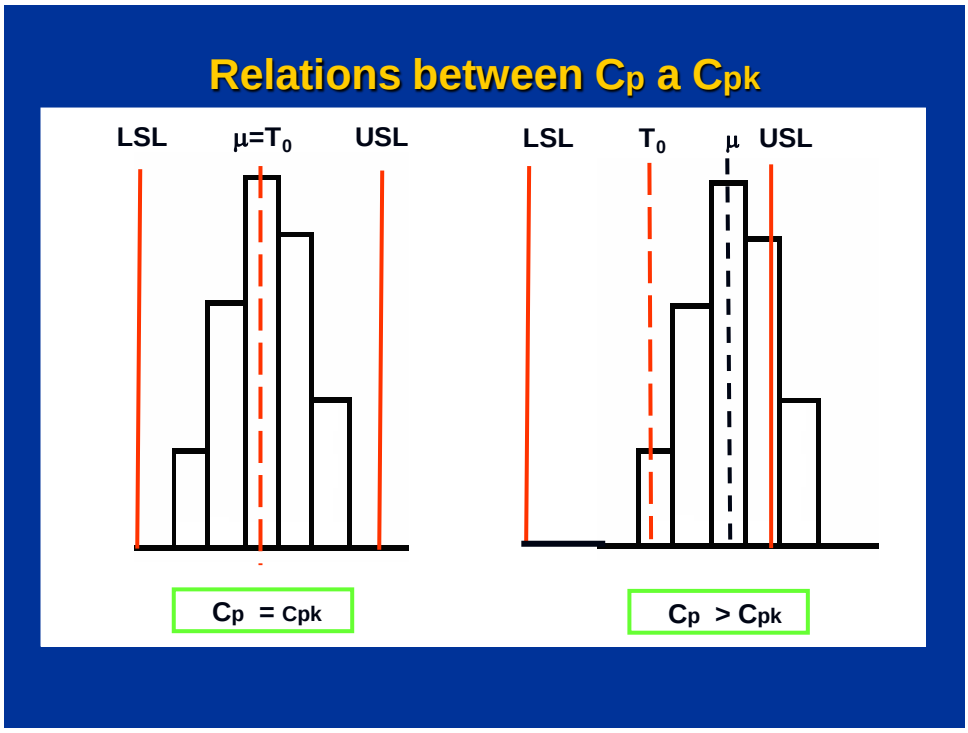


Fig. 3.12 Relations between indices C_p and C_{pk} (drift to USL)

Compared to the situation depicted in Fig. 3.12 on the left (process is centered on the middle of the specification zone), the process on the right of this figure was drifted to the upper specification limit.

Fig. 3.13 covers the clarification of the fact why it is not enough to compute only the index C_p. This index does not reflect the process drift. As it can be seen on the next figure, the Cp equals to high value of 2.0 in spite of the gradual process drift outside the USL (C_p expresses potential capability for the situation when the process is centered in the middle of the zone between specifications). The process drift is reflected by the value of index C_{pk}. As it can be seen, values of the C_{pk} are changing with the process drift to the USL, i.e. from 2.0 to – 0.5. In general, computation of the both indices is the best practice.

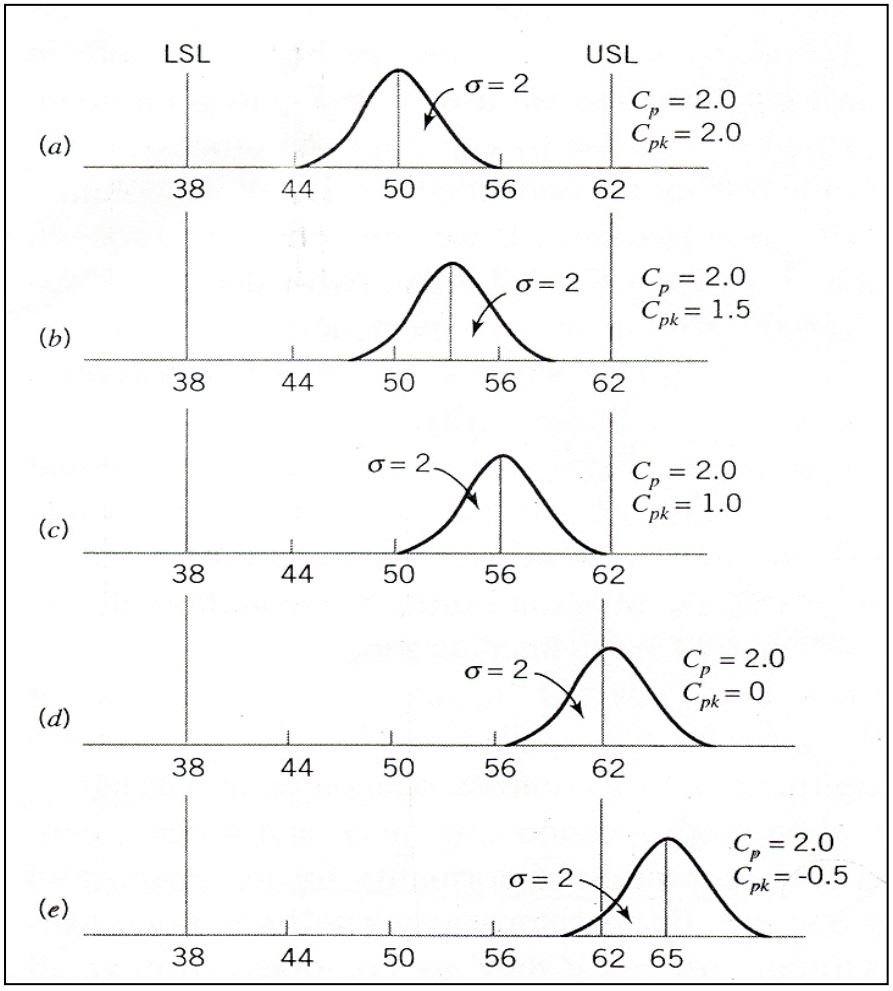


Fig. 3.13 Relations between C_p and C_{pk}

In the table below, the relation between the value of Cp (supposing that the process is centered in the middle of the specification zone) and a portion of noncoforming units is presented.

Tab. 3.3 Relations between C_P and portion of noncoforming units

Width of specification zone in number of standard deviations	C_P	Portion of nonconforming units	Number of nonconforming units
±2σ	0.67	0.0455	5 ze 100
±3σ	1.0	0.0027	3 z 1000
±4σ	1.33	0.0000636	6 ze 100 000 63 ppm
±5σ	1.67	0.00000028	3 z 10 mil. 0.3 ppm
±6σ	2.0	0.00000000099	9 z 10 mld

Notes

As it was mentioned above, traditional capability indices Cp and Cpk suppose that the monitored variable has a normal distribution. For the case of a non-normally distributed variable, the following modificate indices can be applied:

$$C_p = \frac{USL - LSL}{x_{0.99865} - x_{0.00135}}, \tag{3.1}$$

$$C_{pk} = \min\left(\frac{x_{0.5} - LSL}{x_{0.5} - x_{0.00135}}, \frac{USL - x_{0.5}}{x_{0.99865} - x_{0.5}}\right), \tag{3.2}$$

where

$x_{0.00135}$ … is 0.135% quantile of the corresponding distribution of the monitored variable,

$x_{0.99865}$ … is 99.865% quantile of the corresponding distribution of the monitored variable,

$x_{0.5}$ ... is an estimate of median.

For attributes the following indices can be applied:

$$C_{pk} = \frac{p_{max} - \bar{p}}{3\sigma_p} \qquad \sigma_p = \sqrt{\frac{(1-\bar{p})\bar{p}}{\bar{n}}}, \tag{3.3}$$

where

p_{max} … acceptable limit value of portion of nonconforming units,

$\bar{p}$ … defined average value of portion of nonconforming units,

σ_p … standard deviation of portion of nonconforming units,

$\bar{n}$ … average sample size.

Types of capability analysis

Capability analyses can be divided as follows:

I. Pre-batch production

- short term capability study
- preliminary capability study

II. *Batch production*

- long term capability study

Short term capability study

This type of capability study is suitable for these cases:

- when an adequate quantity of products is not available;
- for the preparatory capability evaluation;
- before delivery of a machine to a customer;
- after installation of a new machine;
- after a machine repair.

For successful implementation of the short term capability analysis, stable conditions must be ensured (the same operator, material, machine setting, stable operating process parameters, etc.). Collection and data processing should be carried out as follows:

- minimally 50 consecutive products must be evaluated;
- values of the quality characteristics measured on these products are divided into subgroups with 5 units;
- analysis of the process statistical stability is carried out using standard control charts;
- machine capability indices C_m and C_{mk} are computed (the same formulae as for C_p and C_{pk}).

Machine is considered to be capable when $C_{mk} > 1.67$.

Preliminary capability study

During this type of analysis standard conditions of the batch production must be ensured and as many variability causes as possible should impact the process.

Collection and data processing should be realized as follows:

- in constant time intervals minimally 20 subgroups of 3 products (better 25 subgroups of 5 products if possible) are sampled;
- using measurements from these products, the analysis and ensuring of the statistical stability is carried out by means of the standard control charts;
- indices C_p a C_{pk} are computed.

Long term capability study

It represents the capability assessment in real process conditions. For that reason it must cover a longer time period to enable all causes of the process variability to impact the process (at least 20 days). The procedure is the same as for the preliminary capability study.

IV. Phase of ongoing process control

In this phase the process is maintained statistically stable and capable. To give a signal about new failures in the statistical stability, to identify and eliminate them are the main goals of this phase. Control charts then operate with control limits computed during the II. phase (see Fig. 3.14 on the left), with respect to results of the capability analysis of the III. phase). These limits have a long-term character (see Fig. 3.14 on the right) and they are valid until the significant change of the process with long-term influence and with identificable cause.

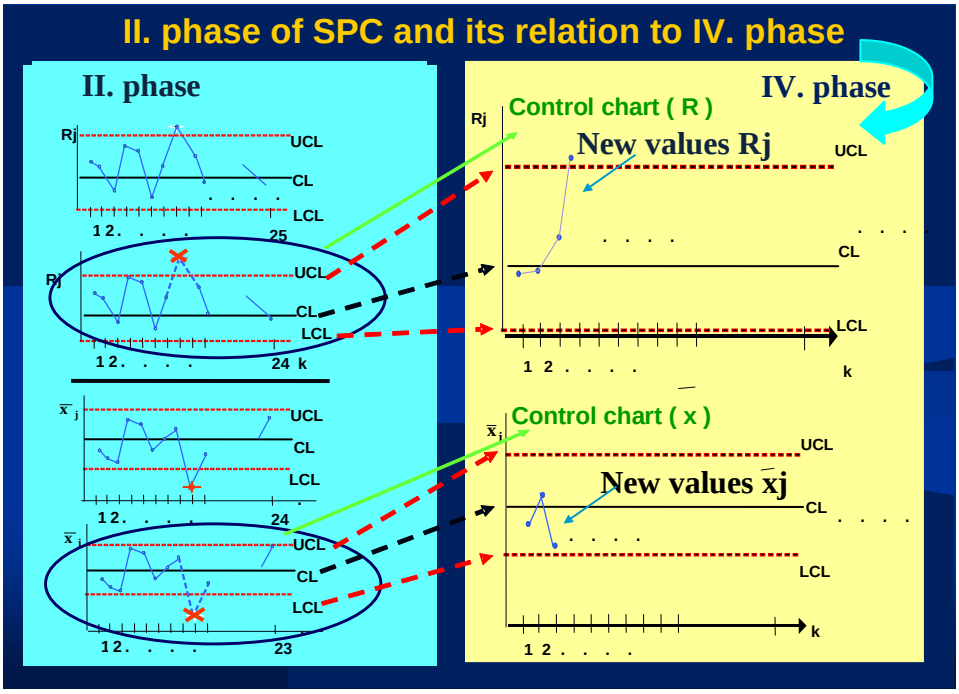


Fig. 3.14 Relations between II. and IV. phase of SPC

Summary of terms

After studying this chapter you should understand these terms:

- **statistical process control (SPC)**
- **control chart**
- **statistically stable and unstable process**
- **upper control limit**
- **lower control limit**
- **central line**
- **specification limit (tolerance)**
- **nonrandom patterns**
- **control action**
- **phases of SPC**

Questions

1. **What is the main goal of statistical process control (SPC)?**
2. **What is the basis idea of SPC?**
3. **What is the main tool of SPC?**

4. **What is the difference between control limits and tolerances (specification limits)?**
5. **What is the difference between UCL and UWL, resp. LCL and LWL?**
6. **How to construct a control chart?**
7. **How to interpret the results obtained from a control chart?**
8. **Which non-random patterns do you know? What is their interpretation?**
9. **When control limits must be recomputed?**
10. **What are the activities of the I. phase of SPC?**
11. **What is the main goal of the II. phase of SPC?**
12. **How to realize the II. phase?**
13. **What is the goal of the III. phase of SPC and how to realize it?**
14. **What is the goal of the IV. phase of SPC and how to realize it?**
15. **What is the main difference between short-term, preliminary and long-term capability study?**
16. **How were the values in Tab. 3.3 computed?**

Additional study resources

[1] GRIFFITH, G. K. *Statistical Process Control Methods For Long and Short Runs*. Milwaukee, Wisconsin: ASQC Quality Press, 1996. 250 p. ISBN 0-87389 315-X.

[2] MONTGOMERY, D. C.: *Statistical Quality Control. A Modern Introduction*. New York : J. Wiley & Sons, 2012. 768 p. ISBN: 978-1-1183-2257-4.

[3] ISO 8258: *Shewhart Control Charts*. 1991.

[4] TOŠENOVSKÝ, J. – NOSKIEVIČOVÁ, D.: *Statistické metody pro zlepšování jakosti*. Ostrava: Montanex, 2000. 362 p. ISBN 80-7225-040-X.

[5] WHEELER, D. A CHAMBERS, D.: *Understanding Statistical Process Control*. 3rd Ed. Knoxville, Tennessee, SPC Press Inc., 2010. 978 p. ISBN 978-0-945320-69-2.

Topics for the supplementary self-study

- computation of CL, LCL and UCL in Shewhart control charts;
- derivation of formulae for the computation of CL, LCL and UCL in Shewhart control charts;
- explanation of selection of a particular Shewhart control chart in detail;
- computation of values in Tab. 3.3.

4 SPC AS A STATISTICAL HYPOTHESIS TESTING

Chapter structure

Topics for review

Time for learning

Goal

Lecture

- **Conception of SPC as a statistical hypothesis testing**
- **SPC and the probability of type I and type II errors**
- **Computation of the risk of false and missing signals**
- **ARL**

Summary of terms

Questions

Examples

Additional study resources

Topics for the supplementary self-study

Solution to examples

R

Topics for review:

- **testing statistical hypotheses**
- **process variability**
- **process statistical stability**

Time for learning: 240 min

Goal: after studying this chapter

- **you will understand the SPC as the application of the statistical hypothesis testing;**
- **you will know which errors shall be considered when applying the SPC;**
- **you will be able to compute the risk of a false signal (alarm) and the risk of a missing signal (alarm);**
- **you will understand the term ARL and its connection to the risks of false**

and missing signals;

- **you will be able to set the value of ARL(0) and ARL(δ) for the selected Shewhart control charts.**

Lecture

Conception of SPC as a statistical hypothesis testing

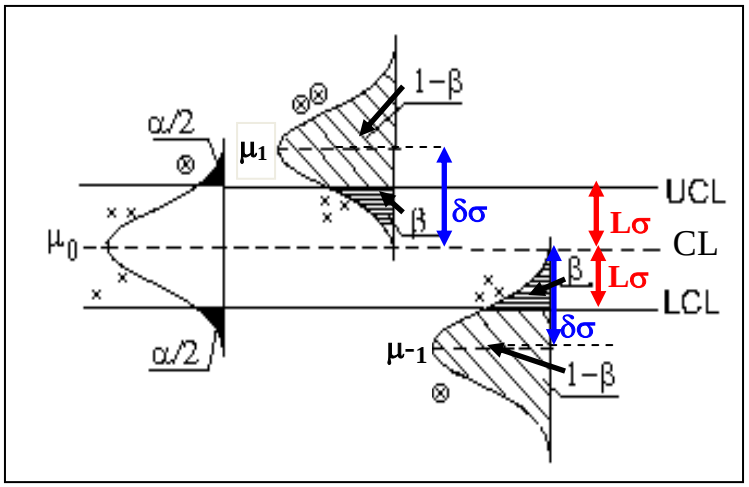
For the controlled variable (monitored quality characteristic or technological parameter) as a random variable, the hypothesis about the values of parameters of its probability distribution is being formulated. This null hypothesis should be formulated in the way that the process meets the quality requirements when this hypothesis is true (so that the process could be considered statistically stable). This null hypothesis is repetitively tested based on the regularly repeated, mostly small samples (rational subgroups). Rejecting the null hypothesis (points outside the control limits, trends or some non-random patterns) is the signal that the process with high probability deviated from the supposed state (it means that the process is out of control (it is not statistically stable) and some control action must be accepted and implemented. Control action equals to the identification and partial or total elimination of the assignable cause that caused the signalled undesirable changes in the process behaviour.

Hypothesis H_0 in SPC means that the process is statistically stable; *alternative hypothesis* H_1 means that the process is not statistically stable. The area between control limits LCL and UCL in the control chart constitutes the domain of acceptance of the null hypothesis, and the area outside the control limits is the domain of rejection of the null hypothesis. Values of the control limits LCL and UCL are called critical values depending on the significance level α, i.e. the probability of the type I error.

SPC and the probability of type I and type II errors

In SPC the probability of type I error (denoted as α) is called the *risk of a false signal*. It represents probability of the vain search for an assignable cause based on the symptom of instability in the control chart (point outside the control limit or some non-random pattern) even if the process, in fact, has not changed (see Fig. 4.1 a)). This incorrect result is associated with the costs of searching for a non-existing problem.

In SPC the probability of type II error (denoted as β) is called the *risk of a missing signal*. It is the probability that a control chart is not able to detect significant change of the process immediately after its appearance (there is no point outside the control limit, no instability pattern in a control chart). This incorrect result leads to the costs due to the missing control action. In Fig. 4.1 b), c) this situation is depicted for a significant shift of the level of the controlled variable from the desirable target μ_0 to the undesirable (critical) level μ_1 or μ_{-1}. The value $(1 - \beta)$ is generally called the test power. It is the probability that the critical shift from μ_0 to μ_1 (i.e. $\delta\sigma$ that is expressed in standard deviations) will be revealed in the first subgroup after its appearance.



a) b) c)

Fig. 4.1 Risk of the false signal α and risk of the missing signal β

Practical implementation of this testing can be described as follows:

- Mostly - in regular control intervals - the samples of size n (the so called rational subgroups) are selected.
- In every unit of every sample the values of the monitored quality characteristic or the process parameter is measured or counted.
- Value of the applied testing criterion is computed from all n values in each sample (for instance the sample average, sample standard deviation, sample median, sample range, fraction of nonconforming, number of nonconformities, etc.). For $n = 1$ the testing criterion is represented by individual measurements (for variables SPC).

- The control chart is a graphical depiction of the applied testing criterion as a function of time. On the X-axis, numbers of the samples (their order) are kept. On the Y-axis, there is the scale for the applied testing criterion.
- State of the statistical stability is then judged through analyzing the position of the points in relation to the CL, LCL, UCL. The X-coordinate of every point corresponds to the sample order and its Y-coordinate corresponds to the value of a testing criterion for each sample.

Computation of the risk of false and missing signals

Computations are shown on the case of the average and individuals control charts.

Computation of the risk β

Let the process standard deviation σ be known and constant. Suppose that the process level moves from the value μ_0, corresponding to the statistically stable process, to the critical value $\mu_1 = \mu_0 + \delta\sigma$ (see Fig. 4.1). The probability that this drift will not be revealed with the following sample, i.e. risk of the missing signal β, can be then expressed as follows:

$$\beta = P(LCL \le \bar{x} \le UCL | \mu = \mu_1 = \mu_0 + \delta.\sigma, \tag{4.1}$$

as $\bar{x} \sim N(\mu, \sigma^2/n)$,

$UCL = \mu_0 + L\sigma/\sqrt{n}$ (see Fig. 4.1) and $LCL = \mu_0 - L\sigma/\sqrt{n}$, (see Fig. 4.1),

β can be computed as follows:

$$\beta = \Phi\left(\frac{UCL - \mu_1}{\sigma/\sqrt{n}}\right) - \Phi\left(\frac{LCL - \mu_1}{\sigma/\sqrt{n}}\right) =$$

$$= \Phi\left(\frac{\mu_0 + L\sigma/\sqrt{n} - (\mu_0 + \delta\sigma)}{\sigma/\sqrt{n}}\right) - \Phi\left(\frac{\mu_0 - L\sigma/\sqrt{n} - (\mu_0 + \delta\sigma)}{\sigma/\sqrt{n}}\right), \tag{4.2}$$

where Φ is the distribution function of the standardized normal distribution.

After modifying the formula we obtain

$$\beta = \Phi(L - \delta\sqrt{n}) - \Phi(-L - \delta\sqrt{n}) \tag{4.3}$$

Note:

If $n > 1$, calculation of the risk β is related to the average control chart. If $n = 1$, the risk β of the control chart for individuals is computed.

Computation of the risk α

As the risk of the false signal α is related to the process which is statistically stable, i.e. it is centred on the desirable value μ_0 (no deviation from this value has come); the standardized deviation $\delta = 0$ and the formula (4.3) can be changed into the following form:

$$1-\alpha = \Phi(L-0\sqrt{n}) - \Phi(-L-0\sqrt{n}) = \Phi(L) - \Phi(-L).$$

The risk α can be then computed as follows:

$$\alpha = 1 - [\Phi(L) - \Phi(-L)] \tag{4.4}$$

ARL

ARL is an acronym for *Average Run Length*. It is the next indicator for the control chart performance.

ARL definition

ARL is the average number of points that are recorded into the control chart before some point goes outside the control limit. For uncorrelated data, the ARL for Shewhart control charts can be in general expressed as follows:

$$ARL = \frac{1}{p}, \tag{4.5}$$

where … p is the probability that some point will be outside the control limit.

Indicator ARL(0)

ARL(0) is the average run length for the statistically stable process. It can be computed using this formula:

$$ARL(0) = \frac{1}{\alpha}. \tag{4.6}$$

Indicator ARL(δ)

ARL(δ) is the average number of points between the moment when the process change $\delta.\sigma$ has occurred and the moment when a point outside the control limit signalizes this change. Formula for computing this indicator is as follows:

$$ARL(1) = \frac{1}{1-\beta}, \tag{4.7}$$

where (*1 - β*) ... is the probability that the critical shift from μ_0 to μ_1 (i.e. $\delta\sigma$ expressed in standard deviations) will be revealed by the first subgroup after its appearance (the point representing this subgroup will be outside the control limit).

Note:

ARL for Shewhart control chart is a random variable having a geometric distribution with parameter *p*. The mean of this distribution is *1/p*.

Indicator ATS

ATS is the acronym for Average Time to Signal. It can be computed using the value of ARL:

$$ATS = ARL.h, \tag{4.8}$$

where h …. is the length of the control interval.

Indicator I

ARL can be expressed in a number of inspected units. Then we work with the indicator I, i.e. average number of units inspected before any point goes outside the control limit.

$$I = n.ARL, \tag{4.9}$$

Where n …. is the sample size.

Summary of terms

After studying this chapter you should understand these terms:

- **desirable process level**
- **critical process level**
- **critical process shift**
- **risk of false signal**
- **risk of missing signal**
- **average run length for statistically stable process**
- **average run length for statistically unstable process**
- **average time to signal**

Questions

1. **How is the null hypothesis defined in SPC as a statistical hypothesis testing?**
2. **What are the critical values in SPC as a testing hypothesis?**
3. **What is the domain of acceptance of the null hypothesis in SPC as a testing hypothesis?**
4. **What is the domain of rejection of the null hypothesis in SPC as a testing hypothesis?**
5. **What is the risk of false signal?**
6. **What is the risk of missing signal?**
7. **What is ARL(0)?**
8. **What is ARL(δ)?**
9. **How are these indicators computed for the Shewhart control charts?**
10. **What is ATS?**
11. **Is it desirable to have ARL(0) as short as possible or as long as possible?**
12. **Is it desirable to have ARL(δ) as short as possible or as long as possible?**

Examples

Example 1

Compute the risk of false signal of the Shewhart control chart with obvious 3σ control limits.

Example 2

Let us assume the Shewhart average control chart with obvious 3σ control limits and sample size $n = 5$. Compute the probability that the shift from the mean μ_0 to $\mu_1 = \mu_0 + 2\sigma$ will be revealed by the first sample after its appearance.

Example 3

Let us assume the Shewhart average control chart with obvious 3σ control limits.

Compute ARL(0).

Example 4

We want to reveal the change of the process level 1.5σ sized. The risk of missing signal $\beta = 0.75$. Compute ARL(δ).

Additional study resources

[1] MONTGOMERY, D.C.: Introduction to Statistical Quality Control. J.Wiley & Sons, New York, 2001. ISBN 0-471-31648-2.
[2] TOŠENOVSKÝ, J. – NOSKIEVIČOVÁ, D.: *Statistické metody pro zlepšování jakosti*. Ostrava: Montanex, 2000 (in Czech).

Topics for the supplementary self-study

- computation of ARL for other control charts (see [1]).

Solution to examples

Example 1

Shortened assignment:

L=3; $\delta = 0$

α = ?

Solution:

We substitute into the formula (4.4):

$\alpha = 1-[\Phi(3)-\Phi(-3)] = 1-0.9973 = 0.0027$

The risk of false signal for the Shewhart control chart with obvious 3σ control limits is 0.0027.

Example 2

Shortened assignment:

L=3; n=5; $\delta = 2$

$1-\beta$ = ?

Solution:

We substitute into the formula (4.3):

$\beta = \Phi(3-2\sqrt{5}) - \Phi(-3-2\sqrt{5}) = 0.0708$

$1-\beta = 1-0.0708 = 0.9292$.

The probability that the shift from the mean μ_0 to $\mu_1 = \mu_0 + 2\sigma$ will be revealed by the first sample after its appearance for assigned conditions is 0.0292.

Example 3

Solution:

In example 1 we computed the probability that a point will exceed the control limit and, although the process is statistically stable, it is 0.0027 (the risk of false signal α). Therefore,

ARL(0) = 1/0.0027 = 370

It means that in spite of the fact that the process continues to be statistically stable, the control chart will signal instability with every 370 samples in average.

Example 4

Shortened assignment:

$\delta = 2$; $\beta = 0.75$

Solution:

- probability of revealing the change of the defined size by the first sample after this change: 1 - β = 0.25;
- probability of revealing the change of the defined size by the second sample after this change: β.(1 - β) = 0.75(0.25) = 0.19;
- probability of revealing the change of the defined size by the third sample after this change: β.β.(1 - β) = (0. 75^2)0.25 = 0.14;
- probability of revealing the change of the defined size by the r-th sample after this change: β^{r-1}.(1 - β).

$$ARL(\delta) = \sum_{r=1}^{\infty} r\beta^{r-1}(1-\beta) = \frac{1}{1-\beta}.$$

For the assignment $ARL(\delta) = 1/0.25 = 4$.

5 EFFICIENCY OF CONTROL CHART

Chapter structure

Topics for review

Time for learning

Goal

Lecture

- **Definition of the operating characteristic (OC) of the control chart**
- **Construction of OC**
- **Definition of the ARL curve**
- **Construction of the ARL curve**

Summary of terms

Questions

Examples

Additional study resources

Solution to examples

Topics for review:

- ARL

- risk of false signal

- risk of missing signal

(see Chapter 4 of this textbook)

Time for learning: 240 minutes

Goal: After studying this chapter

- **you will be able to construct and analyze OC and ARL curves for the selected Shewhart control charts;**
- **you will be able to select an optimal control chart based on the assessment of the efficiency of this control chart using the OC and ARL curve.**

Lecture

Definition of the operating characteristic of a control chart

Operating characteristic of a control chart (OC) expresses the efficiency of the control chart, i.e. its ability to reveal changes in the process parameters.

Construction of OC

In this chapter the attention is paid only to the OC for standard Shewhart variables control charts for controlling the process level.

When constructing the OC for a control chart, selected sizes of process parameters changes, which shall be revealed to reveal as soon as possible, are plotted on x-axis (this change/deviation is expressed in a number of standard deviations marked (δ) and mostly used values are 0; 0.5; 1; ...; 3). Values of the risk of missing signal β are plotted on y-axis. β is computed for selected sizes of the process change/deviation using the formula 4.3 in chapter 4.

An example of such OC is provided in the following figure.

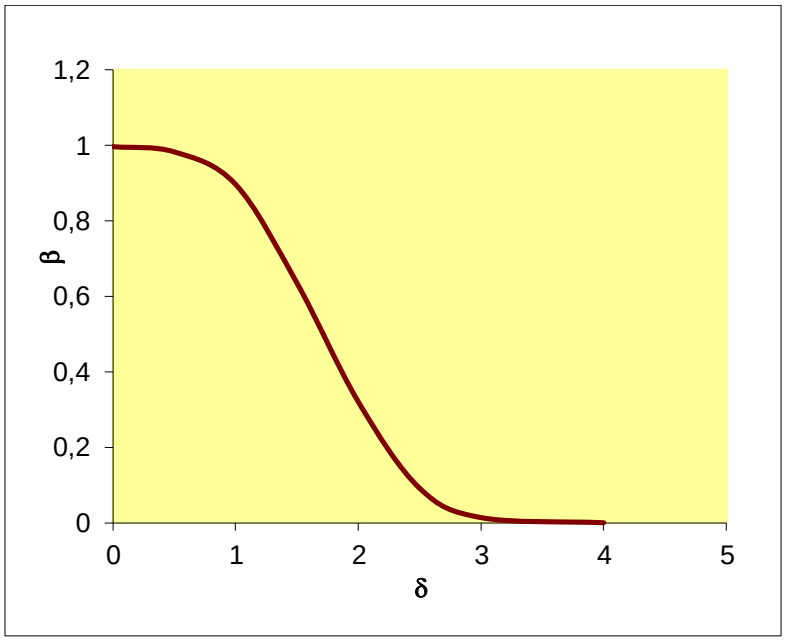


Fig. 5.1 Example of OC for an average control chart

Definition of the ARL curve

The ARL curve is a depiction of another function expressing the efficiency of a control chart when revealing the changes in process parameters.

Construction of the ARL curve

In this chapter the attention is paid only to the ARL curves for standard Shewhart variables control charts for controlling the process level.

When constructing the ARL curve for a control chart, selected sizes of process parameters changes, which shall be revealed as soon as possible, are plotted on x-axis – the same as with the OC construction. Values of ARL(δ) are plotted on y-axis. The ARL(δ) is computed for selected sizes of the process change/deviation using the formula (4.7) in chapter 4.

An example of such ARL curve is provided in the following figure.

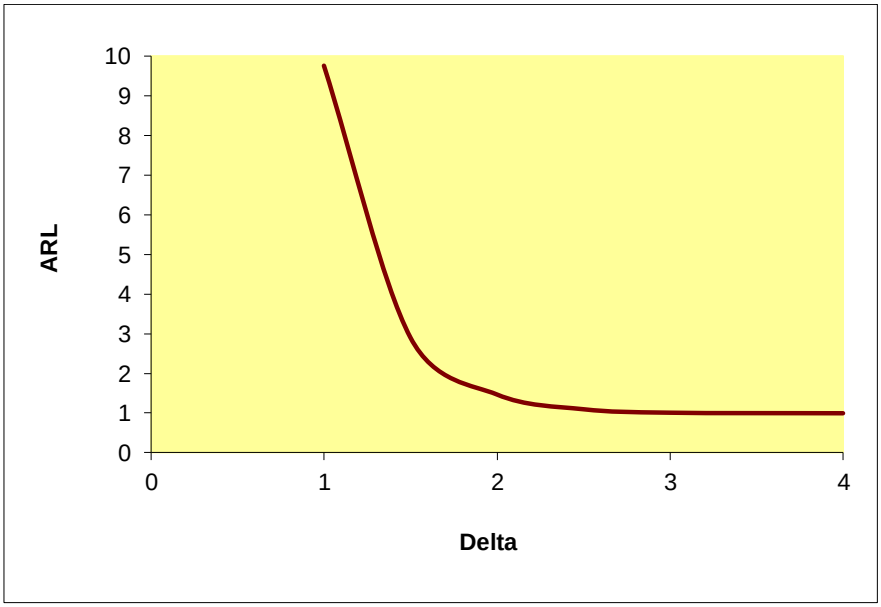


Fig. 5.2 Example of the ARL curve for an average control chart

Summary of terms

After studying this chapter you should understand these terms:

- **efficiency of a control chart**
- **operating characteristic of a control chart**
- **ARL curve**

Questions

1. **What can be read from the OC of a control chart?**
2. **What does the ARL curve express?**
3. **What is plotted on the y-axis of the ARL curve?**
4. **What is plotted on the x-axis of the OC for a control chart?**
5. **How do we determine values on the x-axis of the OC and ARL curve?**
6. **How do we determine values on the y-axis of the OC and ARL curve?**
7. **Compute ARL(δ) for $\delta = 0$. Does this value depend on the sample size *n*?**
8. **(see the formula for the computation of β)? Is this value in relation to the risk α?**
9. **Is the OC for the more efficient control chart steeper than for less efficient chart?**
10. **How to interpret the OC of a control chart?**
11. **What can be read from the ARL curve?**

Examples

Example 1

Construct the OC for the Shewhart average control chart with 3σ control limits and sample size n = 3.

Example 2

For the same situation construct the ARL curve.

Additional study resources

[1] TOŠENOVSKÝ, J. – NOSKIEVIČOVÁ, D.: *Statistické metody pro zlepšování jakosti*. Ostrava: Montanex, 2000 (in Czech)

[2] MONTGOMERY, D.C.: Introduction to Statistical Quality Control. J.Wiley & Sons, New York, 2001.

Solution to examples

Example 1

Shortened assignment:

L = 3

n= 3

Solution:

1. We select suitable values of the shift of the process level δ (the shift expressed in a number of standard deviations), for instance the values 0 – 3 with the step 0.5.

2. For particular selected values of δ we calculate the risk β using the formula:

$$\beta = \Phi(L - \delta\sqrt{n}) - \Phi(-L - \delta\sqrt{n}).$$

For instance for δ = 1 the computation is as follows:

$\beta = \Phi(3 - 1\sqrt{3}) - \Phi(-3 - 1\sqrt{3})$ = 0,8976.

In the following table the values of the risk β for all selected values of δ are presented:

Tab. 5.1 Values of the risk β

Shift δ (in a number of standard deviations)	**Risk β**
0	0.9973
0.5	0.9835
1	0.8976
1.5	0.6361
2	0.3211
2.5	0.0917
3	0.0138

3. The points with coordinates [δ; β] are placed upon the graph and interconnected by a continuous curve – see Fig. 5.3.

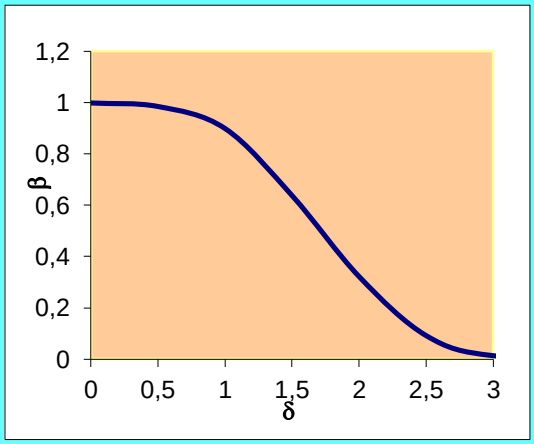


Fig. 5.3 OC for the Shewhart average control chart with 3σ limits and for n=3

Example 2

Shortened assignment:

L = 3
n = 3

Solution:

1. We select suitable values of the shift of the process level δ (the shift expressed in a number of standard deviations), for instance the values 0 – 3 with the step 0.5.
2. For particular selected values of δ we calculate the value of ARL using the formula:

$$ARL(\delta) = \frac{1}{1-\beta}.$$

For instance

for δ = 1: ARL(1) = 1/(1-0.8976) = 9.765

(values of β can be found in tab. 5.1).

In the following table the values of ARL for all selected values of δ are presented:

Tab. 5.2 Values of ARL(δ)

Shift δ (in a number of standard deviations)	**ARL(δ)**
0	370.376
0.5	60.689
1	9.765
1.5	2.908
2	1.473
2.5	1.101
3	1.014

3. The points with coordinates [δ; ARL(δ)] are placed upon the graph and interconnected by a continuous curve – see Fig. 5.4.

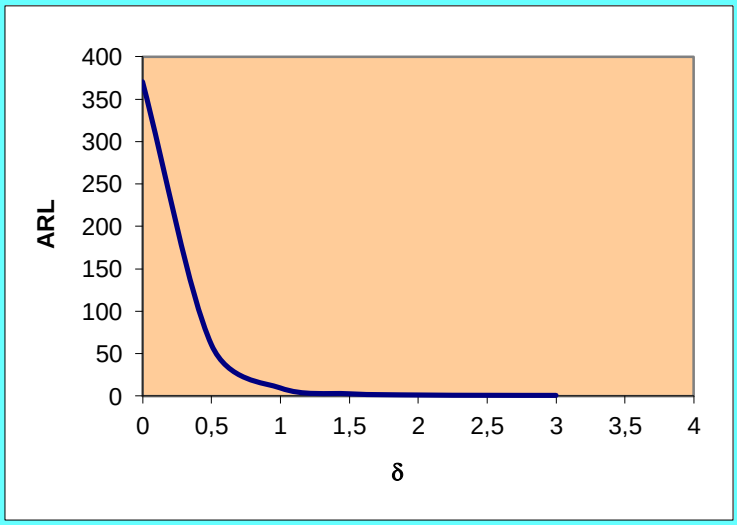


Fig. 5.4 ARL curve for Shewhart average control chart with 3σ limits and for n=3

6 SHEWHART CONTROL CHARTS

Chapter structure

Time for learning

Goal

Lecture

- **Selection of Shewhart control chart**
- **Example of control card**

Summary of terms

Questions

Examples

Additional study resources

Topics for the supplementary self-study

Solution to examples

Time for learning: 720 min

Goal: After studying the chapter

- **you will know what Shewhart control charts are;**
- **you will know statistical preconditions of applying Shewhart control charts;**
- **you will be able to derive formulae for the computation of control limits in the Shewhart control charts;**
- **you will be able to choose the correct type of the Shewhart control chart, to construct and interpret it;**
- **you will know what the control card is and what it contains;**
- **you will know how to achieve the statistically stable process;**

Lecture

Selection of Shewhart control chart

This type of the control chart was designed by W. Shewhart in 1924. He is the founder of the basis for the whole SPC system. The mentioned control charts were created in the period of the fast arrival of the high volume industrial production and application of Taylor's principles of production organization and management. For that reason, a suitable number of samples (subgroups) is the basic precondition for an effective and correct application of control charts of the Shewhart type (for computing the control limits, there are supposed to be at least 20 – 25 subgroups implemented in relatively stable conditions). Products produced during the time of the birth of the control charts were relatively simple and number of the watched quality characteristics was small as compared to the present production which is strongly customized with small production runs and short cycles and with high requests on the precision. Therefore, Shewhart control charts were designed for monitoring only **one quality characteristics** or one process parameter per one chart.

Basic statistical preconditions for the correct application of the Shewhart control charts for variables are as follows: data normality, data independence (they are no autocorrelated), constant mean and constant dispersion. Shewhart control charts belong to the group of control charts without memory because the former values of the used statistics are not taken into consideration when computing current value of it. Therefore, these control charts are suitable expecially for the idenfication of larger deviations in the process (larger than 2σ).

Shewhart control charts are divided into two groups: control charts for variables (see Fig. 6.1) and control charts for attributes (see Fig. 6.2).

For SPC for variables there are four pairs of the control charts (see Fig. 6.1). The first control chart in every pair is used for the evaluation of the process level stability and the second one is used for the evaluation of the stability of the process variation.

In SPC for attributes there exist two control charts for the situation when a number of nonconforming units is monitored and two control charts for the situation when a number of nonconformities is monitored (see Fig. 6.2).

Shewhart variables control charts

Control charts	Description
x_i, R_{kl}	Individuals and moving range charts
$\bar{x}$, R	Average and range charts
$\bar{x}$,s	Average and standard deviation charts
$\tilde{x}$, R	Median and range charts

Fig. 6.1 Shewhart control charts for variables

Shewhart attribute control charts

Control chart	Description
p	Control chart for fraction nonconforming (defective units)
np	Control chart for number nonconforming
u	Control chart for average number of nonconformities (defects) per unit in subgroup
c	Control chart for total number of nonconformities in subgroup

Fig. 6.2 Shewhart attribute control charts

The first question while selecting the correct control chart runs: Is the monitored quality characteristics variable or attribute? If it is variable, we select the pair of charts according to the sample (subgroup) size (see Fig. 6.3). When we work with attributes, first we must answer the question if we count nonconforming units or nonconformities. Then we select the correct control chart considering constancy of the sample size (see Fig. 6.3).

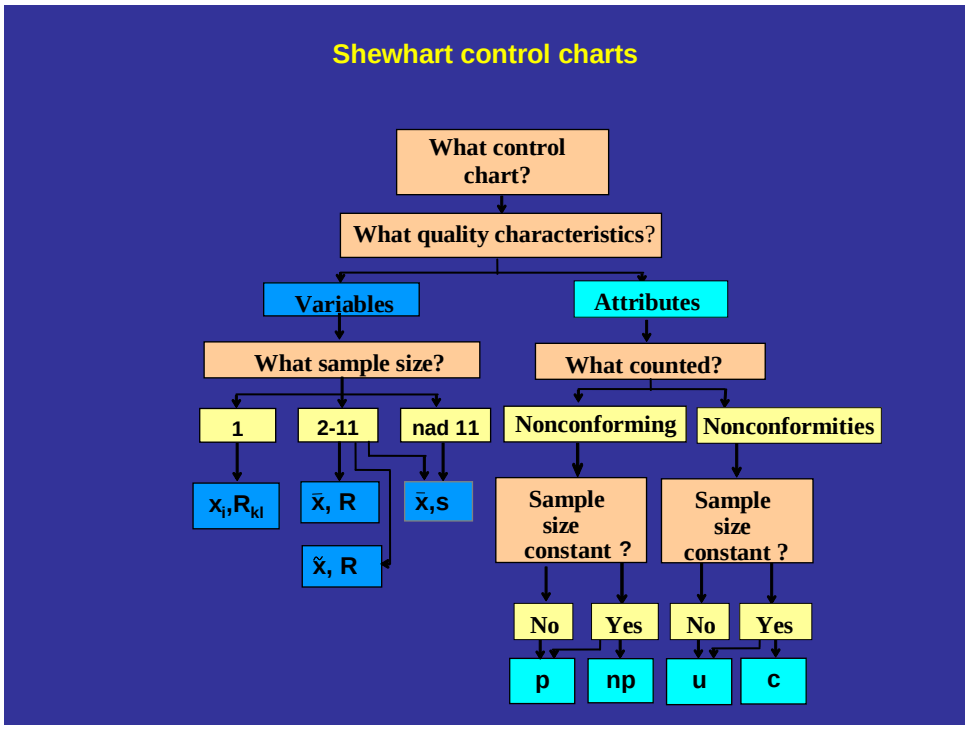


Fig. 6.3 Decision tree for the selection of the Shewhart control chart

In Tab. 6.1 summary of the formulae for computation of CL, LCL and UCL for Shewhart control charts can be found.

Tab. 6.1 Formulae for the computation of CL, LCL and UCL for Shewhart control charts [4]

Control chart	Central line CL	Upper control limit UCL	Lower control limit LCL
$(\bar{x},R)$	$\bar{\bar{x}} = \frac{1}{k}\sum_{j=1}^{k}\bar{x}_j$	UCL = $\bar{\bar{x}} + A_2 . \bar{R}$	LCL = $\bar{\bar{x}} - A_2 . \bar{R}$.
	$\bar{R} = \frac{1}{k}\sum_{j=1}^{k} R_j$,	UCL = $D_4 . \bar{R}$	LCL = $D_3 . \bar{R}$.
$(\bar{x},s)$	$\bar{\bar{x}} = \frac{1}{k}\sum_{j=1}^{k}\bar{x}_j$	UCL = $\bar{\bar{x}} + A_3 . \bar{s}$	LCL = $\bar{\bar{x}} - A_3 . \bar{s}$
	$\bar{s} = \sqrt{\frac{1}{k}\sum_{j=1}^{k} s_j^2}$	UCL = $B_3 \bar{s}$	LCL = $B_4 \bar{s}$
p n_j is not out of interval $\bar{n} \pm 0{,}25\bar{n}$	CL = $\bar{p} = \sum_{j=1}^{k} x_j / \sum_{j=1}^{k} n_j$	UCL = $\bar{p} + 3.\sqrt{\bar{p}(1-\bar{p})/\bar{n}}$	LCL = $\bar{p} - 3.\sqrt{\bar{p}(1-\bar{p})/\bar{n}}$
p n_j is out of interval $\bar{n} \pm 0{,}25\bar{n}$		UCL = $\bar{p} + 3.\sqrt{\bar{p}(1-\bar{p})/n_j}$	LCL = $\bar{p} - 3.\sqrt{\bar{p}(1-\bar{p})/n_j}$
np	CL = $n.\bar{p} = \frac{1}{k}\sum_{j=1}^{k} x_j$	UCL = $n.\bar{p} + 3\sqrt{n.\bar{p}(1-\bar{p})}$	LCL = $n.\bar{p} - 3\sqrt{n.\bar{p}(1-\bar{p})}$.
c	CL = $\bar{c} = \frac{1}{k}\sum_{j=1}^{k} c_j$,	UCL = $\bar{c} + 3.\sqrt{\bar{c}}$	LCL = $\bar{c} - 3.\sqrt{\bar{c}}$
u n_j is not out of interval $\bar{n} \pm 0{,}25\bar{n}$	CL = $\bar{u} = (\sum_{j=1}^{k} c_j) / \sum_{j=1}^{k} n_j$	UCL = $\bar{u} + 3.\sqrt{\bar{u}/\bar{n}}$	LCL = $\bar{u} - 3.\sqrt{\bar{u}/\bar{n}}$
u n_j is out of interval $\bar{n} \pm 0{,}25\bar{n}$		UCL = $\bar{u} + 3.\sqrt{\bar{u}/n_j}$	LCL = $\bar{u} - 3.\sqrt{\bar{u}/n_j}$

Constants A_2, B_3, B_4, D_3, D_4 and others can be found in the following table (their value depends on the sample (subgroup) size).

Tab**.** 6.2 Constants for computation of control limits in Shewhart variables control charts [4]

Sample size n	**Constants for computation of LCL and UCL**							
	A_2	**A_3**	**B_3**	**B_4**	**D_3**	**D_4**	**C_4**	**d_2**
2	1.880	2.659	0.000	3.267	0.000	3.267	0.7979	1.128
3	1.023	1.954	0.000	2.568	0.000	2.574	0.8862	1.693
4	0.729	1.628	0.000	2.266	0.000	2.282	0.9213	2.059
5	0.577	1.427	0.000	2.089	0.000	2.114	0.9400	2.326
6	0.483	1.287	0.030	1.970	0.000	2.004	0.9515	2.534
7	0.419	1.182	0.118	1.882	0.076	1.924	0.9594	2.704
8	0.373	1.099	0.185	1.815	0.136	1.864	0.9650	2.847
9	0.337	1.032	0.239	1.761	0.184	1.816	0.9693	2.970
10	0.308	0.975	0.284	1.716	0.223	1.777	0.9727	3.078
11	0.285	0.927	0.321	1.679	0.256	1.744	0.9754	3.173
12	0.266	0.886	0.354	1.646	0.283	1.717	0.9776	3.258
13	0.249	0.850	0.382	1.618	0.307	1.693	0.9794	3.336
14	0.235	0.817	0.406	1.594	0.328	1.672	0.9810	3.407
15	0.223	0.789	0.428	1.572	0.347	1.653	0.9823	3.472
16	0.212	0.763	0.448	1.552	0.363	1.637	0.9835	3.532
17	0.203	0.739	0.466	1.534	0.378	1.622	0.9845	3.588
18	0.194	0.718	0.482	1.518	0.391	1.608	0.9854	3.640
19	0.187	0.698	0.497	1.503	0.403	1.597	0.9862	3.689
20	0.180	0.680	0.510	1.490	0.415	1.585	0.9869	3.735
21	0.173	0.663	0.523	1.477	0.425	1.575	0.9876	3.778
22	0.167	0.647	0.534	1.466	0.434	1.566	0.9882	3.819
23	0.162	0.633	0.545	1.455	0.443	1.557	0.9887	3.858
24	0.157	0.619	0.555	1.445	0.451	1.548	0.9892	3.895
25	0.153	0.606	0.565	1.435	0.459	1.541	0.9896	3.931

Example of control card

In Fig. 6.4, an example of a control card for range and average control charts can be seen. There are not only control charts (zone A) but also zone B for recording the measurements and values of computed sample statistics (in this example, subgroup ranges and subgroup averages) and zone C containing formulae and computations of CL, LCL and UCL and values of selected constants for computation of control limits corresponding to the used sample size *n*. In the upper part of the control card (zone D), one can find information about the controlled process, product, controlled quality characteristics and about sample size and control frequency. The last marked zone – zone E - is really of a great importance. It contains a place for the date and time of recording the measurements and signature of the worker who has made measurements and records.

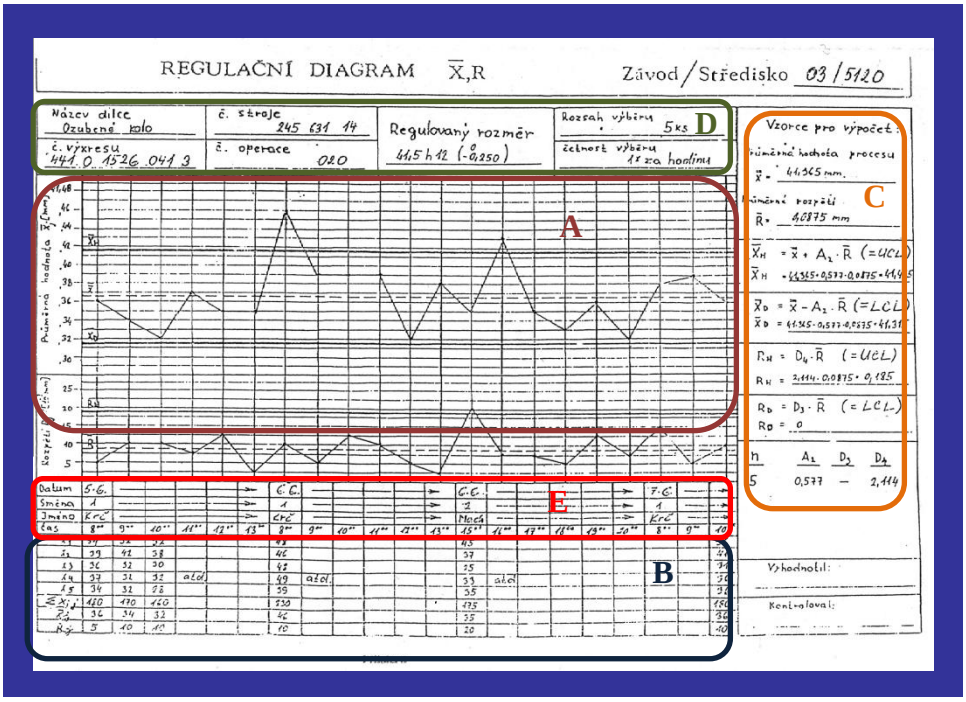
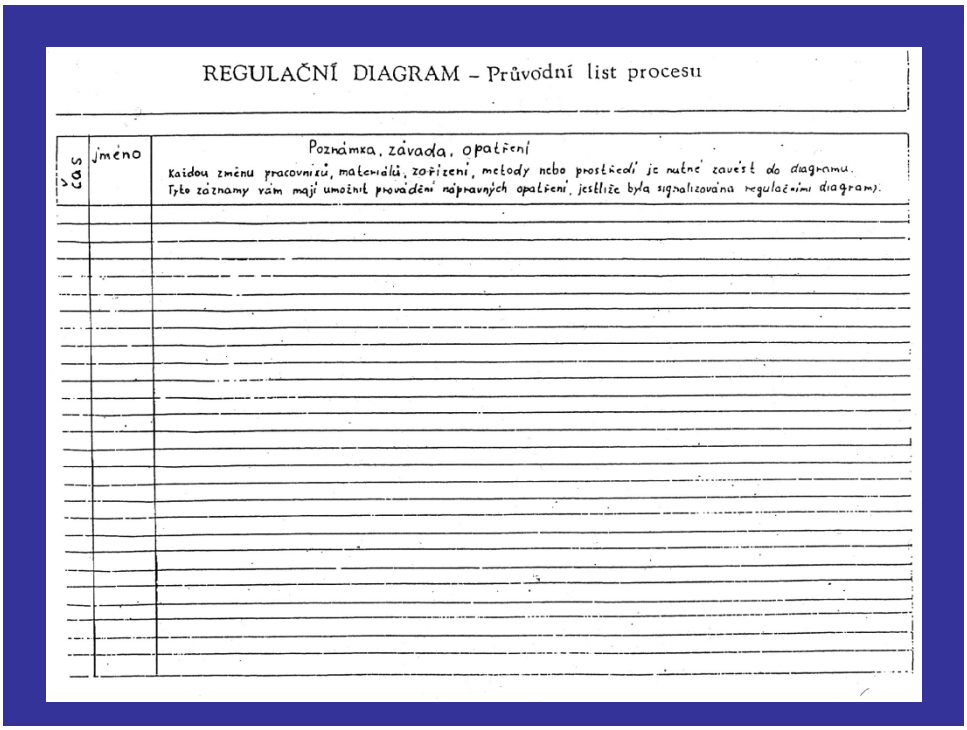


Fig. 6.4 Example of a control card

The control card should contain the so called process dispatch sheet (see Fig. 6.5) where information about every change in the process (change in material, in operator, machine failures, maintenance activities, change of machine adjustment, change of instrument …) should be recorded - including the date and time of the change and the name of a person who identified or carried out some changes.


REGULAČNÍ DIAGRAM – Průvodní list procesu

čas	jméno	Poznámka, závada, opatření Každou změnu pracovníků, materiálů, zařízení, metody nebo prostředí je nutné zavést do diagramu. Tyto záznamy vám mají umožnit provádění nápravných opatření, jestliže byla signalizována regulačními diagramy.

Fig. 6.5 Example of a process dispatch sheet

This information is necessary for an effective and fast identification of assignable causes diagnosed in the control chart and for the fast realization of the correct control action to ensure statistical stability of the controlled process.

Summary of terms

After studying this chapter you should understand these terms:

- **control chart**
- **statistically stable and unstable process**
- **upper control limit**
- **lower control limit**
- **central line**
- **constants for computing control limits**
- **nonrandom patterns**
- **control action**
- **control card**
- **process dispatch sheet**

Questions

1. **What is the main tool of SPC?**
2. **What is the control card?**
3. **What is the process dispatch sheet and what is its purpose?**
4. **What are Shewhart control charts?**
5. **What are statistical preconditions for an effective and correct application of Shewhart control charts, namely for variables?**
6. **What is the decision criterion for selecting the best Shewhart control chart?**
7. **How to determine CL in a particular Shewhart control chart?**
8. **How to determine UCL and LCL in a particular Shewhart control chart?**
9. **Which control charts can have the control limits non-constant? Why?**
10. **What are the combined control limits? Where and when are used?**
11. **Do you know any variable quality characteristics or process parameter?**
12. **What are the attributes?**
13. **Define the difference between a nonconforming unit and nonconformity.**

Examples

Example 1

Data in Table 6.3 represent results of an inspection of all PCs produced during the last 10 days. Verify statistical stability of the process using a suitable control chart. If needed, use combined control limits composed of average and individual limits.

Tab. 6.3 Data for Example 1

Day	Number of inspected units n_i	Number of nonconforming units x_i
1	80	4
2	110	7
3	90	5
4	75	8
5	130	6
6	120	6
7	70	4
8	125	5
9	105	8
10	95	7
Sum		

Example 2

In Tab. 6.4 inspection results of 26 consecutive batches of 100 integrated circuits are summarized.

Select a suitable control chart and evaluate statistical stability of the process. When some assignable causes are indicated, ensure the statistical stability of the process. Then use recalculated limits for the evaluation of the process statistical stability in the following period.

Tab. 6.4 Data for Example 2

Batch No.	Number of nonconformities c_j	Batch No.	Number of nonconformities c_j
1	21	**14**	19
2	24	**15**	10
3	16	**16**	17
4	12	**17**	13
5	15	**18**	22
6	5	**19**	18
7	28	**20**	39
8	20	**21**	30
9	31	**22**	24
10	25	**23**	16
11	20	**24**	19
12	24	**25**	17
13	16	**26**	15
Sum			516

Example 3

Producer of a chemistry product has problems with a filling line where glass bottles with the nominal value of 52.00 g are filled. It was decided to control this process using control charts. Samples (rational subgroups) of the size of 6 units were collected every 5 minutes. Values of 22 samples can be found in Tab. 6.5.

Verify statistical stability of the filling process using suitable control charts. If the process is not statistically stable, describe and carry out procedure of the stabilization of the process and set new control limits. After that, analyze capability of this process using information from the control charts.

Tab. 6.5 Data for Example 3

	Measurements							
Subgr. No.	**1**	**2**	**3**	**4**	**5**	**6**		
1	52,22	52,85	52,41	52,55	53,10	52,47		
2	52,25	52,14	51,79	52,18	52,26	51,94		
3	52,37	52,69	52,26	52,53	52,34	52,81		
4	52,46	52,32	52,34	52,08	52,07	52,07		
5	52,06	52,35	51,85	52,02	52,3	52,2		
6	52,59	51,79	52,2	51,9	51,88	52,83		
7	51,82	52,12	52,47	51,82	52,49	52,6		
8	52,51	52,8	52	52,47	51,91	51,74		
9	52,13	52,26	52,00	51,89	52,11	52,27		
10	51,18	52,31	51,24	51,59	51,46	51,47		
11	51,74	52,23	52,23	51,7	52,12	52,12		
12	52,38	52,2	52,06	52,08	52,1	52,01		
13	51,68	52,06	51,9	51,78	51,85	51,4		
14	51,84	52,15	52,18	52,07	52,22	51,78		
15	51,98	52,31	51,71	51,97	52,11	52,1		
16	52,32	52,43	53	52,26	52,15	52,36		
17	51,92	52,67	52,8	52,89	52,56	52,23		
18	51,94	51,96	52,73	52,72	51,94	52,99		
19	51,39	51,59	52,44	51,94	51,39	51,67		
20	51,55	51,77	52,41	52,32	51,22	52,04		
21	51,97	51,52	51,48	52,35	51,45	52,19		
22	52,15	51,67	51,67	52,16	52,07	51,81		

Additional study resources

[1] GRIFFITH, G. K. *Statistical Process Control Methods For Long and Short Runs*. Milwaukee, Wisconsin: ASQC Quality Press, 1996. 250 p. ISBN 0-87389 315-X.
[2] MONTGOMERY, D. C.: *Statistical Quality Control. A Modern Introduction*. New York : J. Wiley & Sons, 2012. 768 p. ISBN: 978-1-1183-2257-4.
[3] ISO 8258: *Shewhart Control Charts*. 1991.
[4] TOŠENOVSKÝ, J. – NOSKIEVIČOVÁ, D.: *Statistické metody pro zlepšování jakosti*. Ostrava: Montanex, 2000. 362 p. ISBN 80-7225-040-X.
[5] WHEELER, D. A CHAMBERS, D.: *Understanding Statistical Process Control*. 3rd Ed. Knoxville, Tennessee, SPC Press Inc., 2010. 978 p. ISBN 978-0-945320-69-2.

Topics for the supplementary self-study

- computation of CL, LCL and UCL in Shewhart control charts;
- derivation of formulae for the computation of CL, LCL and UCL in Shewhart control charts;
- explanation of selection of a particular Shewhart control chart in detail.

Solution to examples

Solution to Example 1

1. Selection of the control chart

Number of nonconforming units is a discrete random variable, sample (subgroup) size is not constant. Therefore, we select the control chart p for fraction nonconforming – see the decision tree in Fig. 6.6 – orange arrows.

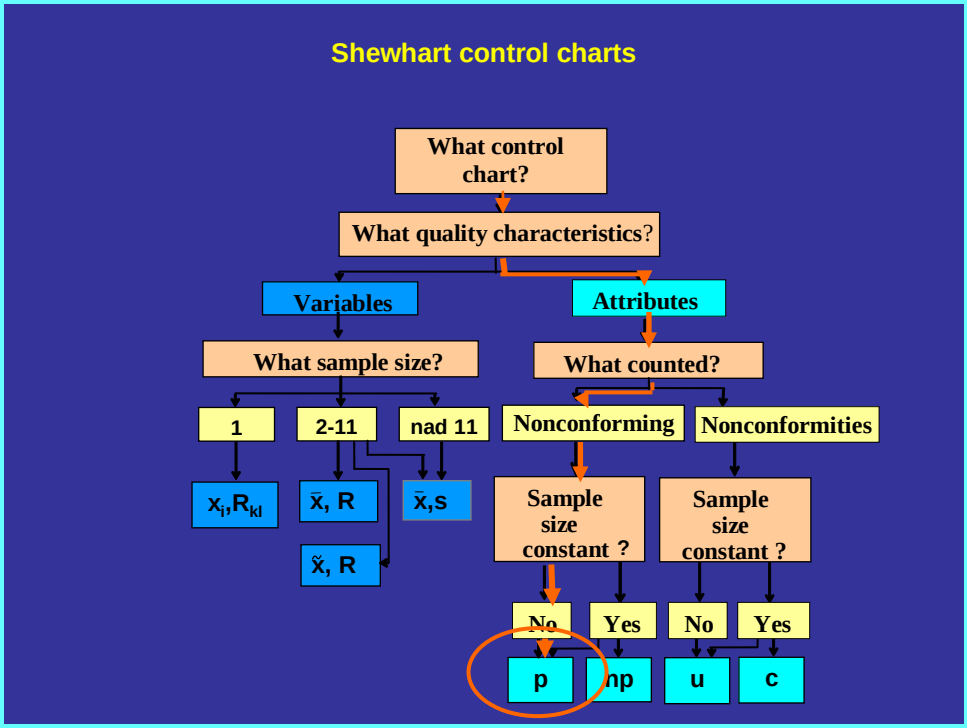


Fig. 6.6 Selection of control chart – Example 1

2. Computation of CL, LCL and UCL (methodical notes – see the block *Notes* in this example below; necessary formulae – see Tab. 6.1 in this chapter and [3], [4]).

Computation of central line CL:

CL = 60/1000 = 0,06 (see the last row in Tab. 6.6).

Computation of control limits:

At first, we must find if we apply only average control limits or individual control limits too. For that reason, we compute the average simple size $\bar{n}$ and limits of the interval $\bar{n} \pm 0.25\,\bar{n}$:

$\bar{n}$ = 1000/10 = 100

Lower interval limit: $\bar{n} - 0.25\,\bar{n} = 75$;

Upper interval limit: $\bar{n} + 0.25\,\bar{n} = 125$.

Samples no. 5 and 7 do not lie in this interval (see Tab. 6.6, column 2).

For these two samples, individual control limits will be used; for other samples, average

limits will be applied.

Setting the average control limits (with $\bar{n} = 100$)

UCL =0.131

LCL = 0.000

Setting individual limits (with $n_5 = 130$ a $n_7 = 70$)

$UCL_5 = 0.123$

$LCL_5 = 0$

$UCL_7 = 0.145$

$LCL_7 = 0$

3. Construction of the control chart (p)

We shall draw the combined control limits and p_j values (fraction of nonconforming units in the j-th sample (subgroup) – see Tab. 6.6, column 4) into the control chart.

We shall connect points together with a line – see Fig. 6.7.

Tab. 6.6 Partial computations for Example 1

Col. 1	**Col. 2**	**Col. 3**	**Col. 4**
Day	**Number of inspected units n_i**	**Number of nonconforming units x_i**	**Fraction of nonconforming units $p_j = x_j / n_j$**
1	80	4	0,05 = **4/80**
2	110	7	0,064
3	90	5	0,056
4	75	8	0,107
5	**130**	**6**	**0,046**
6	120	6	0,05
7	**70**	**4**	**0,057**
8	125	5	0,04
9	105	8	0,076
10	95	7	0,074
Suma	**1000**	**60**	**CL = 0,06**

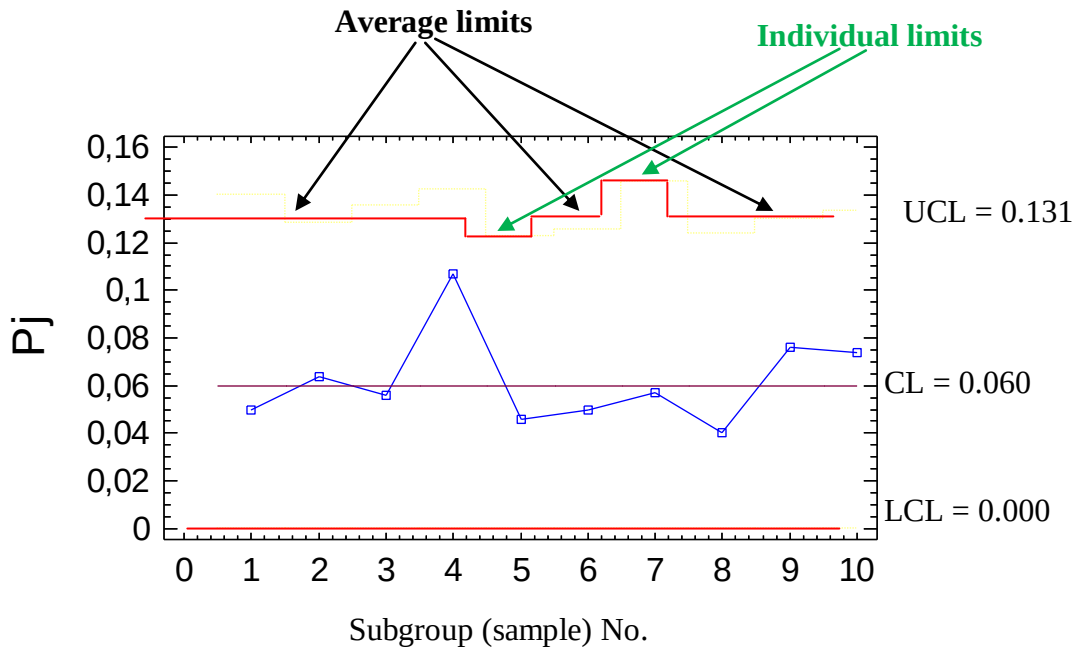


Fig. 6.7 Control chart (p) – Example 1

4. Control chart interpretation

As one can see in Fig. 6.7, no point is outside the control limits and there are no non-random patterns there.

5. Conclusions

The analysed process can be considered statistically stable. No control action is necessary.

Notes:

Combined limits represent a combination of individual and average limits. They can be applied in the control chart (p) and (u).

Average limits are applied for the j-th sample (subgroup) if the sample (subgroup) size n_j lies in the closed interval $\bar{n} \pm 0.25\,\bar{n}$.

Individual limits are applied for the j-th sample (subgroup) if the sample (subgroup) size n_j is not from the closed interval $\bar{n} \pm 0.25\,\bar{n}$ ($\bar{n}$ is the average subgroup (sample) size).

Solution to Example 2

1. Selection of the control chart

Number of nonconformities in a subgroup is a discrete random variable, the sample (subgroup) size is constant (n = 100 units). Therefore, we select the control chart c for a number of nonconformities in a subgroup – see the decision tree in Fig. 6.8 – orange arrows.

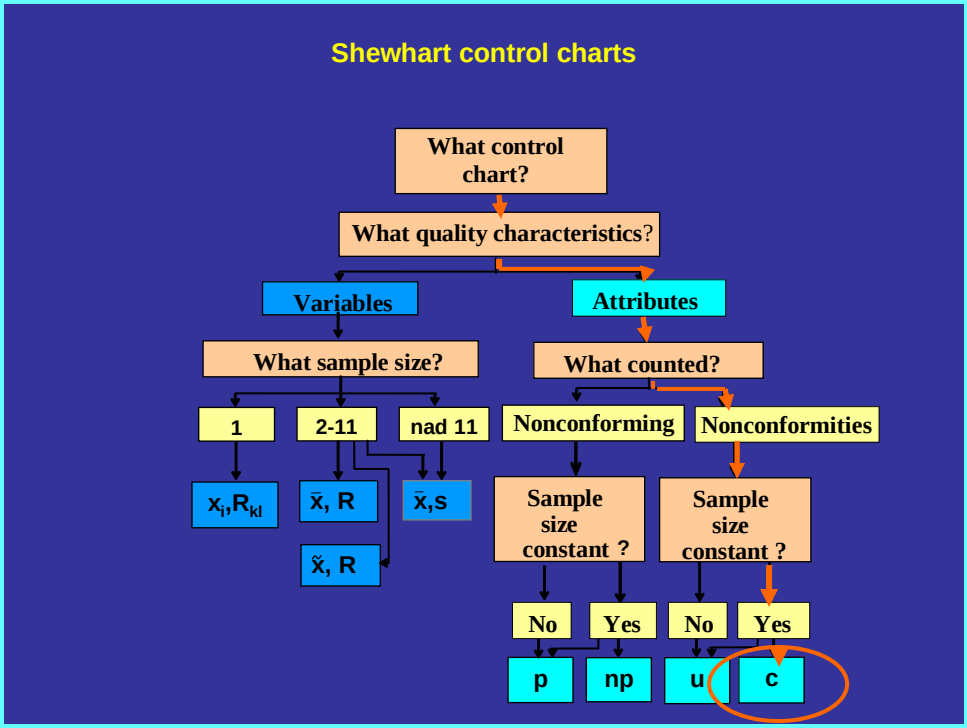


Fig. 6.8 Selection of control chart – Example 2

2. Computations of CL, LCL and UCL (necessary formulae – see Tab. 6.1 in this chapter and [3], [4]).

CL = 516/26 = 19.85

UCL = 33.21

LCL = 6.48

3. Construction of the control chart (c)

We shall draw CL, UCL, LCL and values c_j (number of nonconformities in j-th sample (subgroup) – see Tab. 6.4 in the previous chapter Examples) into the control chart. Then the points are connected together with the line – see Fig. 6.9.

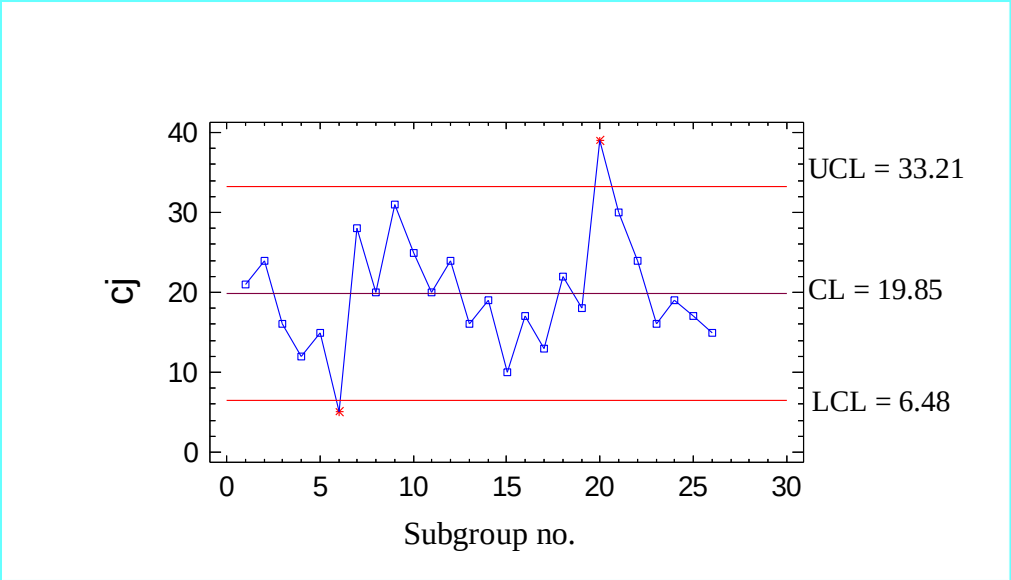


Fig. 6.9 Control chart (c) for Example 2

4. Interpretation of the control chart

As one can see in Fig. 6.9, the points corresponding to the sample 6 and 20 are outside the control limits.

The process cannot be considered statistically stable.

5. Identification of assignable causes

The following analysis of the process revealed that the large deviation in the 6th subgroup had been the result of insufficient experience of the new inspector. He had not identified all different types of defects. That is why every defect had not been recorded. It was immediately decided to train him for this analysis more carefully. Another large deviation in the subgroup 20 was caused by problems with the temperature regulation on the wave soldering. This problem was also immediately solved and eliminated.

6. Recomputation of CL,UCL, LCL

With regard to the fact that the assignable causes were identified and control corrective actions were carried out, it is possible to eliminate subgroups 6 and 20 from next computations and to

recompute the CL, LCL and UCL without them (in fact, it is realization of II. phase of the SPC – see paragraph "Phases of SPC" in the Chapter 3 of this textbook).

After recomputing without the sample 6 and 20

CL = 472/24 = 19,67;

UCL = 32,97;

LCL = 6,37.

After analysing the control chart with recomputed control limits (see Fig. 6.10), the process can be considered statistically stable (it means that by the previous analysis of the process the correct root causes were identified and the accepted control actions were effective. The process has been made statistically stable.

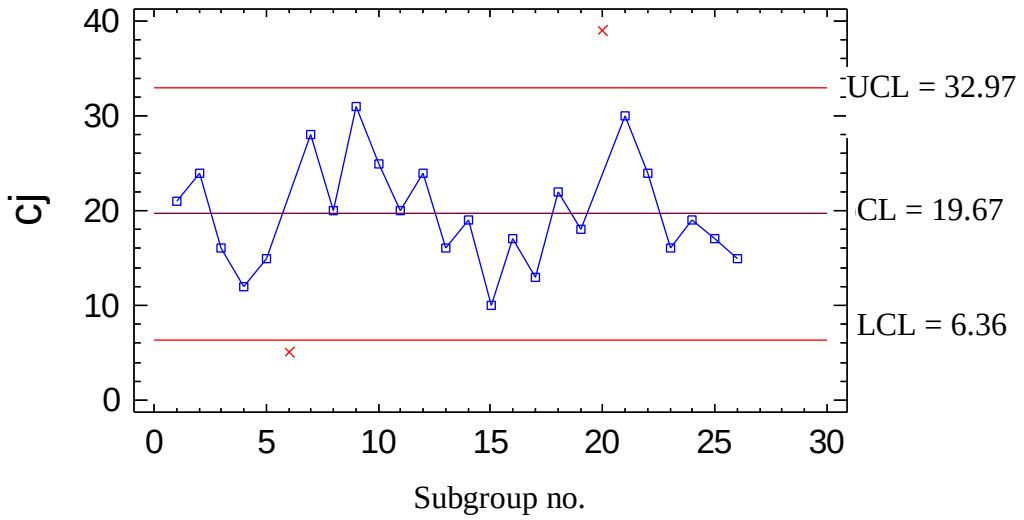


Fig. 6.10 Control chart (c) with recomputed control limits

7. Application of the recomputed CL, LCL and UCL on the consecutive monitoring and controlling the process

The above stated CL, LCL and UCL will be applied on the next monitoring, or control of the process (see IV. phase of the SPC; for more see the paragraph "Phases of SPC" in the Chapter 3 of this textbook). CL, LCL and UCL from the previous control chart will be used in the new control chart. Nonconformities in the new subgroups will be counted and corresponding values c_j will be drawn into this new control chart (see Fig. 6.11). As one can see, the CL, LCL and UCL were transferred from the control chart in Fig. 6.10 and the points drawn into the new control chart in Fig. 6.11correspond to 20 new values c_j recorded in Tab. 6.7.

Tab. 6.7 Data for Example 2

Batch No.	Number of nonconformities c_j	Batch No.	Number of nonconformities c_j
1	14	**11**	19
2	18	**12**	21
3	13	**13**	15
4	15	**14**	23
5	26	**15**	17
6	21	**16**	11
7	27	**17**	15
8	20	**18**	10
9	24	**19**	14
10	18	**20**	21

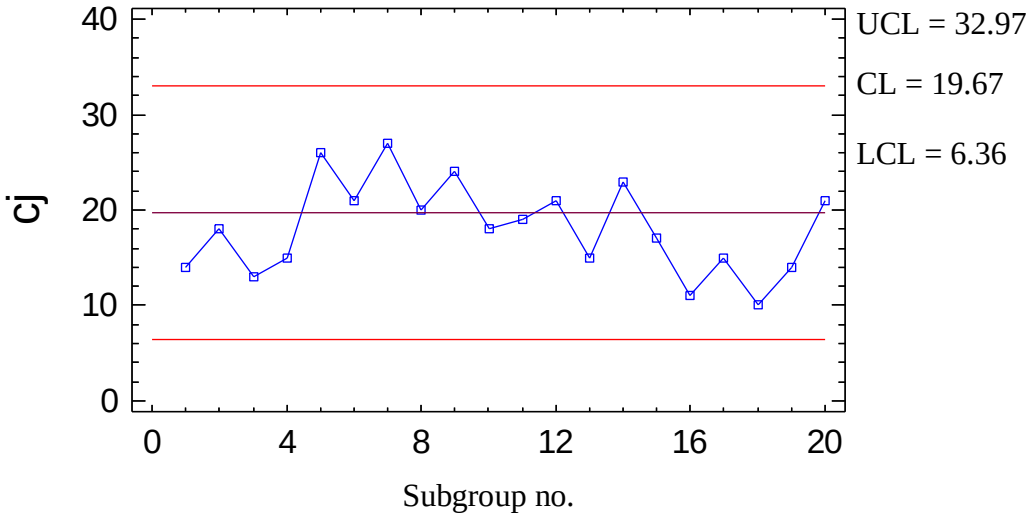


Fig. 6.11 Application of control chart (c) in IV. phase of SPC

8. Conclusions

Based on the analysis of this chart it can be stated that for now the process is statistically stable. In spite of this fact, the number of nonconformities per 100 units is still high. For that reason it needs to do a deeper analysis of the process of wave soldering. It must be found out which types of defects have the highest occurrence (for instance using Pareto analysis) and,

further on, the stress must be put on them. In addition, it should be suitable to do the analysis of the most frequent types of defects according to the types of products. Ishikawa diagram is another suitable tool for analysing the causes of defects and their relations. Using this instrument it is possible to set the variables for designed experiment with the aim to optimize the process of wave soldering.

Solution to Example 3

1. Selection of the control charts

Weight of the chemical product in a bottle is a random variable (continuous variable). Sample (subgroup) size n = 6. For that reason we select a pair of the control charts – the range control chart and the average control chart – see the decision tree in Fig. 6.12 – orange arrows.

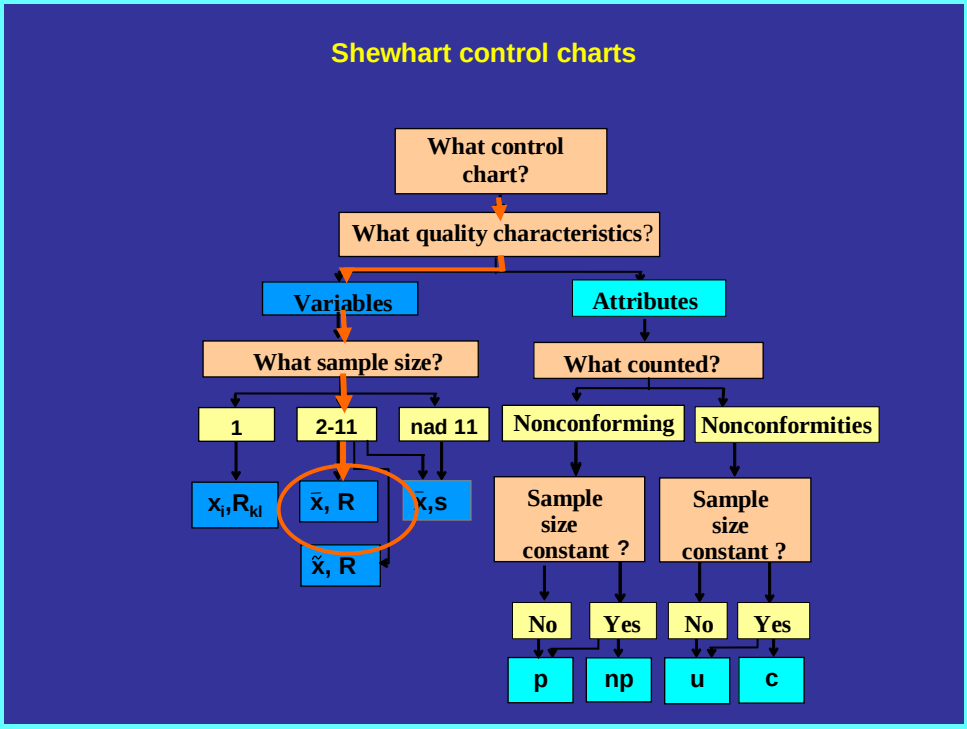


Fig. 6.12 Selection of the control charts for Example 3

2. Computation of sample ranges and averages from measured values – columns 8 and 9 in the following Tab. 6.8 (for formulae see for instance [3], [2], [4].

Tab. 6.8 Data for example 3

Sl. 1	Sl. 2	Sl. 3	Sl. 4	Sl. 5	Sl. 6	Sl. 7	Sl. 8	Sl. 9
	Measurements						Rj	$\bar{x}_j$
Subgr.no.	1	2	3	4	5	6		
1	52,22	52,85	52,41	52,55	53,10	52,47	0,88	52,6
2	52,25	52,14	51,79	52,18	52,26	51,94	0,47	52,09
3	52,37	52,69	52,26	52,53	52,34	52,81	0,55	52,5
4	52,46	52,32	52,34	52,08	52,07	52,07	0,39	52,22
5	52,06	52,35	51,85	52,02	52,3	52,2	0,5	52,13
6	52,59	51,79	52,2	51,9	51,88	52,83	1,04	52,2
7	51,82	52,12	52,47	51,82	52,49	52,6	0,78	52,22
8	52,51	52,8	52	52,47	51,91	51,74	1,06	52,24
9	52,13	52,26	52,00	51,89	52,11	52,27	0,38	52,11
10	51,18	52,31	51,24	51,59	51,46	51,47	1,13	51,54
11	51,74	52,23	52,23	51,7	52,12	52,12	0,53	52,02
12	52,38	52,2	52,06	52,08	52,1	52,01	0,37	52,14
13	51,68	52,06	51,9	51,78	51,85	51,4	0,66	51,78
14	51,84	52,15	52,18	52,07	52,22	51,78	0,44	52,04
15	51,98	52,31	51,71	51,97	52,11	52,1	0,6	52,03
16	52,32	52,43	53	52,26	52,15	52,36	0,85	52,42
17	51,92	52,67	52,8	52,89	52,56	52,23	0,97	52,51
18	51,94	51,96	52,73	52,72	51,94	52,99	1,05	52,38
19	51,39	51,59	52,44	51,94	51,39	51,67	1,05	51,74
20	51,55	51,77	52,41	52,32	51,22	52,04	1,19	51,89
21	51,97	51,52	51,48	52,35	51,45	52,19	0,9	51,83
22	52,15	51,67	51,67	52,16	52,07	51,81	0,49	51,92

3. Computation of CL, LCL and UCL and construction of the range control chart (R)

(for formulae and constants D_3, D_4 see Tab. 6.1, Tab. 6.2 in this chapter or [3], [2], [4] or "Notes". The block "Notes" – methodical notes below in this example).

The range control chart R must be analysed every time as the first chart of the pair.

$D_4 = 2.004$ (for n = 6)

$D_3 = 0$ (for n = 6)

CL = 0,740

UCL = 1,483

LCL = 0

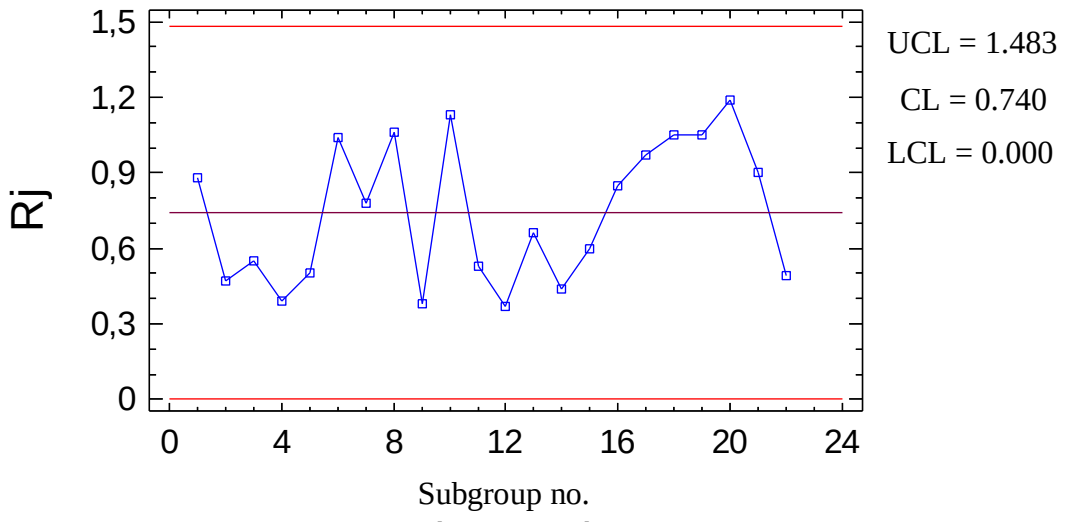


Fig. 6.13 Control chart (R)

4. Interpretation of the control chart (R) – see Fig. 6.13

The control chart (R) does not signal influence of any assignable cause. Therefore, the process can be considered statistically stable as for the process variation and we can continue with the construction and analysis of the average control chart ($\bar{x}$).

5. Computation of CL, LCL and UCL and construction of the average control chart ($\bar{x}$)

(for formulae and constants A_2 see Tab. 6.1, Tab. 6.2 in this chapter; [3], [2], [4]; "Notes" (the block "Notes" – methodical notes below in this example).

$A_2 = 0{,}483$ (for n = 6)

CL = 52,116

UCL = 52,473

LCL = 51,758

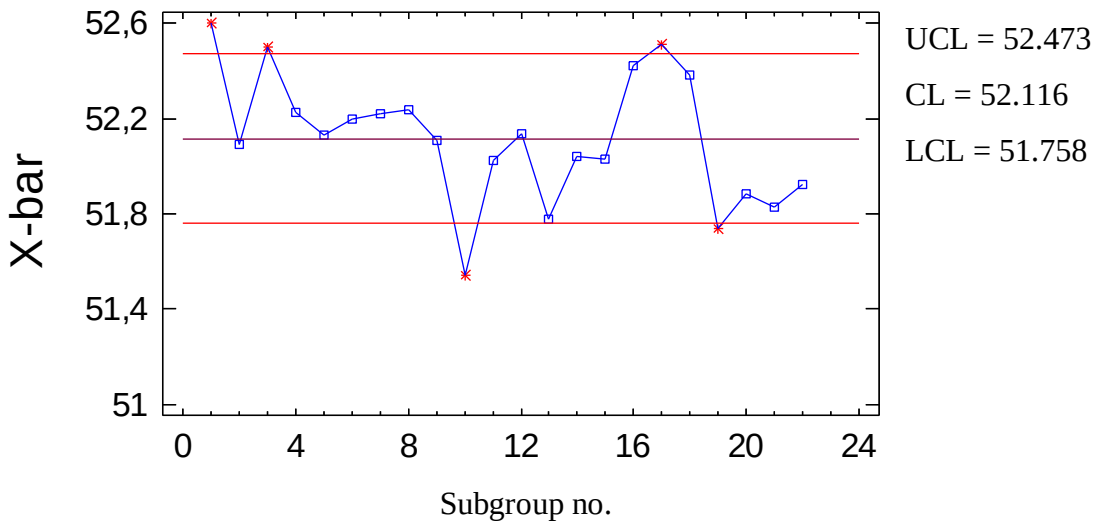


Fig. 6.14 Control chart ($\bar{x}$)

6. Interpretation of the control chart $\bar{x}$

In the control chart in Fig. 6.14 we can see 5 points outside the control limits. Therefore, the process cannot be considered statistically stable. It is necessary:

a) to identify present causes of the deviations in subgroups no. 1, 3, 10, 17 and 19;

b) to accept and carry out control correct actions or improvements to reduce or eliminate effects of these assignable causes.

7. Recomputation of CL and control limits in both control charts

This step can be done only in case that the actions a) and b) from the previous step were implemented.

In our example the process was analysed; concrete corrective actions and one improvement were carried out. Therefore, the CL and control limits can be recomputed without subgroups 1, 3, 10, 17 and 19, again at first in the control chart R (see Fig. 6.15).

CL = 0,688

UCL = 1,379

LCL = 0

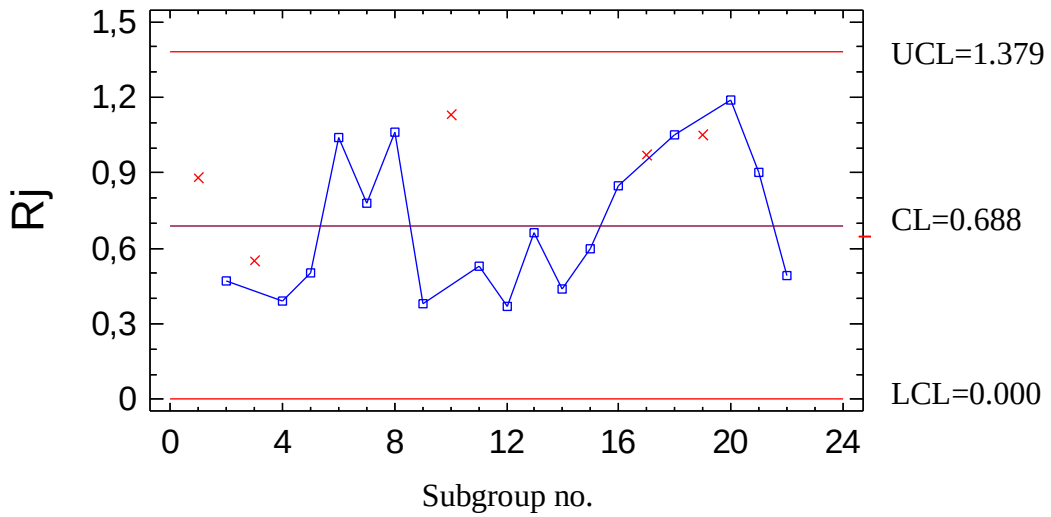


Fig. 6.15 Control chart (R) with recomputed CL and control limits

8. Interpretation of the recomputed control chart (R) and construction of the recomputed chart $\bar{x}$

In the re-computed control chart (R) on Fig. 6.15 there are no symptoms of the process instability. Therefore, it is possible to recompute CL, LCL a UCL in the average control chart $\bar{x}$ and construct it (Fig. 6.16).

CL = 52,097

LCL = 51,765

UCL = 52,43

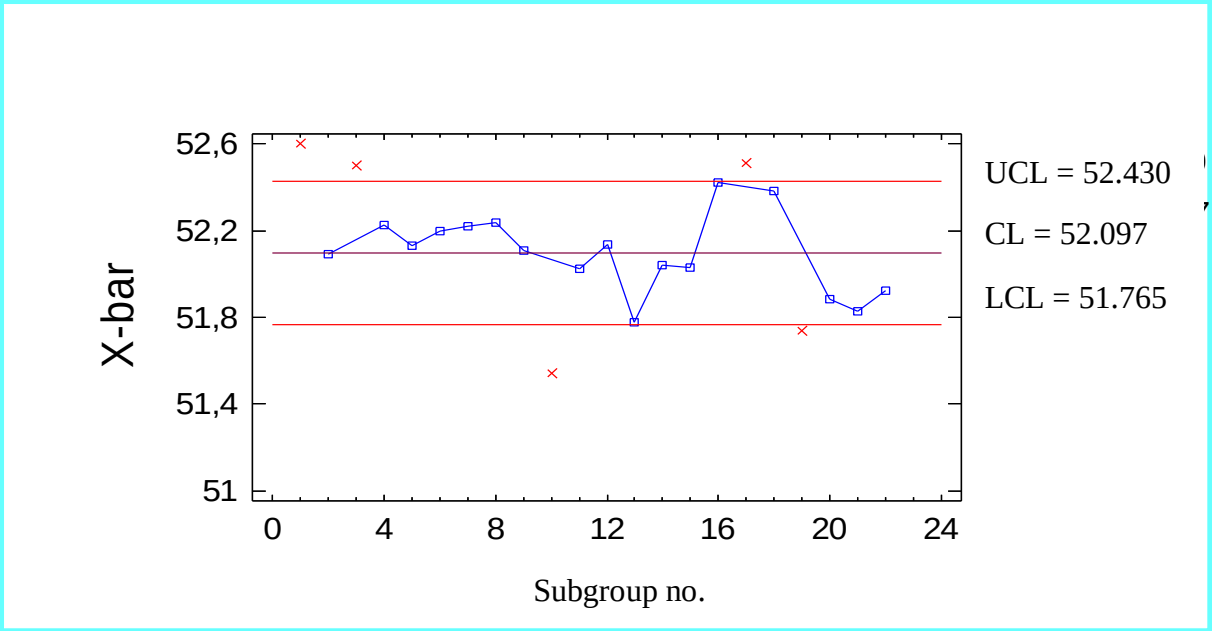


Fig. 6.16 Control chart $\overline{x}$ with recomputed CL, LCL a UCL

9. Interpretation of the recomputed control chart ($\overline{x}$)

Because no points are outside the control limits and no non-random patterns are in Fig. 6.16, the process can be considered statistically stable from the point of view of the level, and we can continue with the process capability analysis.

10. Process capability analysis

One of the main preconditions of the correct capability analysis (statistical stability) has been met (we suppose that other preconditions have been met, too). Therefore, we can compute indices C_p and C_{pk} using information from former control charts (see formulae in the block "Notes" below in this solution).

T_0 (middle of the specification zone) = 52.00 g
USL (upper specification limit) = 52.5g
LSL (lower specification limit) = 51.5g

For the estimation of standard deviation σ we apply the formula working with the value of CL from the control chart (R) showing statistically stable state (Fig. 6.15), i.e. $\overline{R}$ = 0,688. For the

formula for computing this estimation see the block "Notes" below in this solution – it is a part of the formulae for computing C_p and C_{pk} in their denominator).

For the estimation of mean μ we use the value of CL from the control chart ($\bar{x}$) showing statistically stable state (Fig. 6.16), i.e. $\bar{\bar{x}} = 52{,}097$.

Computation of capability indices Cp and Cpk

$\hat{C}_p \cong 0.6137;$

$\hat{C}_{pk} \cong \min(0.7334 , 0.4938) = 0.4938.$

(In fact we compute estimations of capability indices.)

11. Interpretation of results

Both estimations are lower than the standard minimal value 1.33. It means that the process is not capable enough. Value of the index C_p shows that the process has a considerably larger variation than the acceptable variation given by the specifications is prescribed. In addition, the C_{pk} is lower than the C_p. It means that the process is also not centred in the middle of the tolerance zone. Practically it needs to put the stress on the reduction of the process variation at first.

Notes:

D_4 and D_3 are constants for computing the control limits in the chart (R). Their values depend on the sample (subgroup) size *n* (see Tab. 6.2 in this chapter, [3] or [4], [2].

A_2 is a constant for computing the control limits in the chart ($\bar{x}$) in the pair ($\bar{x}$, R). Its value depends on the sample (subgroup) size *n* (see Tab. 6.2 in this chapter, [3] or [4], [2]).

Formulae for computing the indices C_p and C_{pk} using information from the control charts R and ($\bar{x}$):

$$\hat{C}_p = \frac{USL - LSL}{6\hat{\sigma}} = \frac{USL - LSL}{6\dfrac{\bar{R}}{d_2}}, \tag{6.1}$$

$$\hat{C}_{pk} = \min\left\{\frac{USL - \hat{\mu}}{3\hat{\sigma}}, \frac{\hat{\mu} - LSL}{3\hat{\sigma}}\right\} = \left\{\frac{USL - \bar{\bar{x}}}{3\dfrac{\bar{R}}{d_2}}, \frac{\bar{\bar{x}} - LSL}{3\dfrac{\bar{R}}{d_2}}\right\}, \tag{6.2}$$

where

USL is the upper tolerance (specification) limit,

LSL is the lower tolerance (specification) limit,

$\bar{\bar{x}}$ is the CL from the control chart ($\bar{x}$) showing a statistically stable state,

$\bar{R}$ is the CL from the control chart (R) showing a statistically stable state,

d_2 is the Hartley constant depending on the sample size *n* (see Table 6.2 in this chapter, [3], [2], [4]).

7 UNCONVENTIONAL SPC

Chapter structure

Topics for review

Time for learning

Goal

Lecture

- **Conditions for the birth and effective application of conventional Shewhart control charts**
- **Changes in economic conditions in the 1980s and 1990s**
- **Prerequisites for an effective application of Shewhart control charts for variables**
- **Assumption of the unchangeability of the probability distribution in time**
- **Assumption of the data non-autocorrelation**
- **Assumption of the high level of the process repeatibility**
- **Assumption of the control chart sensitivity to large process parameters changes**
- **Assumption of monitoring one characteristic per one product**
- **Selection of the control chart with respect to assumptions about the data properties - summary**

Summary of terms

Questions

Additional study resources

Topics for the supplementary self-study

O

Topics for review:

- Shewhart control charts

Time for learning: 240 minutes

Goal: **After studying the chapter**

- **you will be acquainted with the prerequisites for a correct application of control charts;**
- **you will become aware of the necessity to know and verify assumptions about the data by selecting the right control chart;**
- **you will obtain knowledge of some non-traditional methods of the SPC;**
- **you will be able to select an optimal control chart with respect to the given situation.**

Lecture

Conditions for the birth and effective application of conventional Shewhart control charts

Conditions for the birth and effective application of conventional Shewhart control charts can be summarized as follows:

- discrete mass production with a low innovation rate;
- production of relatively simple products;
- low variability of customers´ quality requirements;
- lower requirements as to the production precision;
- manual data processing;
- manual quality inspection.

Changes in economic conditions in the 80´s and 90´s of the 20th century

During the 20th century many economic and technological changes occured; these changes also had an influence on the application of statistical methods and approaches to the quality.

- increasing rate of production automation;
- drift to product customization;
- shortening of innovation cycles and product life cycles;
- increase in production speed;

- higher customer requirements on product quality;
- higher demands on production precision;
- considerable increase in product complexity;
- greater abilities for computer processing of large data sets.

Prerequisites for an effective application of Shewhart control charts for variables

For an effective application of variables SPC, some general, statistical and other prerequisites shall be met.

1. General prerequisite
   - suitable capability of the measurement system (it must be met even when the SPC for attributes has been applied);
2. Basic statistical prerequisites
   - normal distribution of the quality characteristic with a constant mean and dispersion;
   - no data autocorrelation;
3. Other prerequisites
   - sufficient quantity of the data;
   - sensitivity only to larger process changes;
   - monitoring of only one quality characteristic per production unit.

Assumption of the unchangeability of the probability distribution in time

In six figures below, six process models different from the point of view of the probability distribution type and its parameters (the process type A, A1, B, C, C1, D) can be found and every situation is discussed with respect to suitable control charts. The process type A (Fig. 7.1) supposes normal distribution of quality characteristics with a constant mean and constant dispersion. Based on a large study of 825 processes carried out in Germany in 1990s, only 1.8 % of the analysed processes could be considered the process type A. It means that only for such little portion of the real processes the conventional Shewhart control charts have been the best ones.

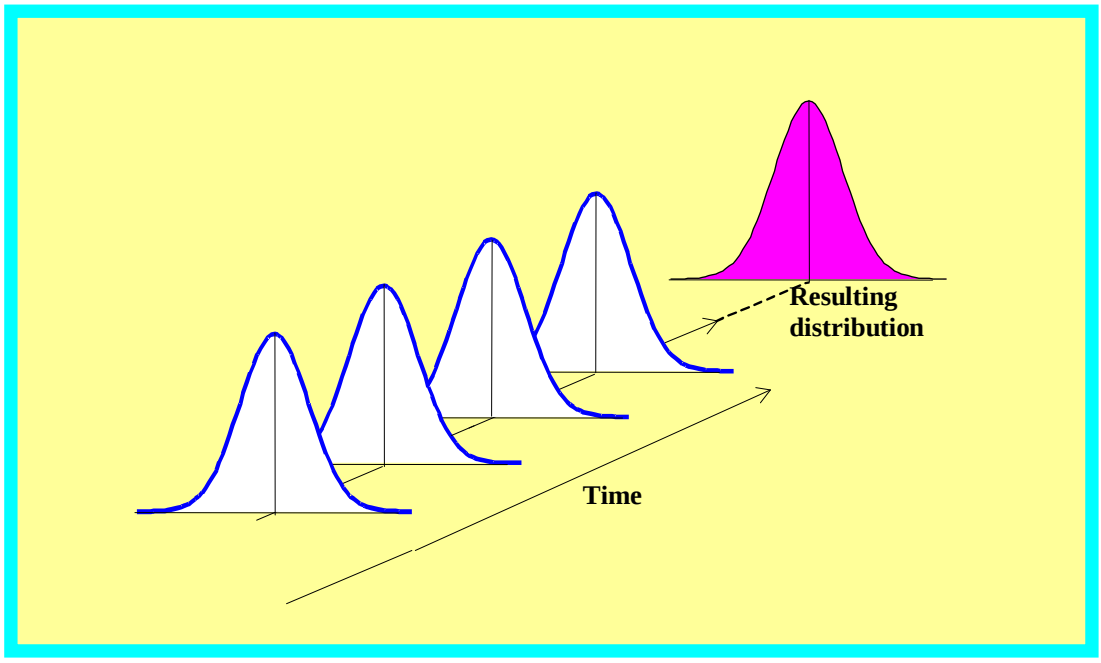


Fig. 7.1 Process type A

Model A1 (Fig. 7.2) differs from the previous one by the probability distribution. It is based on the non-normal distribution with a constant mean and constant dispersion. Based on the results of the study mentioned above, it can be applied to a little portion of real processes (about 2.4 %). Conventional Shewhart control charts are not suitable for this situation. The so called control charts with retransformed control limits are one of the solutions for this situation (for more information see [2], [3]).

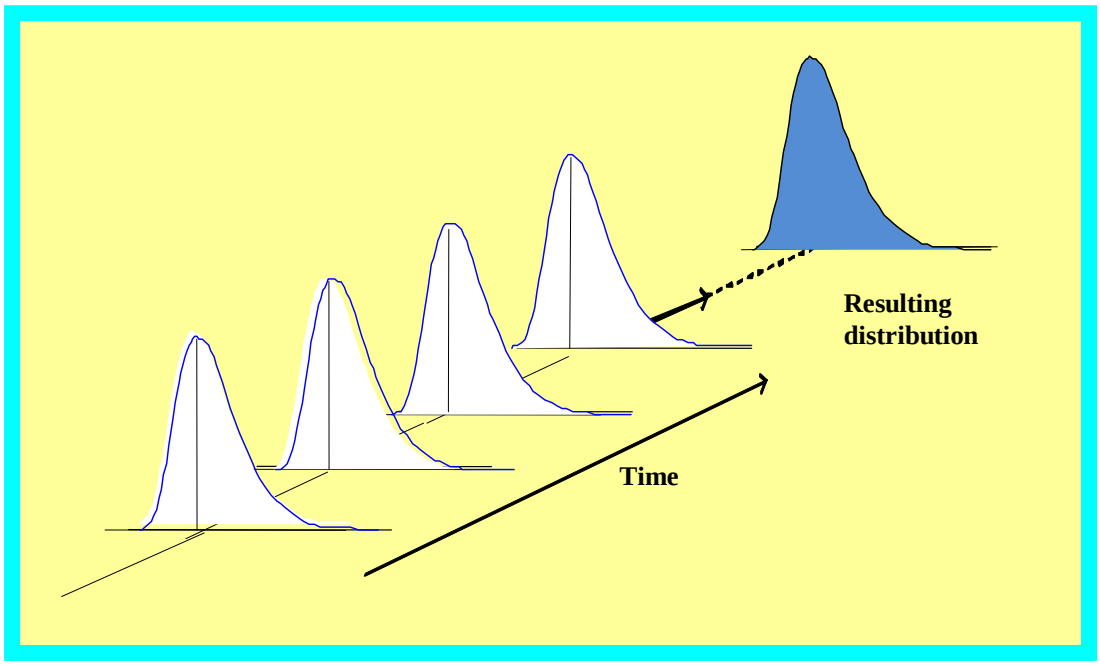


Fig. 7.2 Process type A1

Model B (Fig. 7.3) supposes data coming from normal distribution with a constant mean but non-constant dispersion. Based on the results of the study mentioned above, it can be applied to a very little portion of real processes (about 0.4%). Conventional Shewhart control charts are not suitable for this situation due to the assumption of the normal distribution with a constant mean and dispersion as a suitable model for the data. A standard deviation control chart or a range control chart with relaxed limits are suitable charts for this situation (for more information see [2]).

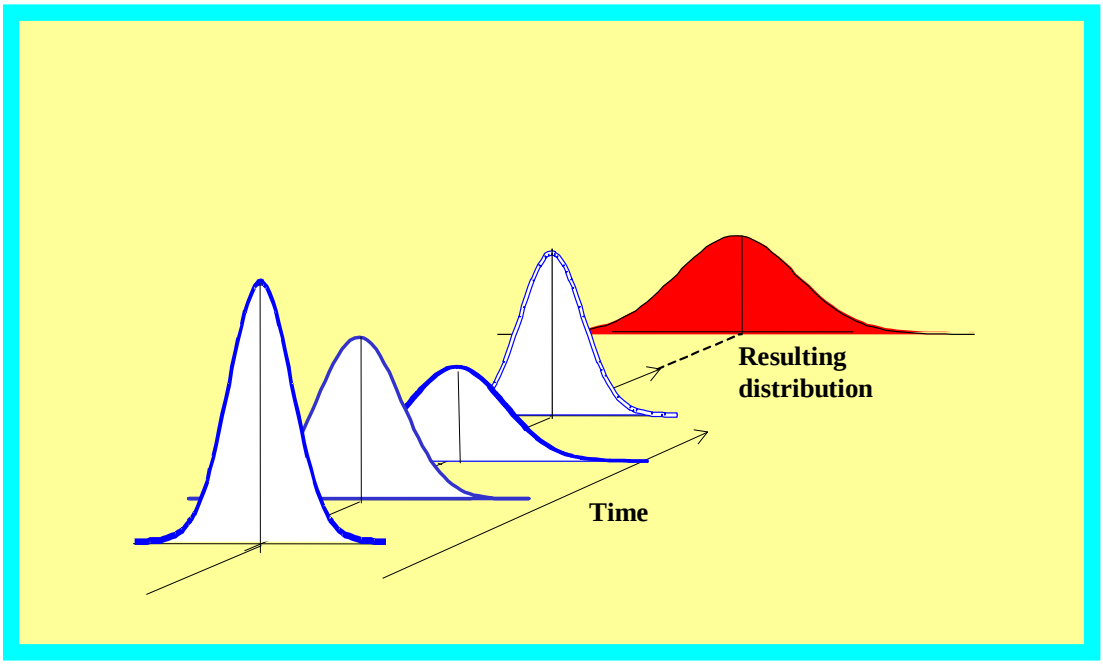


Fig. 7.3 Process type B

Model C (Fig. 7.4) supposes data coming from normal distribution with a constant dispersion but with a randomly changing mean. It is suitable for processes with a random component of the variability between subgroups. Based on the results of the study mentioned above, it can be applied to a quite big portion of real processes (about 35.8 %). But conventional Shewhart control charts are not suitable for this situation due to the assumption of the normal distribution with a constant mean and dispersion as a suitable model for the data. An average control chart with relaxed limits is a suitable chart for this situation (for more information see [2]).

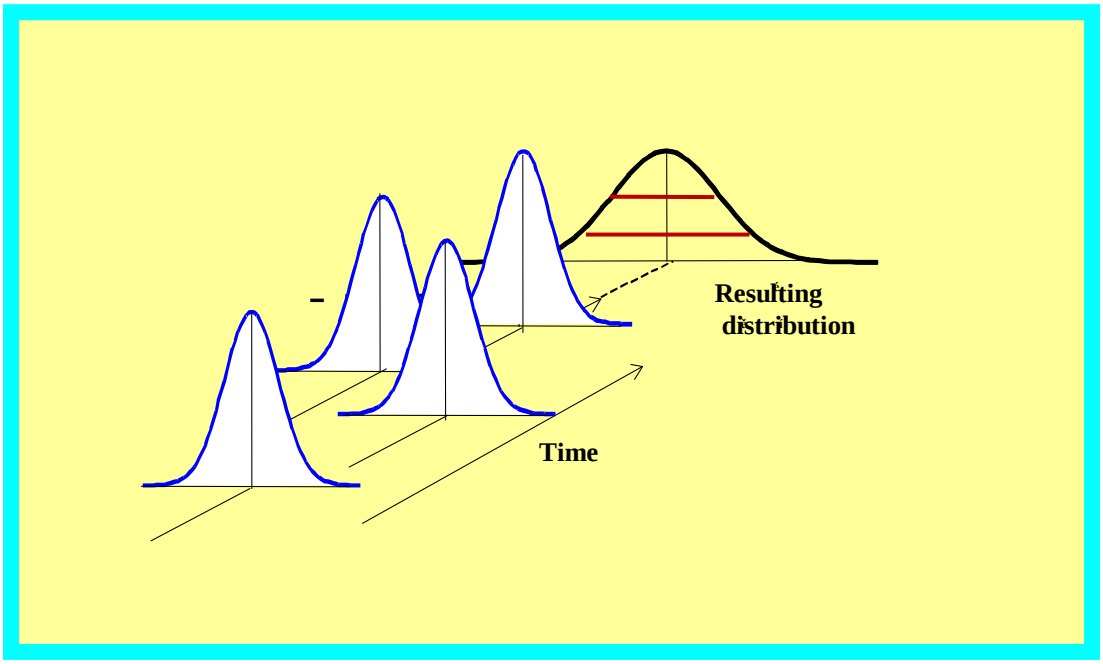


Fig. 7.4 Process type C

Model C1 (Fig. 7.5) also supposes data coming from normal distribution with a constant dispersion but with a systematically changing mean. Based on the results of the study mentioned above, it can be applied to a quite big portion of real processes (about 4.7 %). But conventional Shewhart control charts are not suitable for this situation due to the assumption of the normal distribution with a constant mean and dispersion as suitable model for the data. For the solution of such situation several types of control charts were designed: a modified control chart, acceptance control chart, regression control chart and average control chart with relaxed limits (for more information see [2]).

The last model D (Fig. 7.6) also supposes data coming from normal distribution but with a non-constant dispersion and non-constant mean. Based on the results of the study mentioned above, it can be applied to the largest portion of real processes as compared to all previous models (about 55 %). But again, conventional Shewhart control charts are not suitable for this situation. For the solution of such situation an average control chart and standard or range control charts with relaxed limits are suitable (for more information see [2]).

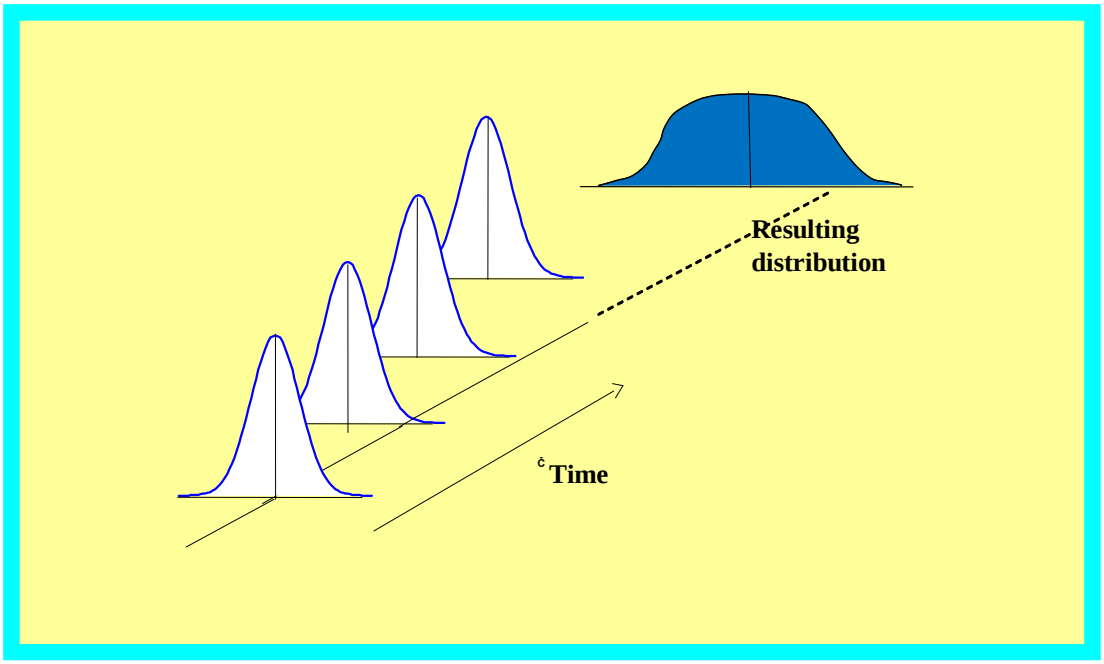


Fig. 7.5 Process type C1

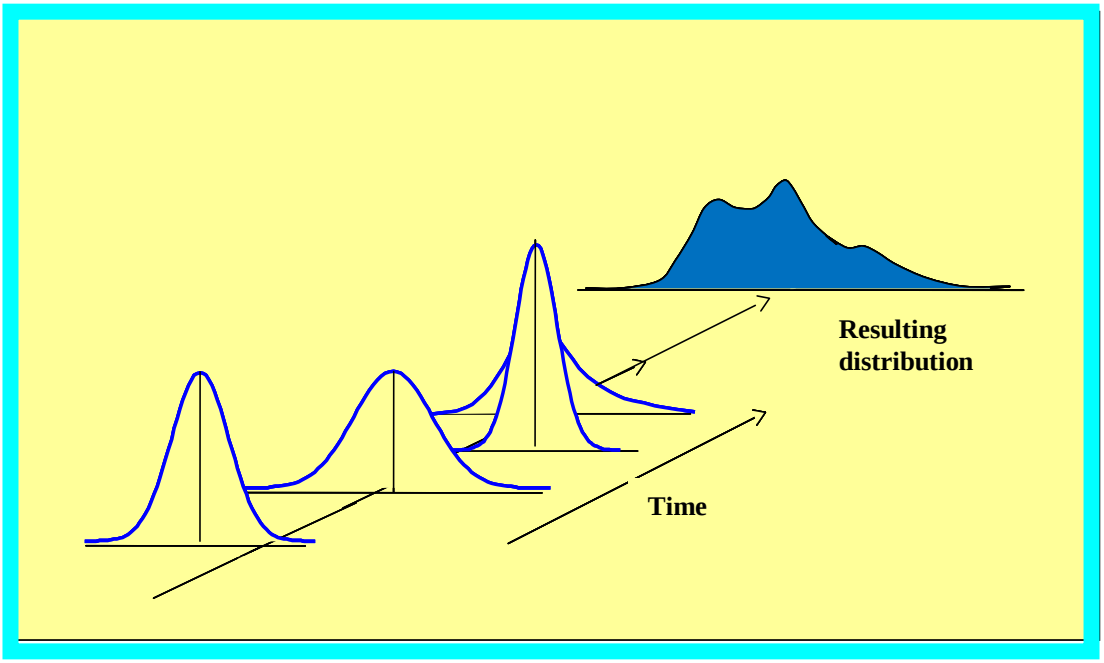


Fig. 7.6 Process type D

Assumption of data non-autocorrelation

This is the most serious prerequisite. When data are auto-correlated, it is not possible to use conventional Shewhart control chart.

For such situation there are four basic ways for how to solve it:

1. Method of the control interval extension;
2. Application of ARIMA models;
3. Approximate method using EWMA statistics;

   Dynamic control chart (see [2], [3]).

   The first three variants have a common base (see Fig. 7.7).

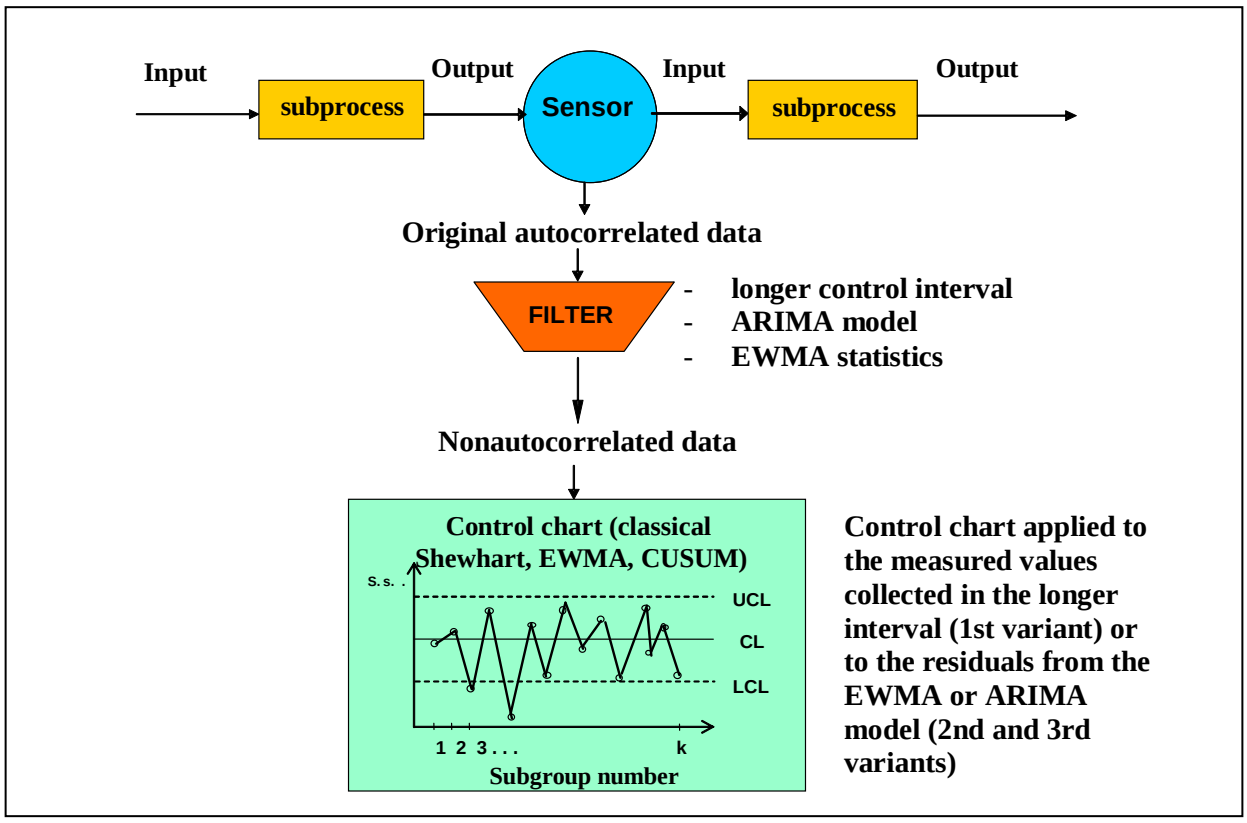


Fig. 7.7 Principle of the solution of the data autocorrelation: variants 1-3

The 4th variant – dynamic EWMA control chart – can be found in Fig. 7.8. As it can be seen, this chart has non-constant both the CL and control limits (for more information see [2]).

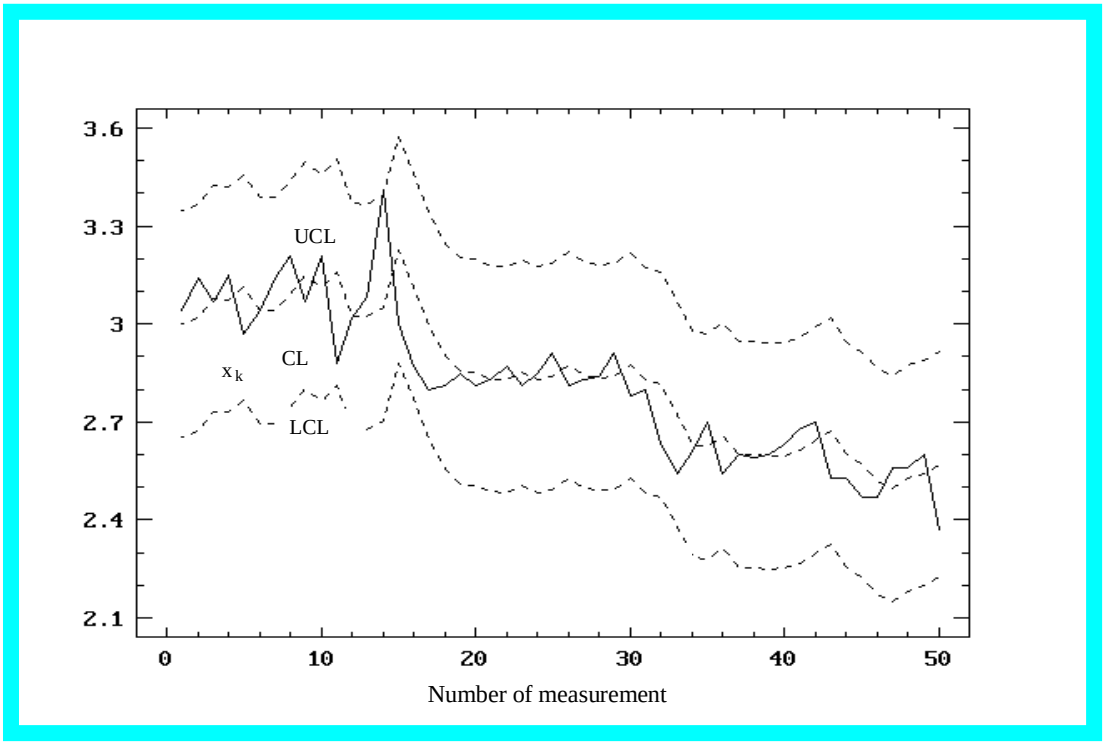


Fig. 7.8 Example of dynamic EWMA chart

Assumption of the high level of the process repeatability

W. Shewhart designed standard control charts for mass production conditions. At present, the production has been more customized so this prerequisite is often not met. In such situation, two types of nonconventional charts – the so called short run charts - can be applied:

1. Target control charts (see Fig. 7.9);
2. Standardized control charts (see Fig. 7.10).

Target control charts can be applied when the process dispersion is consistent for various products. The data transformation used in these charts is done only in the charts controlling the process level (average chart, chart for individuals) [1].

Standardized control charts are suitable when dispersion is quite different for various products (more about these control charts can be found in [1]). The data transformation used in these charts is done both in the charts for controlling stability of the process level and in the charts for controlling stability of the process dispersion (standard deviation chart, variance chart, range chart).

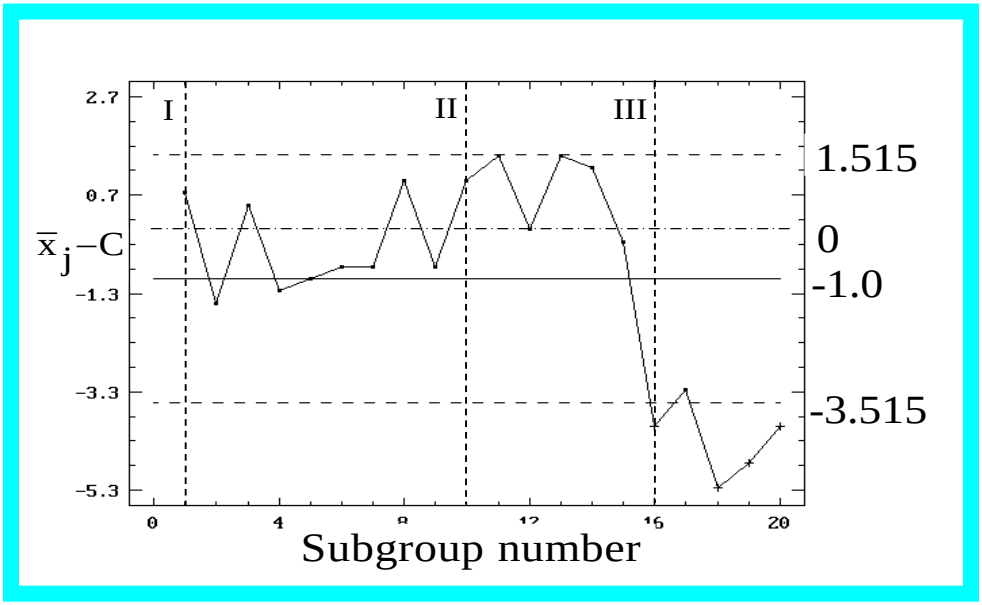


Fig. 7.9 Example of target control chart for averages

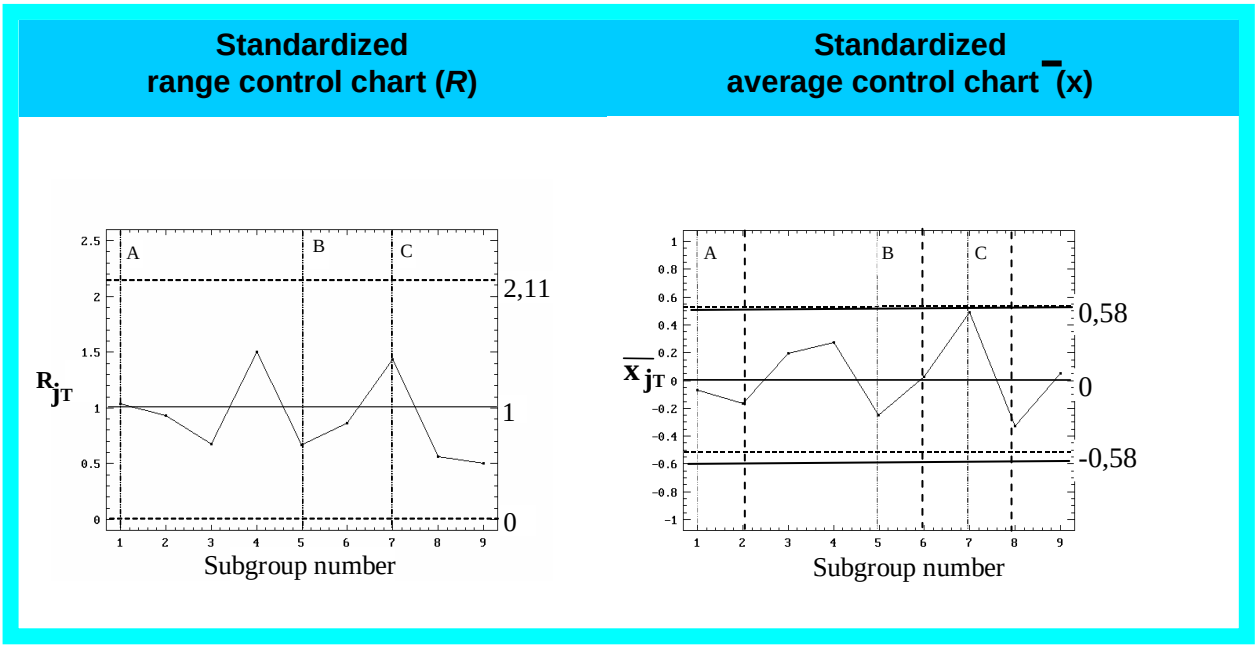


Fig. 7.10 Example of standardized range and average control charts

Assumption of the control chart sensitivity to large process parameters changes

Conventional Shewhart charts were designed for a quick detection of large changes in a process, i.e. changes larger than 2σ. In the processes with a low level of variation (changes lower than 2σ), Shewhart control charts are not effective. The so called control charts with the memory are the solution for such situation:

- MA (moving average), MS (moving standard deviation), MR moving range charts;
- CUSUM (cumulative sums) chart – see Fig. 7.11 (more in [2], [3]);
- standard EWMA (Exponentially weighted moving average) chart – see Fig. 7.12 (standard as opposed to the dynamical – see the chapter "Assumption of the data non-autocorrelation").

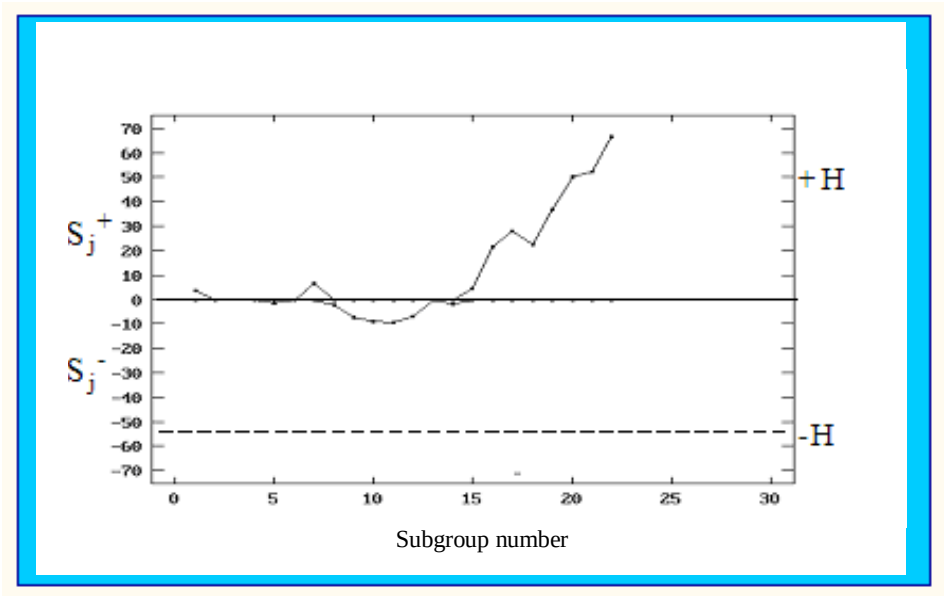


Fig. 7.11 Example of CUSUM chart

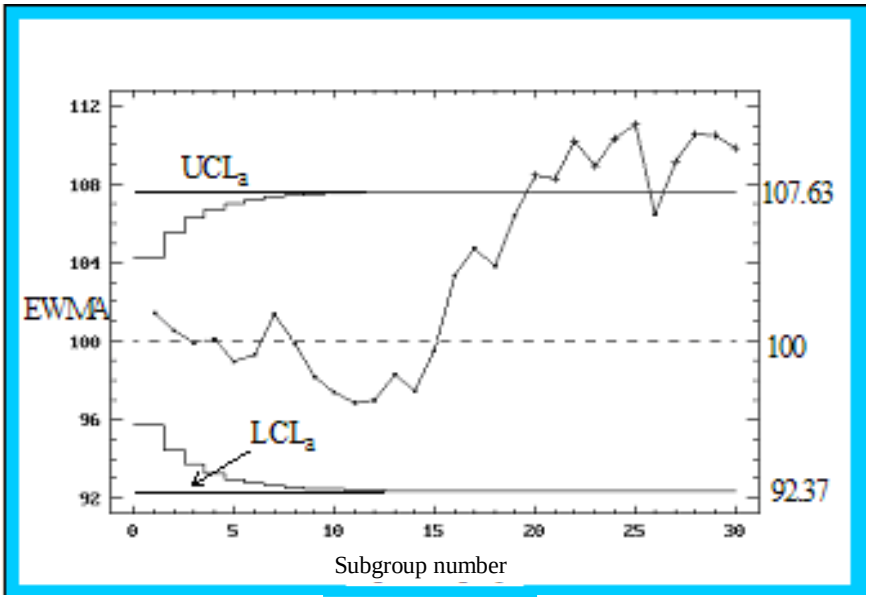


Fig. 7.12 Example of classical EWMA chart

Assumption of monitoring one characteristic per one product

Conventional Shewhart control charts suppose monitoring of only one quality characteristic per one product. In the present world of complex products it is necessary to monitor more than one characteristic. From the point of view of SPC it means that conventional Shewhart control charts shall not be applied successfully. In such situation some multivariate form of the control chart must be used. When the monitored quality characteristics are even correlated, so called Hotelling control chart can be applied – see Fig. 7.13 [2], [3].

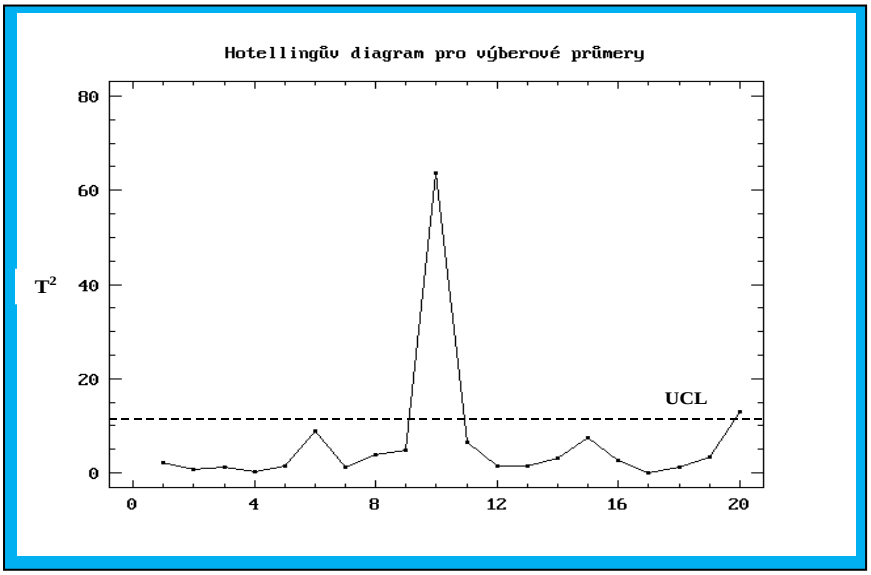


Fig. 7.13 Example of Hotelling chart for averages

Selection of the control chart with respect to assumptions about the data properties – summary

In table 7.1 list of possible and suitable control charts in relation to violated prerequisite can be found.

Tab. 7.1Violated prerequisites and adequate control charts

Violated prerequisite	***Control chart***
No	- Conventional Shewhart control charts
Data normality	- Control chart with retransformed control limits
Invariability of dispersion parameters	- Modified control chart - Acceptance control chart - Regression control chart - Control chart with relaxed limits
Non-autocorrelated data	- ARIMA chart - Chart for residua of the EWMA model - Dynamic EWMA chart
High level of the process repeatibility	- Target chart - Standardized chart
Low sensitivity to process changes	- CUSUM - Standard EWMA chart - MA, MS, MR
Monitoring of only one characteristics	- Hotelling chart

Summary of terms

After studying this chapter you should be familiar with the following terms:

- **process type A**
- **process type A1**
- **process type B**
- **process type C**
- **process type D**
- **dynamic EWMA chart**
- **target control chart**
- **standardized control charts**
- **CUSUM chart**
- **standard EWMA chart**
- **Hotelling chart**

Questions

1. **How did economic and production conditions change at the end of the 20th century and how did it affect SPC methods?**
2. **Which prerequisites shall be met for a correct application of conventional Shewhart control charts for variables?**
3. **Describe particular types of the processes.**
4. **Which SPC methods have been designed for the data which are not normally distributed?**
5. **What to do when data are normally distributed but with a non-constant dispersion or mean?**
6. **What is the solution for the situation when a mean is changing systematically?**
7. **What is the difference between target and standardized control charts?**
8. **What is the difference between standard and dynamic EWMA chart?**
9. **When should the Hotelling chart be applied?**

Additional study resources

[1] GRIFFITH, G. K. *Statistical Process Control Methods For Long and Short Runs*. Milwaukee, Wisconsin: ASQC Quality Press, 1996. 250 p.
[2] MONTGOMERY, D. C.: *Introduction to Statistical Quality Control*. J.Wiley& Sons, New York, 2001. 768 p.
[3] TOŠENOVSKÝ, J. – NOSKIEVIČOVÁ, D.: *Statistickémetody pro zlepšováníjakosti*. Ostrava: Montanex, 2000. 362 p (in Czech).

Topics for the supplementary self-study

- Box-Jenkins methodology for the time series modelling (ARIMA models) [2];
- SPC for nonnormal data [2], [3];
- SPC for detection of small process changes – CUSUM chart [2], [3];
- SPC for detection of small process changes – standard EWMA chart [2], [3];
- SPC for short runs – target control charts [1], [3];
- SPC for short runs – standardized control charts [1], [3];

8 DESIGN OF AN OPTIMAL EWMA CONTROL CHART

Chapter structure

Topics for review

Time for learning

Goal

Lecture

- **Description of EWMA control chart**
- **ARL and influence of parameters K and λ**
- **Design of EWMA control chart**

Summary of terms

Questions

Example

Additional study resources

Solution to example

Topics for review:

- unconventional SPC (see chapter 7 of this textbook)

Time for learning: 300 min

Goal: After studying this chapter

- **you will know when to apply the EWMA control chart;**
- **you will be aware of the differences between control charts without and with memory;**
- **you will know what the parameters of an EWMA control chart are;**
- **you will be able to design optimal parameters for an EWMA control chart;**
- **you will be able to construct an EWMA control chart.**

Lecture

Description of EWMA control chart

The EWMA (exponentially weighted moving average) control chart is a solution for the situation when we are interested in detecting small shifts (about 1.5σ or less) supposing uncorrelated data. For correlated data there exists a special type of the EWMA control chart – the dynamic EWMA control chart. In this chapter we are not going to deal with this special EWMA control chart.

The test criterion in the EWMA chart Y_k is defined as follows:

$$y_k = (1-\lambda)^k . Y_0 + \lambda . \sum_{j=1}^{k} (1-\lambda)^{k-j} . f(x_j) \text{ for j = 1, 2, ..., } k \text{ and for } 0 < \lambda < 1, \tag{8.1}$$

where $f(x_j)$ - is value of a sample measure,

k - is a subgroup order,

Y_0 - is a target level of the parameter of the controlled quantity distribution.

If $Y_0 = \mu_0$ (μ_0 is the process target, desirable value of the process level) and function $f(x_j)$ is a sample mean $\bar{x}_j$, then we have the EWMA control chart for a sample. When the subgroup size n = 1, then we have the EWMA control chart for individuals.

EWMA control charts are charts with memory which is unlimited and unequal. Properties of this memory are determined by the parameter λ ($0 < \lambda \leq 1$).

Central line in the EWMA control chart for the sample means $CL = \mu_0$. Control limits for this control chart are as follows:

$$UCL = CL + K.\sigma_{EWMA} = \mu_0 + K.\sigma_{EWMA} \tag{8.2}$$

$$LCL = CL - K.\sigma_{EWMA} = \mu_0 - K.\sigma_{EWMA} \tag{8.3}$$

Standard deviation σ_{EWMA} is:

$$\sigma_{EWMA} = \frac{\sigma_0}{\sqrt{n}} . \sqrt{\frac{\lambda}{2-\lambda} . \left[1-(1-\lambda)^{2k}\right]}, \tag{8.4}$$

where σ_0 - is a target level of a standard deviation of the controlled quantity,

n - is a subgroup (sample) size,

K - is a constant for setting control limits.

As opposed to the Shewhart control charts, control limits in the EWMA chart depend on the sample time moment k but they approach steady state values quite quickly

$$UCL_a = CL + K.\sigma_a = \mu_0 + K.\sigma_a,$$
$$LCL_a = CL - K.\sigma_a = \mu_0 - K.\sigma_a, \tag{8.5}$$

where we compute σ_a as follows

$$\sigma_a = \frac{\sigma_0}{\sqrt{n}}.\sqrt{\frac{\lambda}{2-\lambda}}. \tag{8.6}$$

To be effective against a small process shift the EWMA control chart must be correctly designed. That means we must design a suitable combination of parameters λ and *K* so that this combination gives a suitable ARL performance for detecting of a small shift of the predetermined size.

Design of EWMA control chart

In this paragraph optimal design of the EWMA chart using the method which is the combination of nomograms (see [2], [3]) and SW Statgraphics is shown.

ARL and influence of parameters *K* and λ

ARL is one of the measurements for describing the performance of a control chart. The ARL (average run length) is the average number of points that must be plotted in the control chart before a point indicates an out-of-control state.

When data are uncorrelated, then the ARL for any Shewhart control chart can be computed as follows:

$$ARL = \frac{1}{p}, \tag{8.7}$$

where *p* is the probability that any point falls outside the control limit in a control chart.

For instance in the control chart for variables where we suppose normal distribution of the controlled quantity and 3σ control limits, the probability that the point will lie between these control limits is 0.9973. Thus the probability that any point will lie outside the limit is 1-0.9973 = 0.0027 (= *p*). From this ARL(0) = 1/0.0027 ≅ 370. It means that even if the process remains in control, an out-of-control signal will be given out to every 370th sample (points in a chart) on average.

For the EWMA chart the ARL can be calculated as follows assuming that *L(u)* is the ARL and the EWMA starts with $EWMA_0 = u$ [3]*:*

$$L(u) = 1 + (1/\lambda)\int_{LCL}^{UCL} L(y) f\{[y-(1-\lambda)u]/\lambda\}dy \tag{8.8}$$

where
- *UCL* and *LCL* are the control limits,
- *f(x)* is the $N(\mu, \sigma^2/n)$ density function,
- μ represents the true process mean,
- σ is the nominal process standard deviation,
- *n* is a sample size.

The ARL corresponding to the risk of a false signal α (i.e. probability that a point in a control chart falls outside the limits even if the process is in control) is called the *ARL(0).* It is the average number of points before an out-of-control signal is given when the process is actually in control. We need to have the risk α as low as possible in order to have the *ARL(0*) as high as possible. In practice decision about the *ARL(0)* depends on economic factors such as costs for the process interruption and looking for the out-of-control state causes costs. When we want to have the EWMA control chart corresponding to 3σ limit Shewhart control chart with $\alpha = 0.0027$, then we use ARL(0) = 370.

The ARL corresponding to the risk of a missing signal β (i.e. probability of not detecting the shift in the process mean of the given size δ on the first subsequent sample) is called the *ARL(*δ*)*. It is the average number of samples taken before the shift is detected. For instance if $\mathrm{ARL}(\delta) = 8$, it means that the average number of samples, taken to detect the the shift δσ, is 8. We want the *ARL(*δ*)* to be minimal.

When we compute the *ARL(0)* in the EWMA Chart Options dialog box in Statgraphics Plus v. 5.0 [5] (see Fig. 8.1), we must have number 0 in the field next to the word ARL.

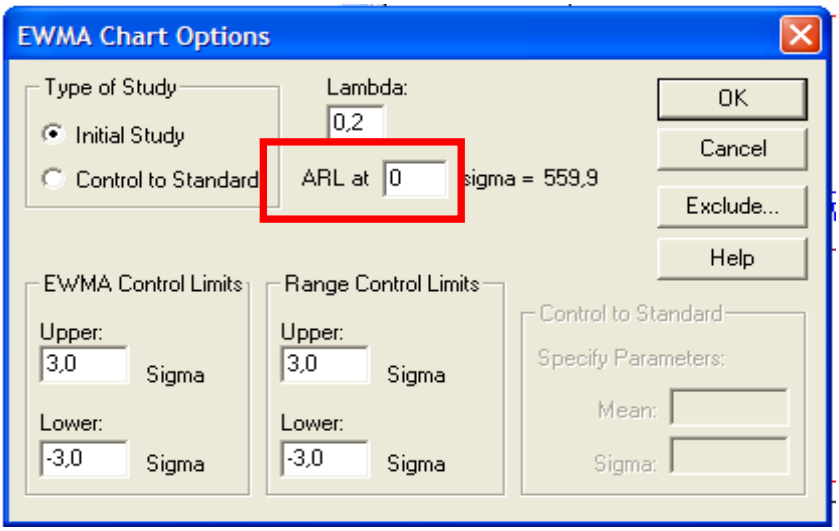


Fig. 8.1 Chart Options dialog box [5]

When we want to compute the *ARL(δ)*, we must enter the value of δ into this field. δ is the shift size expressed in the process standard deviation σ which we want to detect as soon as possible (critical size of the shift). Its magnitude will likely depend on factors such as process capability relative to specifications and costs of adjusting the process [1]. As we can see in Fig. 8.2, values of the ARL change with the changes of parameter λ and upper and lower sigma constant *K* for setting the EWMA control limits.

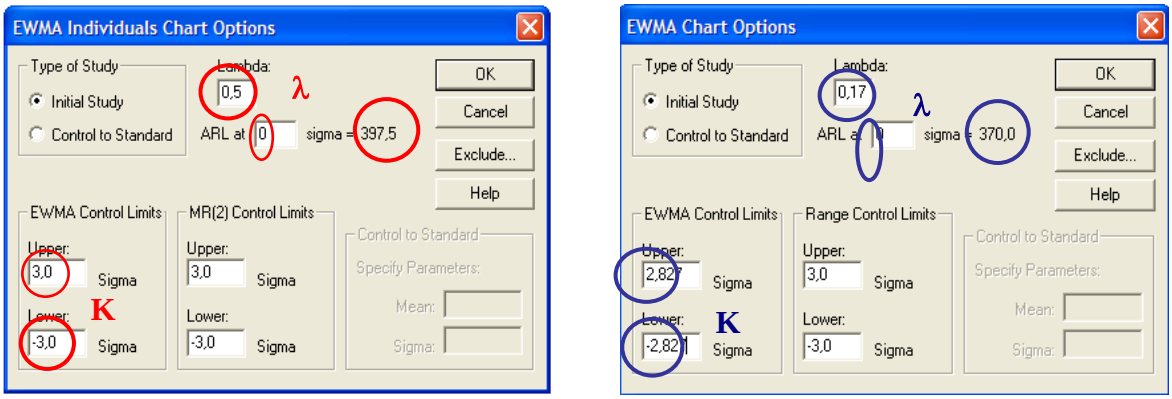


Fig. 8.2 Influence of changes of λ and upper and lower sigma for setting EWMA control limits K on the ARL value

Combination of $\lambda = 0.5$ and K = 3 gives ARL(0) nearly 398 (see the dialog box on the left in Fig. 8.2), combination of $\lambda = 0.17$ and K = 2.827 gives ARL(0) = 370 (see the dialog box on the right in Fig. 8.2). How to set parameters λ and *K* to obtain desirable value of the ARL will be explained in the following paragraph.

Setting optimal combination of parameters λ and K

Parameter λ $(0 < \lambda \leq 1)$ is a smoothing parameter which determines properties of the EWMA control chart memory. Parameter K is the constant for setting control limits.

Combination of parameters λ and K is considered to be optimal in the sense that for a fixed false alarm α this combination produces the smallest possible risk β for a specified shift in the process mean. For setting the optimal values of λ and K we can use nomograms [1].

We recommend the following steps for setting the optimal values:

1. Select the smallest acceptable *ARL(0)*.
2. Decide what magnitude of the shift in the process must be detected as quickly as possible (critical shift δ), i.e. small ARL ($ARL(\delta)$).
3. Select the parameter λ which produces a minimum ARL for the critical shift using Crowder's nomogram [3] (Fig. 8.3 for ARL(0) = 370 or 250 or 100 or 50 and for shift from 0.25 to 4 where the shift is expressed as a multiple of the standard deviation of the sample average Δ ($\Delta = \delta.\sqrt{n}$, where n is the sample size).

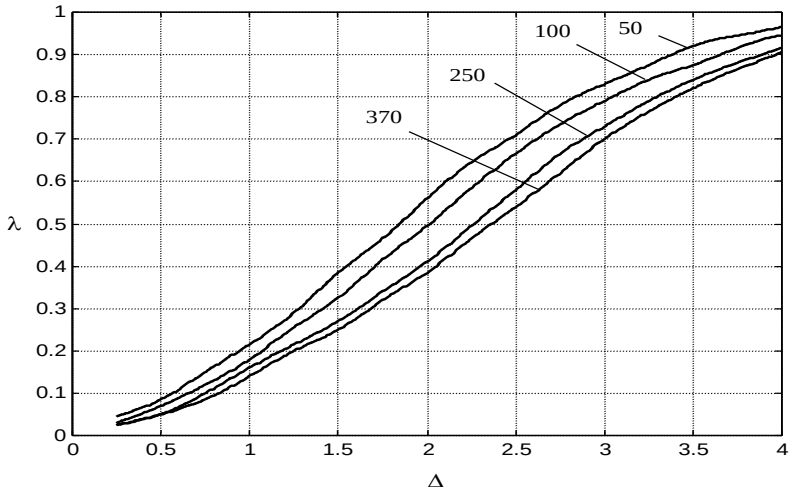


Fig. 8.3 Nomogram for optimal λ vs. shift Δ

Optimal λ is read from the nomogram so that we find the critical shift δ expressed as Δ on the x-axis and we read the optimal value of λ through the curve for ARL(0), set in the step 1, on the y-axis.

4. Find optimal parameter K for setting the control limits in the EWMA chart which satisfies the in-control *ARL(0)* set in the step 1. Use the Crowder's nomogram [3] below (Fig. 8.4).

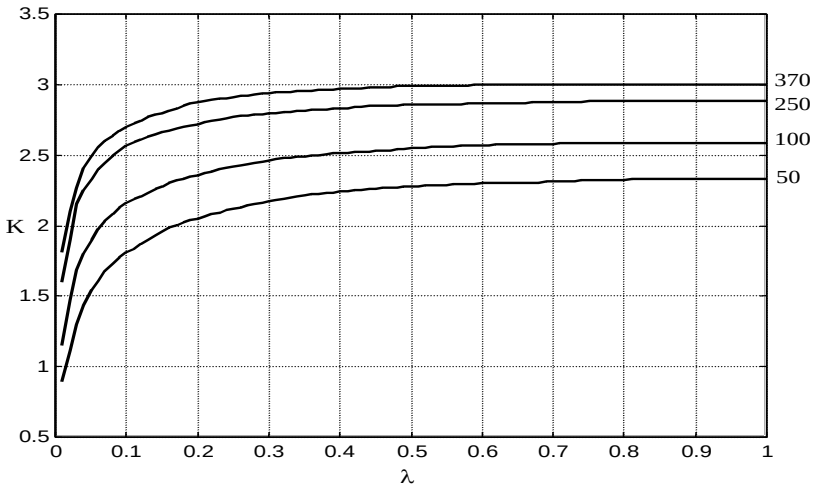


Fig. 8.4 Nomogram of *K* vs. λ

Each curve in this nomogram represents all possible combinations of λ and *K* ($\lambda \geq 0.01$) producing the in-control *ARL(0)* associated with that curve. To illustrate it there is the table 8.1 in which you can see combinations of λ and *K* (with the most preferred values of λ) that produce approximately ARL(0) = 370.

Tab. 8.1 Combinations of λ and K for the same ARL(0) [1]

λ	0.05	0.10	0.15	0.20	0.25	0.4
K	2.490	2.701	2.8005	2.859	2.898	2.959
ARL(0)	370.3	370.0	370.3	370.0	370.4	370.5

To find the optimal *K*, first we find the value of λ on the x-axis as set in the previous step and through the curve for *ARL(0)*, according to the step 1, we can read the optimal value of *K* on the y-axis.

5. Enter this optimal λ and *K* to the EWMA Chart Options dialog box and read if this combination gives the *ARL(0*) as set in the step 1.
6. As the reading from nomograms need not to be precise enough, it is possible that there could be a difference between the target *ARL(0)* (step 1) and the *ARL(0)* in Statgraphics. In that case we must try to change precision of the parameters λ and *K* to obtain the target value of the *ARL(0).*

7. Compute the $ARL(\delta)$ for the critical shift so that you change number 0 in the field next to the ARL (in the Statgraphics Chart Options dialog box) to δ - critical shift expressed as a multiple of the process standard deviation σ.

After these steps computations and construction of the EWMA control chart with determined parameters are implemented. These steps are shown in the example in the following subchapter.

Summary of terms

After studying this chapter you should understand these terms:

- **EWMA chart**
- **control chart memory**
- **optimal design of EWMA**
- **parameter *K***
- **parameter λ**

Questions

1. **Which situations is the EWMA control chart suitable for?**
2. **What does it mean that the EWMA control chart is a control chart with memory?**
3. **What are the characteristics of the EWMA chart memory?**
4. **What is the purpose of the control chart memory?**
5. **What is the role of parameter K in the EWMA chart?**
6. **What is the role of parameter λ?**
7. **How can the optimal EWMA chart be designed using suitable SW?**

Example

We have 80 measurements of quality characteristics (in mm). The sample interval is 10 minutes. We want to detect the shift from $\mu_0 = 950\text{mm}$ (target value of the process mean) to $\mu_1 = 962\text{mm}$ (critical value of the process mean with which the process produces unacceptable portion of nonconforming units) as soon as possible. The target process standard deviation σ_0 = 12 mm. We want to apply the

two-sided EWMA control chart for averages corresponding to the ARL(0) performance of a standard two-sided Shewhart chart for averages based on the target values of μ_0 and σ_0.

Additional study resources

[1] MONTGOMERY, D. C.: Introduction to Statistical Quality Control. J. Wiley & Sons, New York, 2001.

[2] TOŠENOVSKÝ, J. – NOSKIEVIČOVÁ, D.: *Statistické metody pro zlepšování jakosti*. Ostrava: Montanex, 2000.

[3] CROWDER, S. V. Design of Exponentially Weighted Moving Average Schemes. *Journal of Quality Technology*, 1989. Vol. 21, No. 3, pp. 155-162.

[4] Statgraphics Centurion XV. *On-line User Manual*. Available from www.statgraphics.com/documents.htm.

[5] Statgraphics Plus Version 5.0.

Solution to example

I. First we must select the EWMA Individuals Chart Options, then the dialog box the Control to Standard within the Type of Study in Statgraphics, and then – within the Specify Parameters – select the Mean ($\mu_0 = 950$mm) and Sigma (σ_0 = 12mm) – see Fig. 8.5, numbers I.

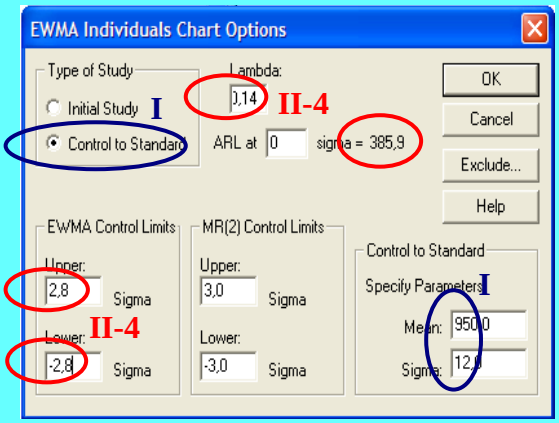


Fig.8.5 Steps I and II-4

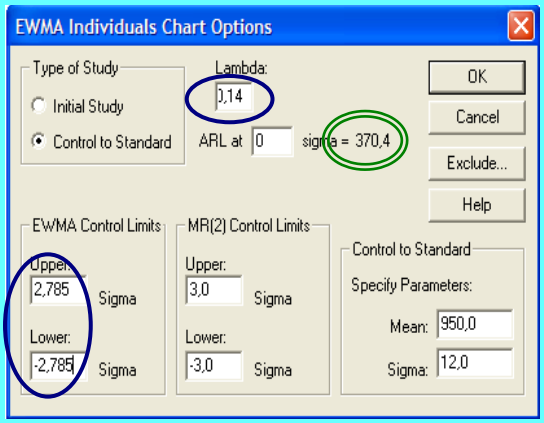


Fig. 8.6 Steps II-5

Then we find an optimal λ and K applying the steps 1- 6 described in the previous chapter.

1. We have decided to select ARL(0) = 370. It corresponds to the *ARL(0)* of the 3σ limits Shewhart control chart.

2. The critical shift which we want to detect in the control chart as soon as possible is 12 mm ($\mu_1 - \mu_0$), i.e. $\delta = (\mu_1 - \mu_0)/\sigma_0 = 12/12 = 1;\ \ \Delta = \delta.\sqrt{n} = 1.\sqrt{1} = 1.$

   For $\Delta = 1$ through the curve for ARL(0) = 370 in the nomogram in Fig. 8.3 we have found that the optimal parameter λ is approximately 0.14.

3. For optimal $\lambda = 0.14$ through the curve for ARL(0) = 370 in the nomogram in Fig. 8.4 we have read that the optimal parameter K is approximately 2.8.

4. We entered the read values of λ and K into the EWMA Chart Options dialog box and the program computed ARL(0) = 385.9 (see Fig. 8.5, character II-4).

5. As the obtained *ARL(0)* is different from the value 370, we try to change precision of the parameter K to produce the *ARL(0)* closer to 370. We leave $\lambda = 0.14$ and we change K from 2.8 to 2.785 and the *ARL(0)* is nearly 370 (see Fig. 8.6). We consider $\lambda = 0.14$ and K = 2.785 to be the optimal values that give the minimal $ARL(\delta)$, i.e. the minimal ARL(1).

6. We change the value next to the acronym ARL from 0 to 1 (see δ in step 2) and the program immediately computes $ARL(\delta) = 9.6$ (see Fig. 8.7). It means that the EWMA chart for the individuals chart with $\lambda = 0.14$ and K = 2.785 will give a signal about the shift $1\sigma_0$ - 10 samples on average - after the shift occurrs. Considering the sample interval of 10 minutes the chart will give the signal about this shift via a point outside the control limit 100 minutes on average after the critical shift occurrs.

   For now it is possible to run computation of the control limits and construction of the EWMA control chart using optimal parameters $\lambda = 0.14$ and K = 2.785 (see Fig. 8.8).

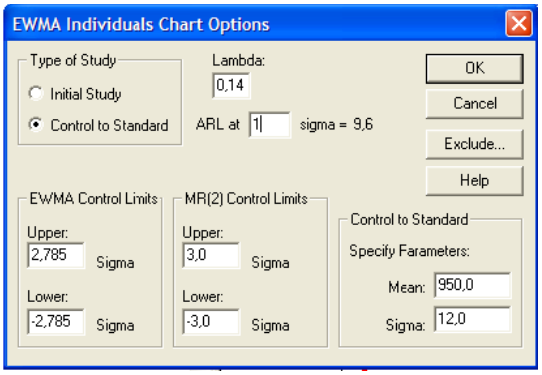


Fig. 8.7 Step II-6

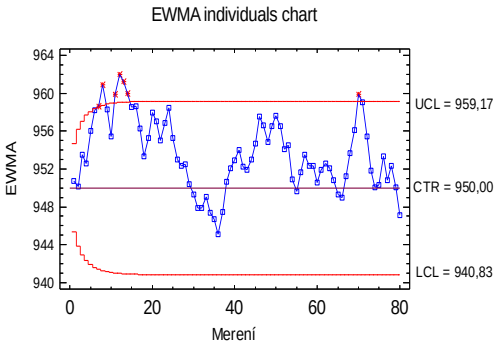


Fig. 8.8 EWMA control chart

Based on Fig. 8.8 we can conclude that the analyzed process is not statistically stable (it is out of control).

9 SOME PRACTICAL ASPECTS OF SPC

Chapter structure

Topics for review

Time for learning

Goal

Lecture

- **Complex and effective application of SPC**
- **SPC and problem-solving**

Summary of terms

Questions

Additional study resources

R

Topics for review:

- phases of SPC implementation

- goals of SPC

- general structure of the problem-solving process

- dispatch process sheet

- Pareto analysis

- Ishikawa diagram

Time for learning: 240 min

Goal: After studying this chapter

- **you will remind the phases of SPC;**
- **you will understand what the effective application of SPC is;**
- **you will know what the main aspects of the complex application of SPC are;**
- **you will understand which various factors shall be taken into**

consideration when applying the SPC;

- **you will know what the SPC cycle is;**
- **you will understand SPC as a problem-solving process;**
- **you will know what the content of the separate sub-processes of SPC as a problem-solving is.**
- **you will know how to implement SPC as a problem-solving for various types of processes differing in number of the effecting factors and dynamics.**
- **you will know which methods and tools to use for different variants of the methodology for implementing the SPC as a problem-solving process.**

Lecture

Complex and effective application of SPC

Complex application of SPC is an application that is realized in the frame of the following four phases:

I. Preparatory phase;
II. Phase of verification and ensuring the process statistical stability;
III. Phase of verification and ensuring the process capability;
IV. Phase of ongoing statistical process control.

In Tab. 9.1 there is a broad list of possible factors mentioned above that are identified in the frame of particular steps of the SPC cycle (see Fig. 9.1).

However, in practice there are many of the factors mentioned above that are not considered when implementing the SPC. It results from misunderstanding of the main goal of the SPC.

Process monitoring as such is not enough for meeting this goal, and the SPC must be implemented in the form to be able to offer correct and as quick as possible identification of assignable causes and implementation of adequate actions for improvement.

Tab. 9.1 Factors of the effective SPC implementation

Step	Factor
Preparation of SPC implementation	• Understanding the principles and goals of the SPC at all managerial levels • Overcoming the old thinking according to the spirit of technical variability concept • Leadership encouragement • Motivation of involved workers • Delegation of roles, competences, responsibilities • Preparation and realization of training • Quality and specialization of training courses • Understanding statistical principles • Ability to interpret correctly the control charts • Comprehension of economic consequences of correct procedures keeping • Verification and ensuring measurement system capability • Selection of a measurement method • Defining the way of collecting and recording data • Building the defence mechanism against non-keeping of the defined procedures • Organization of the problem-solving process • Building the information system aiding the problem-solving step • Selection of the process, process parameters and quality characteristics • Analysis of the process and its variability • Defining possible assignable causes and adequate actions • Creation of rules for interpreting the control charts • Ability to assign a correct cause to signal in the control chart and to choose correct action • Simplicity and comprehension of the SPC system • Time intensity of the designed SPC system • Complexity of application • Selection of SW • Selection of the control chart • Design of the control chart • Ability to access new knowledge of industrial statistics
Control chart construction	• Construction method • Time intensity • Mistake proofing
Data collection	• Observance of the defined procedure for data collection (control interval, rational subgroups forming) • Observance of the defined procedure for the data finding and recording • Mistake proofing

Tab. 9.1 Factors of the effective SPC implementation – continuation

Step	Factor
Computation and recording of statistics into control chart	• Way of computation and statistics recording • Complexity of statistics computation and recording • Keeping the procedure for computations and recording • Mistake proofing
Control chart interpretation	• Rules existence • Clear determination of competencies and responsibilities • Ability to „read“ in the control chart • Skills of responsible workers
Assignable causes identification	• Clear determination of competencies and responsibilities • Existence of the system of recording and processing historical data concerned in the process variability causes • Deep knowledge of the process variability and its causes • Identification speed • Existence of the system for decision making and problem-solving support • Motivation for making identification • Costs of identification • Ability to assign cause to signal non-stability in the control chart • Leadership encouragement • Time sources • Communication abilities • Experience
Selection of action for improvement	• Clear determination of competencies and responsibilities • Knowledge of the process • Database of causes and possible or accepted actions • Costs of implementing particular actions • Existence of information about efficiency of accepted actions • Managerial encouragement • Speed of the implementation of selected actions • Realization of balancing costs and benefits of possible actions • Experience
Implementation of action for improvement	• Clear determination of competencies and responsibilities • System for evaluation of the accepted action impact and for recording the results • Resources for implementation • Experience

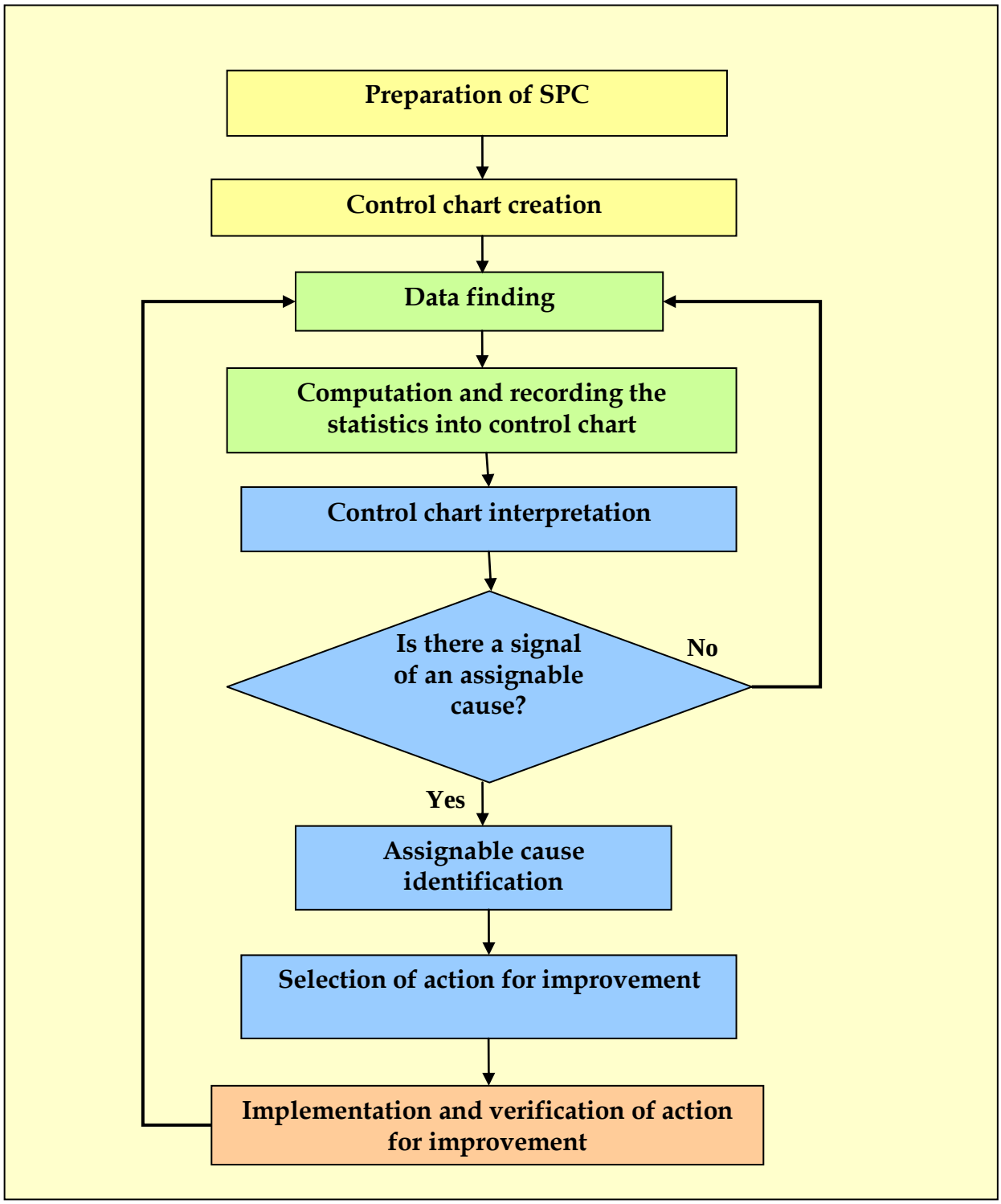


Fig. 9.1 SPC cycle

SPC and a problem-solving

SPC must be built as a problem-solving instrument. In the phase of the SPC design the general structure of the problem-solving process must be respected and the sequence of sub-

processes "Out-of-control signal revelation – Root cause identification – Corrective action acceptance – Verification of corrective action" must be the axis of the SPC application. This axis must be assured with many other actions to really operate.

General structure of the problem-solving process

In this chapter basic sub-processes of a properly operating problem-solving process are defined. Structure of this process can be seen in Fig. 9.2.

Decision-making is a part of a wider problem-solving process and it covers activities such as identification, defining and diagnosis of a problem, identification of the problem causes, solving alternatives generation, then evaluation and choice of the alternative which meets preset criteria best. The decision-making sub-process is followed by the implementation of the chosen alternative. Monitoring and evaluation of the chosen alternative effectiveness together with a relevant revision are activities forming the last sub-process.

The whole process is cyclic; the last step goes back to the first one.

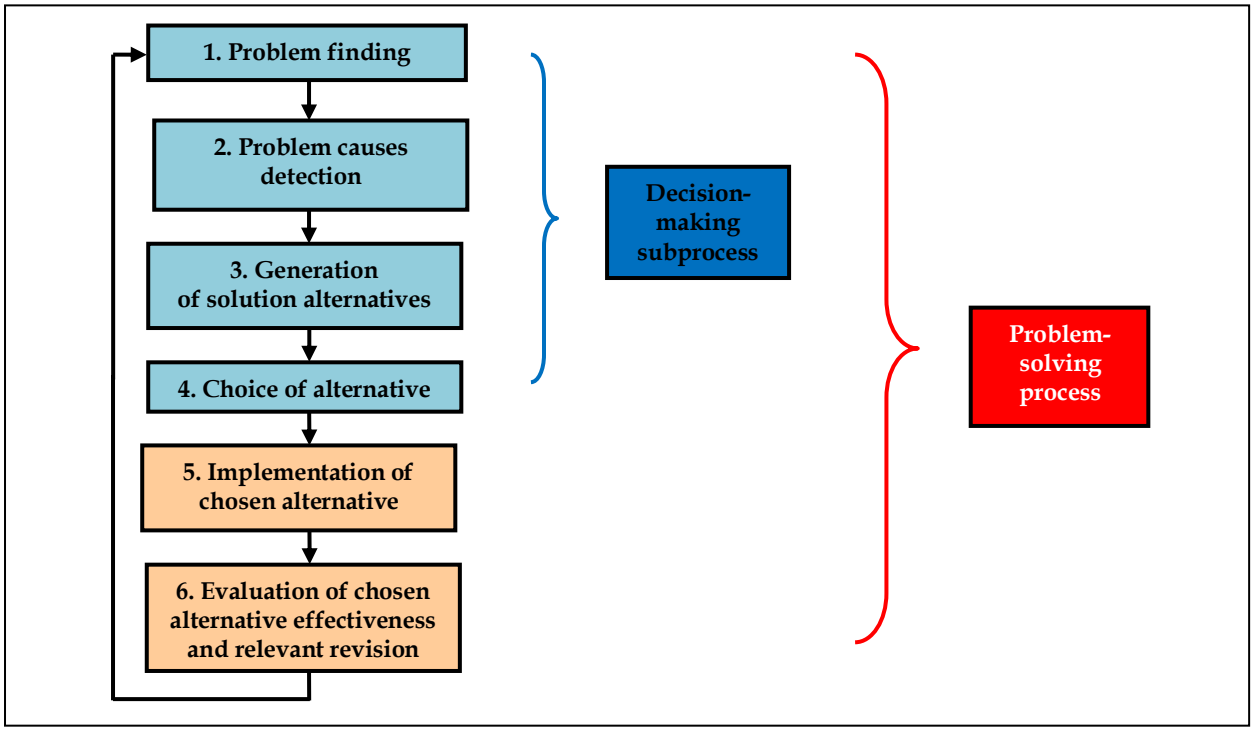


Fig. 9.2 General structure of the problem-solving process

SPC as a problem-solving process

Respective problem-solving process in the frame of the SPC follows the preparation of the SPC implementation and data collection and processing (see the relaxed SPC cycle in Fig. 9.3).

The following paragraphs define the content of separate sub-processes of the SPC as a problem-solving process (numbering of sub-processes is the same as it was used in the previous chapter).

1. Problem finding sub-process

In the frame of SPC this sub-process covers activities connected with the control chart analysis on the basis of diagnostic rules defined during the preparatory phase (points out of control limits, non-random patterns). One or more points outside the control limits or presence of some non-random pattern inside limits constitute the problem as such.

2. Sub-process of the problem cause identification

Identification of the root cause which has led to the signal of non-stability in the control chart is the next sub-process of the problem-solving process in the frame of SPC. This sub-process has a fundamental influence on the effectiveness of the SPC and practical experience shows that its undervaluation can result in failing of the whole SPC implementation. Without effective realization of this sub-process the main goal of SPC (reduction of variability and improvement of the process) cannot be met. It needs many supportive activities and creation of a quality information system using both simple methods and instruments (Ishikawa diagram, Pareto diagram, check lists, data stratification in control charts) and more complex methods in case of a complex process (regression analysis, DOE, discriminant analysis, fuzzy logic**,** expert systems).

3. Sub-process of the solution alternatives generation

In the frame of this sub-process it is necessary to define all possible activities that can result in the elimination of the identified assignable cause or at least to its restriction. This step also needs a quality database containing the list of all possible assignable causes with assigned possible corrective actions. In literature, recommendation to create the so called OCAP, or a properly structured expert system, can be found.

4. Sub-process of the solution choice

In SPC it covers activities such as choice and design of the corrective action or the action for improvement. Realization of this sub-process can also be more effective via creating and using a suitable database. Its basis could be formed by the list of already accepted corrective or improvement actions with relevant identification information. This information can be treated using the Pareto analysis for a particular assignable cause. A staff member responsible for

choosing the most suitable corrective or improvement action could read from this analysis which actions were accepted most frequently. This information could speed up one's decision. Speed of this process is another basic criterion that determines the effectiveness of the SPC implementation. In more complex cases this database could be a part of the mentioned expert system.

5. Implementation of the chosen solution

This subsystem represents true intervention into the process that could result in a real reduction of the process variability and its improvement. It can represent organizational, technological, technical and other solutions.

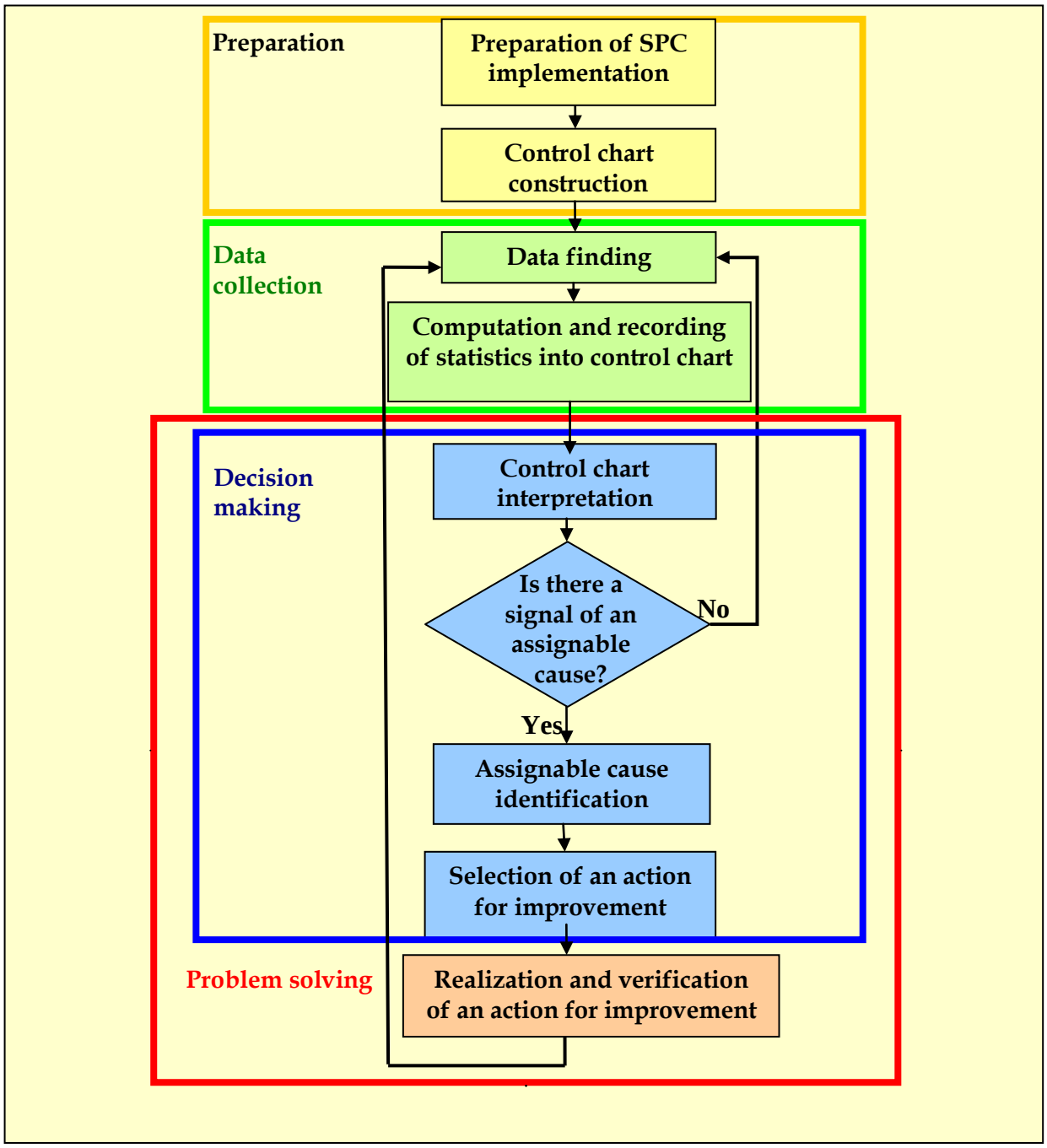


Fig. 9.3 Relaxed SPC cycle

6. Evaluation of corrective or improvement action and eventual revision

Monitoring and evaluation of the effectiveness of accepted corrective or improvement actions must form the integral part of the problem-solving process. The obtained information can help to speed up the future decision-making in the frame of the 4th sub-process. The effectiveness can be evaluated using the indicator of the corrective or improvement action effectiveness which expresses how many % of such realized actions resulted in the process variability reduction.

When $E(RA)_{ij}$ is the i^{th} kind of action accepted for the solving of the j^{th} cause, then its efficiency can be determined using the following formula:

$$E(RA)_{ij} = RA_{ij}(e)/RA_{ij}(c) \; . \; 100, \tag{9.1}$$

where

$RA_{ij}(c)$ … is the whole number of the i^{th} kind of corrective action realized to solve j^{th} cause;

$RA_{ij}(e)$ … is a number of realized actions of i^{th} kind resulting in the process variability reduction; $i = 1, 2, \ldots, m$;

m …… is a number of various kinds of actions accepted for the solution of j^{th} cause, $j = 1, 2, \ldots, n$;

n …… is a number of various kinds of causes so far identified in connection to a specific symptom of nonstability.

This information is also important from the point of view of selecting a correct action because so far most often realized action does not need to be the most effective one. For this reason when applying Pareto diagram for the analysis of the most often realized actions, it should also contain an indicator of the efficiency of every kind of the so far realized actions.

Specificity of the problem-solving process in the frame of SPC

The problem-solving process in the frame of SPC is characterized by some specificity that also requires specific access to its realization.

- Corrective or improvement actions must be realized as soon as possible after a signal of influence of an assignable cause, given by the control chart, occurs.
- Many actions are carried out by operators within limited time.
- The influence of human factor cannot be excepted in full.

These specificities must be considered during the SPC implementation.

Methodology of realization of SPC as the problem-solving process

Methodology is structured according to the number of factors (inputs) that have an influence on the analyzed quality parameter of the product (output) and according to the rate of the process dynamics. Based on the combination of these two criteria there have been defined 4 variants of the methodology (see Tab. 9.2).

Because of the necessity to react to the signal of working of assignable cause, given by the control chart, as soon as possible a lot of information for decision-making must already be taken

in the preparatory phase. The designed methodology contains only activities needed for the realization of the problem-solving process.

Tab. 9.2 Basic variants of the methodology

		Process dynamics	
		Low	**High**
Number	**Small**	Variant I	Variant IV
	Large	Variant II	Variant III

Variant I

Realization of the variant I is schematically visualized in Fig. 9.4. During the preparatory activities it is necessary to create the check sheet for recording the values of the analysed quality characteristic and the document called Dispatch process sheet. It obviously consists of one part for specifications of the process and product, specification of the control chart (type of control chart, sample size, control interval) and one part for records of values of a quality characteristic including specification of time of the data collection and signature of the person responsible for the data recording. In case of hand-made control charts this check sheet is represented by the so called Control card containing even one part with control charts.

The Dispatch process sheet is a document where it is necessary to record every intervention into the process (maintenance activities, machine setting), its changes (change of operator, material batch change) or identified assignable causes including the date and signature of the responsible employee. This document could be carried out as a separate one but it could be a part of the control chart mentioned above.

During preparation of the SPC implementation it is necessary to decide rules for assessing non-stabilities of the process. It is essential to clearly declare and document what is considered to be a signal of the process non-stability (whether only the points outside the control limits or also some non-random patterns). In the second case it is necessary to define which non-random patterns will be applied.

Another task which must be realized before the SPC implementation and which is tightly connected with SPC as a problem-solving instrument is an assignment of potential non-stability

causes that have an influence on the variability of the analysed quality characteristic of a product. A very simple method could be applied to such a task – the Ishikawa diagram.

As in variant I a small number of factors influencing the watched quality characteristic is considered, all these defined causes of variability are considered during the next analysis. These causes are assigned to defined signals of non-stability and corrective or improvement actions are assigned to particular causes. This list is called the OCAP (Out-of Control Action Plan. From the point of view of the main goal of SPC it represents one of the most important information sources.

After recording values to the control chart and records to APS (eventually after translating data into SW) it is necessary to make an interpretation of control charts. That means to find signals of non-stability using predefined rules. When a signal is identified, it is necessary to determine the factual assignable cause using information from the Ishikawa diagram or OCAP.

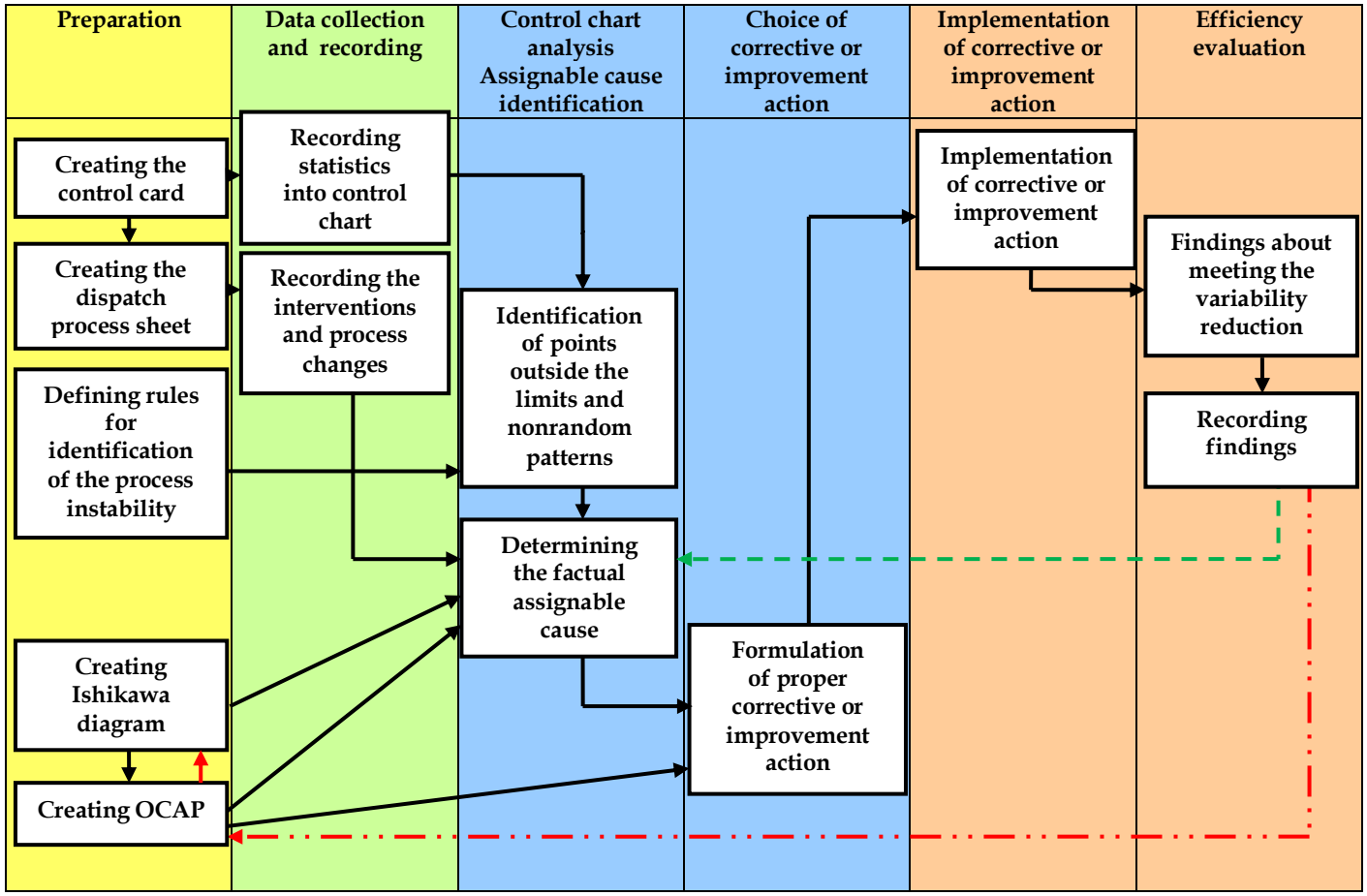


Fig. 9.4 Scheme of variant I

Via the information in OCAP and eventually via records of the efficiency of the accepted corrective or improvement actions, the most suitable corrective or improvement action which must be implemented as soon as possible is then selected.

Based on the subsequent data collection and evaluation of the process stability using a control chart it is necessary to judge the stabilization of the process and the reduction of the process variability. Results must be recorded. If it does not result in positive results, it is necessary to repeat the cause analysis and assignment of the corrective action or to make the new Ishikawa diagram and update OCAP.

Variant II

Second variant of the methodology for the realization of the problem-solving process in the frame of SPC (Fig. 9.5) has been designed for the processes with low dynamics but with a larger number of factors influencing the analyzed quality characteristics. It evokes additional activities within the preparatory phase. From all factors (causes) defined in the Ishikawa diagram the most probable causes must be selected by a score and then the most important causes must be chosen using for instance the Pareto analysis. Then some quantitative indicator should be assign to every important cause. The check sheet for recording the values of these indicators should then be designed in order to make it possible to assign these values to logical subgroups of the analyzed quality characteristic values. In OCAP corrective or improvement actions will be bound to these indicators. It means that both values of the controlled quality characteristic and values of indicators mentioned above must be collected and recorded. When the control chart signalizes non-stability, the analysis of these indicators must be carried out in order to be able to identify the particular cause of non-stability.

Next steps of the problem-solving process are the same as in variant I except for the situation when the SPC does not result in the variability reduction. Then the selection and analysis of the indicators could be repeated.

Variant III

The third variant of the methodology (Fig. 9.6) represents expansion of the second one to the processes with a larger number of factors influencing the controlled quality characteristic and also to the processes with large dynamics. Preparatory steps must be added with the analysis of time lag in the assignable causes treatment.

Some of the causes can actually affect the controlled quality characteristic with time lag and it can evocate problems with assigning the indicator values to the values of sample statistics computed from the output quality characteristic values.

The analysis of particular indicators in the frame of the assignable cause identification must be spread to (using regression and correlation analysis, DOE) the situation when there exists an important correlation between indicators and their connection could have a significant influence on the quality characteristics values.

The forth variant represents combination of variants I and III.

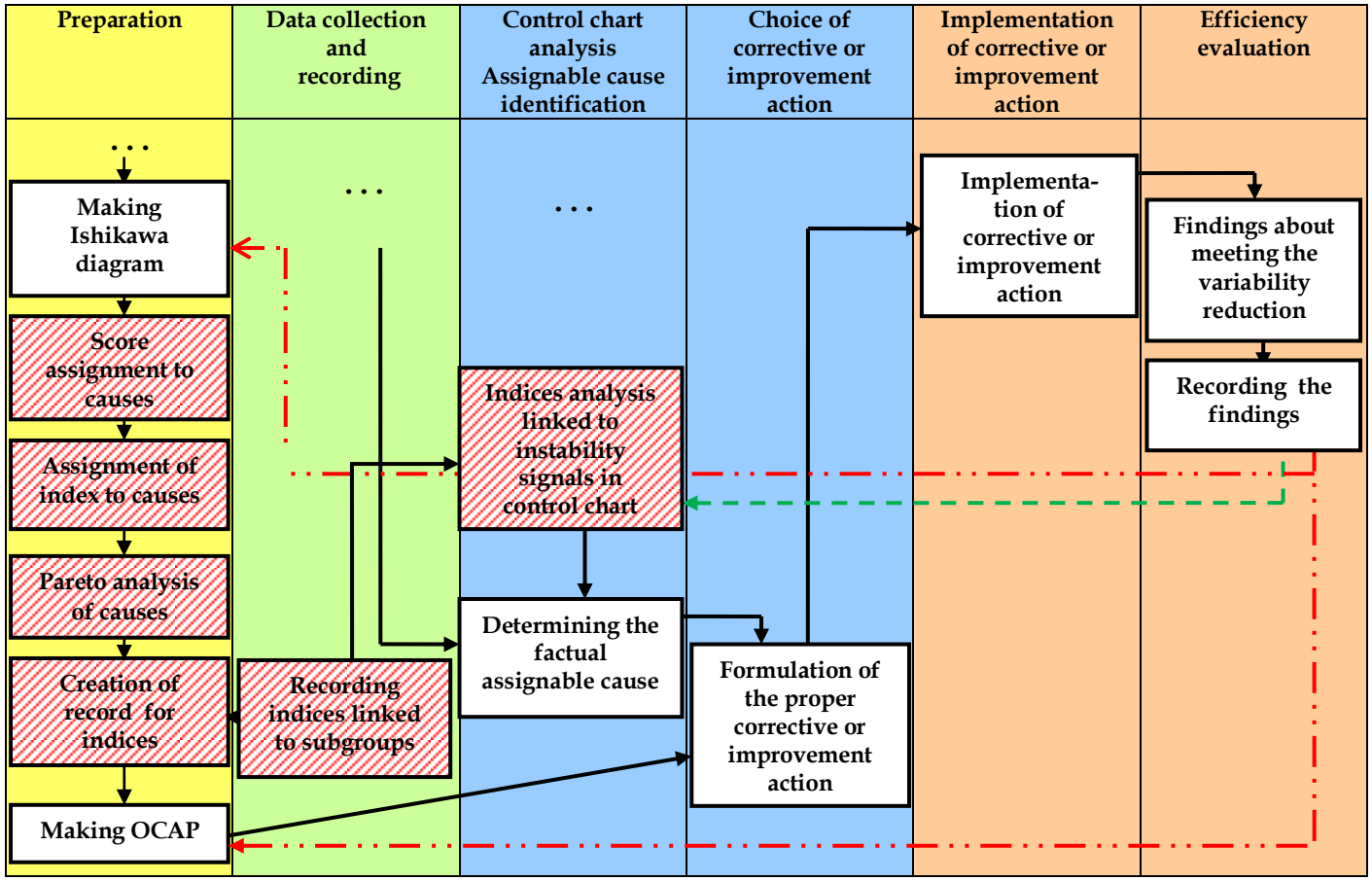


Fig. 9.5 Variant II - new activities (red blocks) and their relations to the selected activities of variant I (white blocks)

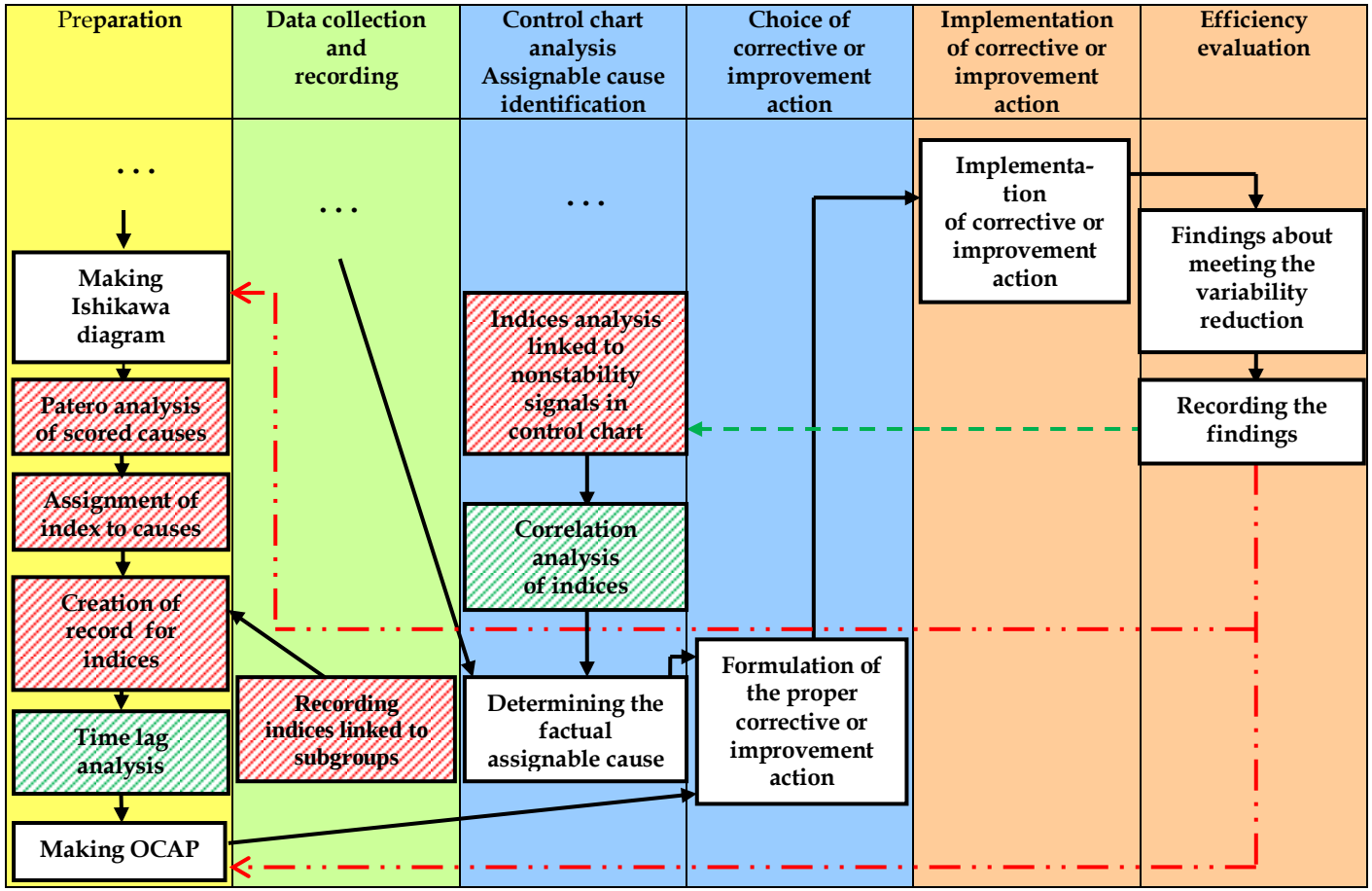


Fig. 9.6 Variant III - new activities (green blocks) and their relations to some activities of variant I (white blocks) and variant II (red blocks)

Summary of terms

After studying this chapter you should understand these terms:

- **problem-solving process**
- **SPC as a problem-solving process**
- **dispatch process sheet**
- **Pareto analysis**
- **decision-making process**
- **Ishikawa diagram**

Questions

1. **What is the effective SPC implementation?**
2. **What is the complex SPC application?**
3. **What is the SPC cycle?**
4. **What types of factors of effective SPC implementation do you know?**
5. **List some of these factors according to the type.**
6. **Describe SPC as a problem -solving process.**
7. **What are the differences among the variants of the methodology for realization SPC as the problem-solving process defined in this chapter?**
8. **Which methods and tools are suitable for different variants of the methodology of realization SPC as the problem-solving process?**

Additional study resources

[1] DASGUPTA, T. Maximizing the effectiveness of control charts: A framework for reacting to out-of-control signals, *Proceedings of Annual Quality Congress*, Vol. 57, No. 0, pp. 327-337, Kansas City, MI, June, 2003, ASQ, Milwaukee.

[2] HOU, S. – TONG, S. Fuzzy Logic Based Assignable Causes Ranking System for Control Chart Abnormality Diagnosis. *Proceedings of IEEE 2008 (FUZZ 2008)*, pp. 49 – 53, ISBN 978-1-4244-1818-3, Hong Kong, June 2008, IEEE, Los Alamitos.

10 CASE STUDY – STATISTICAL DATA ANALYSIS AS A BASE FOR THE CONTROL CHART SELECTION

Chapter structure

Topics for review

Time for learning

Goal

Case study

- **Characterization of the case study**
- **Description of the data analysis**
- **Results and discussion**

Additional study resources

Questions

Topics for review:

- **tests for normality, data independence, homogeneity, constancy of mean and dispersion (see [1]);**
- **unconventional SPC (see chapter 7 of this textbook).**

Time for learning: 120 min

Goal: After studying this chapter

- **you will understand inevitability to base the selection of a control chart on a complex statistical analysis;**
- **you will strengthen your knowledge of the data prerequisites for the control chart selection;**
- **you will learn some graphical and numerical methods for verifying these prerequisities;**
- **you will understand that graphical and numerical methods shall be applied together to provide as complex picture of the reality as possible.**

Case study

Characterization of the case study

Continuous process improvement, people involvement in it and prevention are fundamental principles of the TQM philosophy. Their realization needs the application of statistical thinking [2]. Acceptance and absorbing of statistical thinking is the main precondition for successful application of the statistical methods. Case study shows applying the complex data analysis under conditions of a heat supply station. The main task in this study was to find a suitable effective control chart.

The process of heat production belongs to the continuous type of processes where behaviour of quality characteristics often declines from prerequisites for the application of conventional Shewhart control charts. For that reason it is necessary to pay a big attention to the data analysis leading to selection of a suitable effective control chart.

Description of the data analysis

The main prerequisites that must be verified in connection with the control chart selection are as follows: normality, data independence, homogeneity (see chapter 2). In case that these prerequisites do not comply, then conventional Shewhart control charts can fail.

Former results were applied on the complex data analysis in the heat production [1]. In the process of production of energy the mentioned prerequisites are not often met. The day average of percentage of combustible elements in fly ash is the analyzed quality characteristic. The complex statistical analysis of the process data was carried out using Czech statistical software product QC-Expert.

At the beginning graphical methods were used (Fig. 10.1; Fig. 10.2; Fig. 10.3).

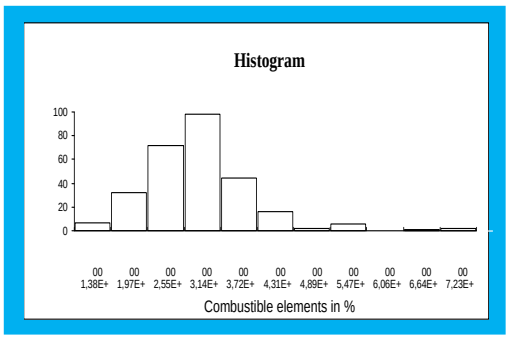


Fig.10.1 Histogram

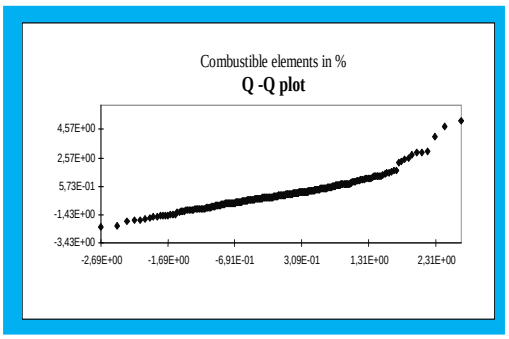


Fig. 10.2 Q-Q plot

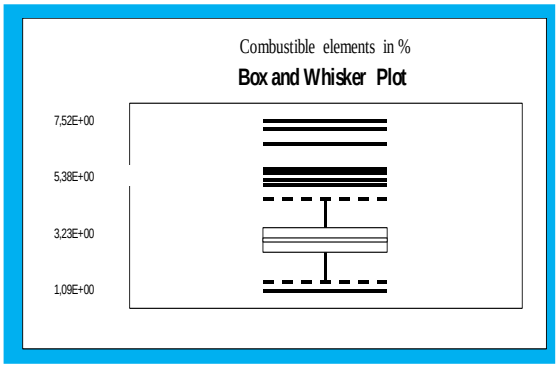


Fig.10.3 Box and Whisker Plot

Then the team of workers and engineers involved in the analyzed process has created the Ishikawa diagram of causes of the outliers [1]. After the solution of the outliers the data normality and homogeneity were verified once more using the same graphical methods and statistical hypothesis testing, in addition. Then data randomness has been verified using statistical hypothesis testing. Based on the results of graphical and numerical analysis of these data the control chart was selected, constructed and interpreted.

Results and discussion

When analyzing the histogram (Fig. 10.1) it can be seen that normality is met but the results are influenced by the presence of outliers. This conclusion has been confirmed by the Q-Q plot (Fig. 10.2) and by Box and Whisker plot (Fig. 10.3).

The origin of the outliers must be analyzed before the decision about their elimination. For that reason team of workers and engineers involved in the analyzed process has created the Ishikawa diagram of causes of the outliers [1] and after a deep analysis the team decided that outliers are possible to be eliminated.

After the elimination of the outliers the data normality and homogeneity were verified once more using the same graphical methods and statistical hypothesis testing, in addition. Then the data randomness has been verified using statistical hypothesis testing. Results of the tests are to be seen in table 10.1.

Tab. 10.1 Results of normality and homogeneity testing [1]

Type of test	Value of test criterion T	Critical value K	Critical signifi-cance level	Acceptance of H_0
Test of standard kurtosis and skewness	3.00035	5.9915	0.2231	Normality is accepted.
Test of trend significance	3.6747	1.9688		Data are not independent.
Test of autocorrelation coefficient significance	0.4245	0.1189	1.27E-13	Data are not independent.

The table shows that the normality can be accepted after the elimination of outliers. Nevertheless, the tests of data randomness did not confirm the data independence.

Based on the previous data analysis the dynamic EWMA control chart [3] was selected (see Fig. 10.4) which is less sensitive to little movements inherent to the process.

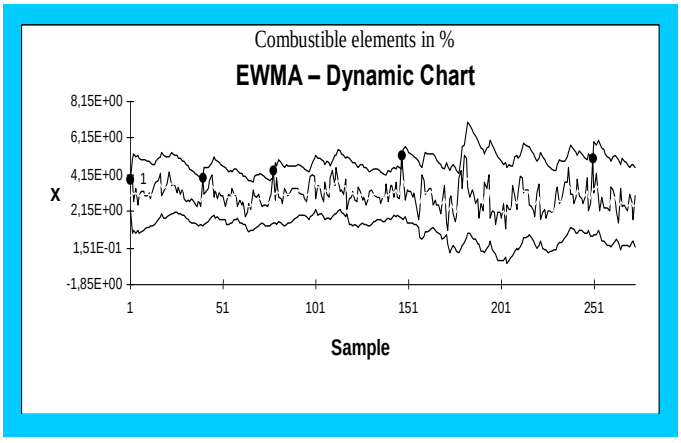


Fig.10.4 Dynamic EWMA Chart

Effective complex data analysis needs a deep analysis of meeting the prerequisites for the right selection, application and results interpretation of the selected statistical method. In our case study the automatic application of a standard control chart could lead to wrong conclusions

about the process. Good results can offer a rarely applied dynamic EWMA control chart in practice.

The case study has shown the way for how to correctly apply statistical methods including statistical process control in non-standard conditions which the production of heat energy represents.

Questions

1. **Why is it inevitable to base the control chart selection on a deep data analysis?**
2. **What does it mean that the control chart fails?**
3. **Is it enough to use graphical methods only? If not, explain it.**
4. **Is it enough to use numerical methods only (testing statistical hypothesis for instance)? If not, expain why.**
5. **Which graphical methods and tests of statistical hypothesis do you know for verifying the data normality?**
6. **Which graphical methods and tests of statistical hypothesis do you know for verifying the data independence?**
7. **Which graphical methods and tests of statistical hypothesis do you know for verifying the data homogeneity?**
8. **How to verify that the selected dynamic EWMA chart was really the best solution (see chapter 12)?**

Additional study resources

[1] BSUMKOVÁ, S. *Implementation of Statistical Methods in MST, Corp. Ostrava (Diploma work)*, VSB-Technical University, Ostrava, 1998 (in Czech).

[2] HARE, L.B.; HOERT, R.W.; HROML, J. D.& SNEE, R. D. The Role of Statistical Thinking in Management. *Quality Progress*, February, 1995, pp. 53-60.

[3] MONTGOMERY, D.C.: *Introduction to Statistical Quality Control*. Wiley, New York, 2009, pp. 151-205.

[4] WOODWARD, P. Accommodation of skewed distributions in SPC. *Quality World*, April, 1997, pp. 308-314.

11 CASE STUDY – APPLICATION OF SPC IN CONDITION OF AUTOCORRELATED DATA

Chapter structure

Topics for review

Time for learning

Goal

Case study

- **Characterization of the case study**
- **Description of the data analysis**
- **Results and discussion**

Additional study resources

Questions

Topics for review:

- **tests for normality, data independence, homogeneity, constancy of mean and dispersion (see [1], chapter 2 of this textbook);**
- **unconventional SPC (see chapter 7 of this textbook);**
- **Box-Jenkins methodology of the time-series analysis [2].**

Time for learning: 120 min

Goal: After studying this chapter

- **you will be able to verify the basic data prerequisites using graphical and numetical methods;**
- **you will know several ways of how to solve the data autocorrelation when applying SPC;**
- **you will become aware of the benefits and weaknesses of these different ways;**
- **you will be able to select the best SPC method for the**

autocorrelated data according to the rate of elimination of the autocorrelation from the data.

Case study

Characterization of the case study

Choice of the proper control chart is one of the most important statistical and methodical factors that really have a great influence on the effectiveness of the SPC implementation. Many metallurgical processes are continuous ones where the problem of data autocorrelation often objectively occurs. In such conditions conventional Shewhart control charts fail. Special approach suitable for the autocorrelated data in current situation must be selected and applied ([1], chapter 7 of this textbook). Solution of this problem is illustrated by the SPC application on the continual casting of aluminium producing aluminium foils. Basic characterization of the analyzed process is as follows:

- *Monitored quality characteristic*: temperature of Al melt in the ladle;
- *Temperature specification:* 680-695 °C;
- *Results of specifications infringement:* temperature > 695 °C results in undesirable changes in mechanical properties of material (strength, ductility), temperature < 680°C results in occurrence of oxides (nonhomogenous material);
- *Main causes of temperature variability:* setting the speed of rolls, infringement of technological regulations, temperature in the gas furnace during pouring, the height of the melt in casting trough;
- *The gauge:* float thermometer;
- *Control interval*: 30 seconds;
- *Subgroup size*: $n = 1$;
- *Size of data set*: 600 values.

Description of the data analysis

After the initial statistical analysis when it was proved that the data were strongly positively autocorrelated (see Fig. 11.1) it was decided to apply the following methods of SPC with the goal to remove autocorrelation from the original data and obtain normally distributed and noncorrelated data [1]:

1. Method of control interval enlargement;

2. Approximate EWMA procedure;
3. ARIMA control chart.

Conventional Shewhart chart for individuals was then applied to the adjusted data.

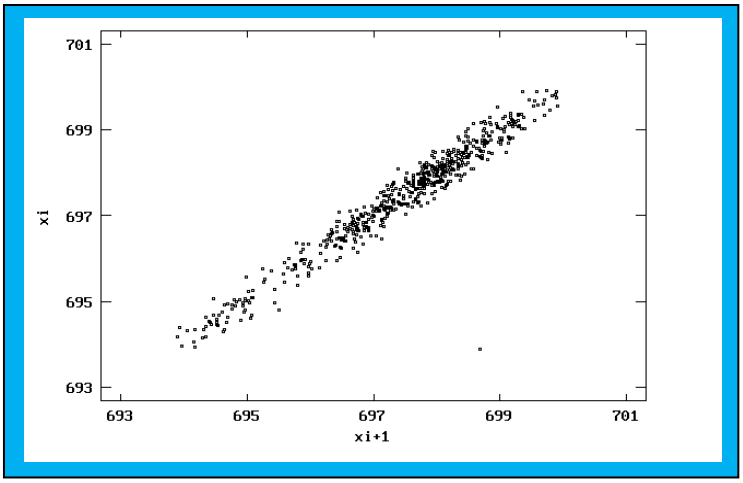


Fig. 11.1 Scatter plot

Results and discussion

Method of control interval enlargement

The initial control interval of 30 seconds was enlarged to 60 seconds, 5 minutes and 10 minutes. Using Autocorrelation function plot ACF, scatter plots and tests for randomness new sets of data were tested for normality and autocorrelation. As it can be seen in Tab. 11.1, autocorrelation was not removed (P-values < assigned value of α). Further enlargement of the control interval was not effective because of the large loss of information about the process.

Tab. 11.1 Results of verifying the data autocorrelation

Interval	Test	Significance level α	P-value	Test result
60 s	Runs above and below median	0.05	0	Data are autocorrelated.
	Box-Pierce	0.05	0	Data are autocorrelated.
5 min	Runs above and below median	0.05	0.00000355	Data are autocorrelated.
	Box-Pierce	0.05	0	Data are autocorrelated.
10 min	Runs above and below median	0.05	0.041	Data are autocorrelated.
	Box-Pierce	0.05	0.00004	Data are autocorrelated.

This simple procedure was not effective for this situation and therefore more complex methods were applied.

Approximate EWMA procedure

Using EWMA statistics for modelling the original data the residuals of this model were computed and verified from the point of view of normality and autocorrelation. Neither this procedure has eliminated the data autocorrelation sufficiently.

ARIMA control chart

First, graphs of the autocorrelation function ACF and partial autocorrelation function PACF were constructed. The graphs confirmed data autocorrelation and process nonstationarity. Such time series could be modelled using ARIMA models [1] and [2]. Before identifying and verifying suitable models the nonstationarity of time series was verified and refuted due to nonconstant variance. This time series was nonstationary due to the fact that only the mean had changed over time. It was solved using the first order differencing. Then three potentially suitable models were selected and verified: ARIMA(1,1,0), ARIMA(0,1,1), ARIMA(1,1,1). Acceptability of these models was verified through the diagnosis of their residuals. As it can be seen in Tab. 11.2, col. 1 and 3, normality and randomness of residuals of all models were not refuted (P-values > assigned value of α =0.05). However, on the basis of the test for parameter significance, the ARMA (1,1,1) model was excepted from the following analysis. Using criteria

AIC, BIC and SBC [2] (Tab. 2, col. 4 - 6), the ARIMA (1,1,0) model was selected as the most suitable model for this SPC application. In statistical program STATGRAPHICS Plus, version 5.0 model parameters using Marquardt nonlinear least squares method were estimated. Resulting format of the ARIMA model is as follows: $X_t = 0.09549 \, . \, X_{t-1} + a_t$.

Tab. 11.2 Tests for diagnosing the model residuals

	P-value			**Criteria for model selection**		
	1	**2**	**3**	**4**	**5**	**6**
Model	***Parameters significance test***	***Test for normality***	***Box-Pierce test***	***AIC***	***BIC***	***SBC***
ARIMA(1,1,0)	AR(1) 0.01931	0.7597	0.7588	-1695.4	-1697.9	-1686.6
ARIMA(0,1,1)	MA(1) 0.02556	0.7612	0.9099	-1695.01	-1697.6	-1686.2
ARIMA(1,1,1)	AR(1) 0.88482 MA(1) 0.93204	0.7605	0.7588			

The Shewhart control chart for individuals was applied (see Fig. 11.2) to the residuals of this model.

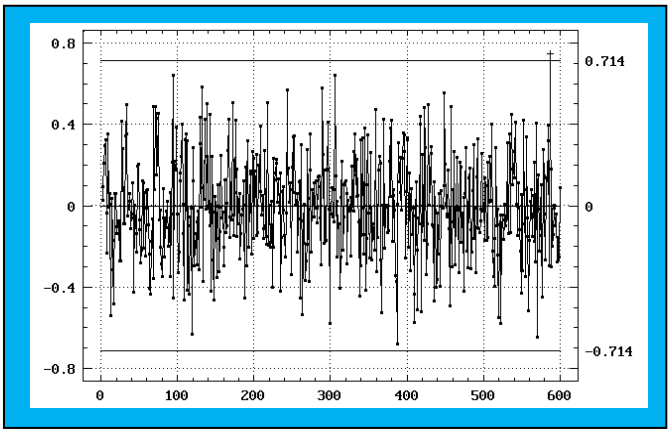


Fig. 11.2 ARIMA control chart

In this application three methods for autocorrelative data were applied. Simpler method of the control interval enlargement and the approximate EWMA procedure turned out to be suitable for the analyzed process. More complex procedure of the ARIMA modelling proved to be the best solution and the SPC application has been based on the suitable ARIMA model. In Fig. 11.2 we can see one point out of the upper control limit and the process can not be considered stable. Assignable cause of this instability must be identified and removed.

Questions

1. **How to verify the data independence?**
2. **Why is the data autocorrelation a problem when applying SPC?**
3. **Which special approaches suitable for autocorrelated data when applying SPC do you know?**
4. **Describe it and explain their differencies.**
5. **How are the criteria AIC, BIC and SBC applied to select the best time-series model?**
6. **Explain how to make sure that the convenient way to remove autocorrelation was selected.**

Additional study resources

[1] MONTGOMERY, D. C. *Introduction to Statistical Quality Control*. 6th ed. New York: Wiley, 2009.
[2] LIU, L. M. *Time series analysis and forecasting*. 2nd ed. Villa Park: Scientific Computing Associates, Corp. 2006.

12 CASE STUDY – COMPLEX AND EFFECTIVE APPLICATION OF SPC

Chapter structure

Topics for review

Time for learning

Goal

Case study

- **Characterization of the case study**
- **Description of the SPC implementation**
- **Results and discussion**

Additional study resources

Questions

Topics for review:

- **tests for normality, data independence, homogeneity, constancy of mean and dispersion (see [1]);**
- **complex and effective application of SPC (see chapter 9);**
- **SPC as a problem solving process (see chapter 9);**
- **phases of SPC (see charter 3).**

Time for learning: 120 min

Goal: After studying this chapter

- **you will understand what the complex and effective implementation of SPC is in practice;**
- **you will discover from the practical point of view what activities shall be carried out in the frame of every SPC phase;**
- **you will become aware of the systematic and repetitive nature of this four-phases cycle (outputs of the previous phase are inputs for the consecutive phase and the phases are repeated whenever it is needed);**

- **you will discover how to implement SPC as a real problem-solving process that runs through all phases;**
- **you will regard the measurement system analysis and capability analysis as integral parts of the SPC process;**
- **you will understand the SPC process as a system of various statistical, technical and economic analyses, different ways of searching for corrective actions and improvements and tools for verification of their effectiveness;**
- **you will accept that the correct SPC application can bring practical results in the form of:**
  - **measurement system improvements;**
  - **optimized algorithm of technological operations;**
  - **economic savings;**
  - **revelation of other improvement potentials.**

Case study

Characterization of the case study

This practical application shows a complex and effective implementation of SPC. ***Complex application*** of SPC is the application that is carried out considering the whole complex of technical, statistical, methodical, organizational and economic factors and that is implemented in the frame of the following four phases as a problem-solving process: preparatory phase, phase of verification and ensuring process statistical stability, phase of verification and ensuring process capability and phase of the ongoing statistical process control (more in chapters 3 and 9). ***Effective application*** of SPC is the complex application which results in the process improvement, i.e. in reduction of the process variability, and reveals the potential for further improvements (more in chapter 1). SPC must be built as a problem-solving process that runs through all four phases defined above (more in chapter 9). The methodology defined above was applied to the lengthwise tonsure rolled plates process on double-sided scissors with the goals to produce rolled plates with the allowance for the width as small as possible (minimization of the production cost) and to reduce the probability of the defective plate occurrence. Further on in

this chapter the description of practical application of SPC and its analysis is accomplished.

Description of the SPC implementation

In the following paragraphs each of the four phases of the complex and effective SPC are described.

I. Preparatory phase

During the preparatory phase the following activities were carried out:

- ***Process definition***: Lengthwise tonsure rolled plates process was selected due to its potential for improvements with indispensable economic benefits.
- ***Defining the controlled quality characteristic***: Allowance for the rolled plate width on the undersurface is crucial for meeting the plate width specifications. Thanks to the technology of lengthwise tonsure the undersurface width is less than the upper one.
- ***Specification of the key possible causes of the process variability***: Based on the knowledge of the process and statistical analysis of some factors the wearing and resetting of the scissors, the measurement system capability, the plates temperature before tonsuring and the way of the operator judgement of the plate width allowance were set as the main potential causes of the process variability.
- ***Realization of MSA***: Due to the revealed non-capability of the applied measurement system during the initial MSA the analysis had to be repeated to verify the effectiveness of accepted corrections and improvements of the measurement system.
- ***Control interval setting***: On the basis of knowledge of the process (slow change of assortment (rolling campaigns), slow wear of scissors, automatic resetting of scissors) 4-hour control interval was set as optimal.
- ***Subgroup size setting***: Due to the long control interval it was decided to make subgroups of 1 piece of the plate.
- ***Target value of the controlled quality characteristic***: On the basis of the knowledge of the process and economic consequences it was set to 3 mm. This value is a compromise between economically motivated production as close as possible to the lower specification and minimization of the probability of the defective plate occurrence.
- ***Verification of the data prerequisites:*** verification of the data preconditions (normality, homogeneity, independence) was carried out to be able to select the right control chart.
- ***Control charts selection***: Based on the subgroup size and verified data preconditions the charts for individuals and moving ranges were selected.

- ***Defining the rules for the process instability evaluation***: It was decided to apply two rules:
  - Any single point outside the control limit;
  - 6 points in a row trending up or down.

II. Phase of verification and ensuring the process statistical stability

Using the selected control charts for individuals and moving ranges and applying the selected instability tests the process statistical stability was verified. As no point was outside the control limits (LCL or UCL) and no trend was detected in these control charts the process could be considered statistically stable. Results of this phase are discussed in more detail in the paragraph "Results and discussion".

III. Phase of verification and ensuring the process capability

Capability indices C_P and C_{pk} were computed and compared with the minimal target value 1.33. Due to the value of C_{pk} the process could not be treated as capable. The analysis of the causes of this situation using Ishikawa diagram was then carried out and the key cause was determined. To remove this problem, the improvement action was designed and implemented. Verification of the process statistical stability and capability was then repeated. Results of this phase are discussed in more detail in the paragraph "Results and discussion".

IV. Phase of the ongoing SPC

As the process statistical stability and capability were ensured during the former phase, it was possible to incorporate the proposed SPC system into the standard control system of the production unit. The improvement remedy (the implemental excel sheet) has been currently applied by operators and statistical stability has been monitored using control charts. The new responsibilities and authorities were delegated to the quality managers and operators.

Results and discussion

The initial MSA showed that the measurement system was not acceptable (the complex index of measurement system capability % GRR is 36.1 % and another indicator ndc = 4).

The rules of thumb for accepting the measurement system are: % GRR $\leq$ 30 %, ndc $\geq$ 5. More detailed rules for % GRR used in the analysed application are in the following table.

Table 12.1 Rules for the measurement system capability evaluation

% GRR	Conclusions
< 10 %	Measurement system is acceptable.
10 % - 30 %	Measurement system may be acceptable based on the application (contingent upon its importance in application, cost of its replacement or its repair).
> 30 %	Measurement system needs improvement (sources of excess variation must be identified and system must be improved).

During the team-made analysis of the causes of the measurement system non-capability four root causes were set using the Ishikawa diagram and Pareto analysis:

- Low measurement scale of the tape line;
- Unsuitable location of the halogen lamp;
- Violation of rectangularity of the tape line location;
- High temperature of the measured plate.

As a result of the former analysis the following actions were set and carried out:

- Application of the new tape line with higher and more marked scale;
- Implementation of the instruction for operators to keep urgently the rectangularity of the tape line location;
- Change of the lighting location;
- Implementation of instruction to minimize time for apposition of the tape line and for making measurement.

MSA was repeated under new conditions and the measurement system improvement was detected (the complex index of the measurement system capability % GRR equals 22.8 %; ndc = 6). All possibilities to improve the measurement system even more and cost consequences of it were analysed. It was concluded that further improvements were not currently possible. After the cost analysis of the present measurement system replacement it was concluded that after improvements described above the existing measurement system could be considered acceptable and the following data analysis could be regarded as reliable enough.

After the process and data analysis the correctly selected control charts (Fig. 12.1 and 12.2) were applied to verify the process statistical stability.

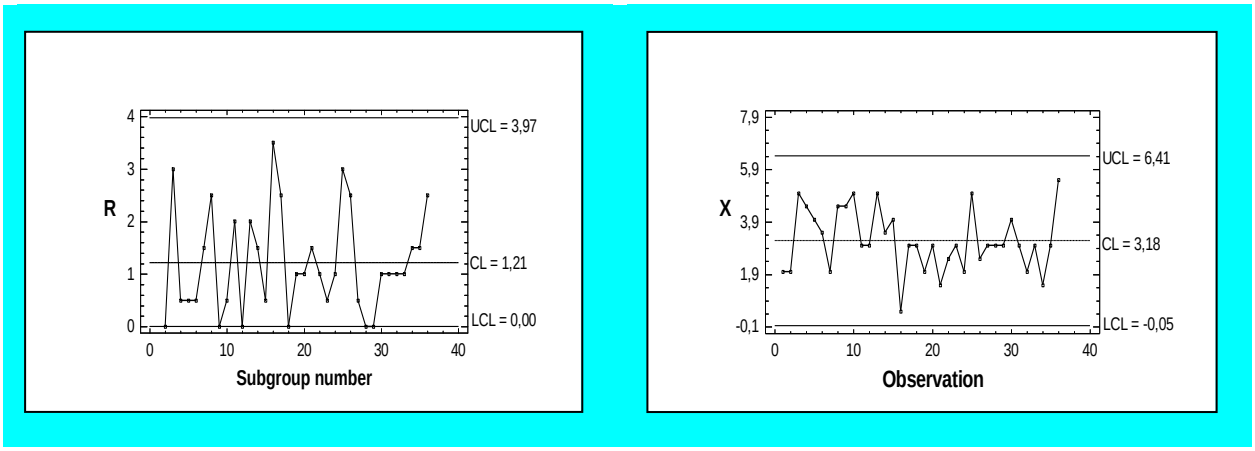


Fig. 12.1 Control chart for moving ranges

Fig. 12.2 Control chart for individuals

The rules for the process instability evaluation defined in the preparatory phase were applied. As no point has been outside the control limits (LCL or UCL) and no trend was identified, the process could be considered statistically stable.

The key condition for the next phase was confirmed and verification and ensuring the process capability could follow.

The capability indices C_p and C_{pk} were computed. C_p is 3.09 and C_{pk} equals 0.98 (1640 nonconforming products per one million products). As the target value for capability indices was set to 1.33, it means that the process could not be considered capable. The analysis of the causes of this situation using the Ishikawa diagram and statistical analysis was then carried out. High variability in the allowance for the width on the upper surface of the plate before cooling of the plate leading to the high variability in the actual width on the undersurface of cooled plates was identified as the key cause. The reason was that the values of the allowance for the width on the upper surface of the plate before cooling were based on operators' own subjective judgement. Every operator had to take into account the temperature, thickness and width of the plate and the previous measurements of the actual plate width over wide assortment of the plates. To remove this problem, an instrument for objectivity increase and standardization of the procedure for this judgment was proposed. The implemental sheet called "Monitoring of the width allowance" for more objective judgment of the plate width and for optimal setting of scissors was designed. After inserting the nominal value of thickness and width and actual temperature of the plate this excel application immediately offers information (to the operator) about the suitable width allowance to reach the optimal value of the width allowance on the undersurface (3 mm) after cooling to 20˚C**.** The design of the implemental sheet allows for two important factors that have a significant influence on the final plate width after cooling: material temperature expansivity

and dependability of the difference between the upper surface and undersurface width on the plate thickness. Mathematical description of this dependability is based on the regression analysis resulting in the following relation:

$$\Delta b = 0.359168 + 0.0865762 \,.\, c, \qquad (12.1)$$

where Δb - is difference of widths,

c - is thickness of plate.

This analysis confirmed the operators' experience that the algorithm of the present computer control system for setting the scissors gap had not been exact.

Verification of the correctness and efficiency of the proposed implemental sheet is based on the hypothesis that standard deviation of values of the allowance of the rolled plate width on the undersurface is less than before the sheet was applied and that the mean of these values is closer to the optimal value of 3 mm. Before confirming the hypothesis by the repeated capability analysis the new evaluation of the process stability using again the control charts for individuals and moving ranges was implemented (see Fig. 12.3 and 12.4). As the analysis of the control charts did not reveal any point outside the control limit nor any trends, the process - after applying the "Implemental sheet" - could be considered statistically stable.

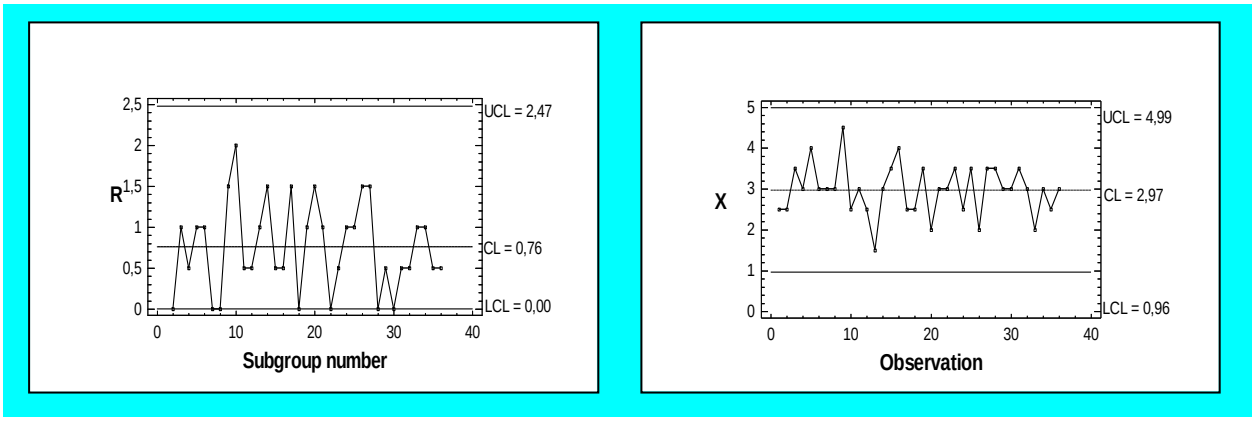


Fig. 12.3 Control chart for moving ranges after improvement

Fig. 12.4 Control chart for individuals after improvement

Then the capability indices were computed: C_p equals 4.96 and C_{pk} equals 1.47. Both indices are greater than 1.33. It means that the process could be considered capable. The increase of capability results from the reduction of standard deviation and from mean lying more closely to the optimal value. It is an evidence of the process improvement.

The economic effect of this improvement is equal to the cost savings in the amount of nearly 2 000 000 CZK / year. It results from the fact that referring to the smaller width allowance the

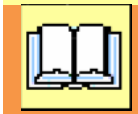
producer gives the consumer less material "gratis". This material stays then with him and could be used as a quality melting charge.

The proposed SPC system was incorporated into the standard control system of the scissors. The implemental excel sheet has been currently applied by operators to support their more precise assessment of the plate width; and statistical stability has been monitored using control charts. New responsibilities and authorities were delegated to quality managers and operators.

The analysis of the SPC application to the lengthwise tonsure rolled plates process showed that it had been implemented in a complex and effective way. It was carried out in all four phases considering many organizational, methodical, technical, statistical and economic factors. SPC was also carried out as a problem-solving process. Owing to cost savings this application can be viewed to be effective, too. During this SPC implementation some other potentials were revealed for future improvement and for further cost savings: change of the gauge for measuring the width of plates, incorporation of the algorithm contained in the implemental sheet "Monitoring of the width allowance" into the automatic control system of the scissors and automatic reading of the plate temperature.

Questions

1. **Can the above analysed SPC implementation be considered complex? If yes, explain it in more detail.**
2. **Can it be considered effective? If yes, explain it in more detail.**
3. **Which activities were carried out in the frame of every phase of SPC?**
4. **Analyse this SPC application from the point of the systematic and repetitive nature.**
5. **Analyse this SPC application from the point of view of the general structure of the problem-solving process.**
6. **Which other tools and methods besides the control charts were used during the analysed SPC implementation?**
7. **What are the real benefits from this SPC implementation?**



Additional study resources

[1] MONTGOMERY, D.C.: Introduction to Statistical Quality Control, Wiley, New York, 2009.